Container terminal management

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Outline

- Container terminals
- The Tactical Berth Allocation Problem
- Column generation & Branch-and-price
- Two-stage column generation
- Conclusions

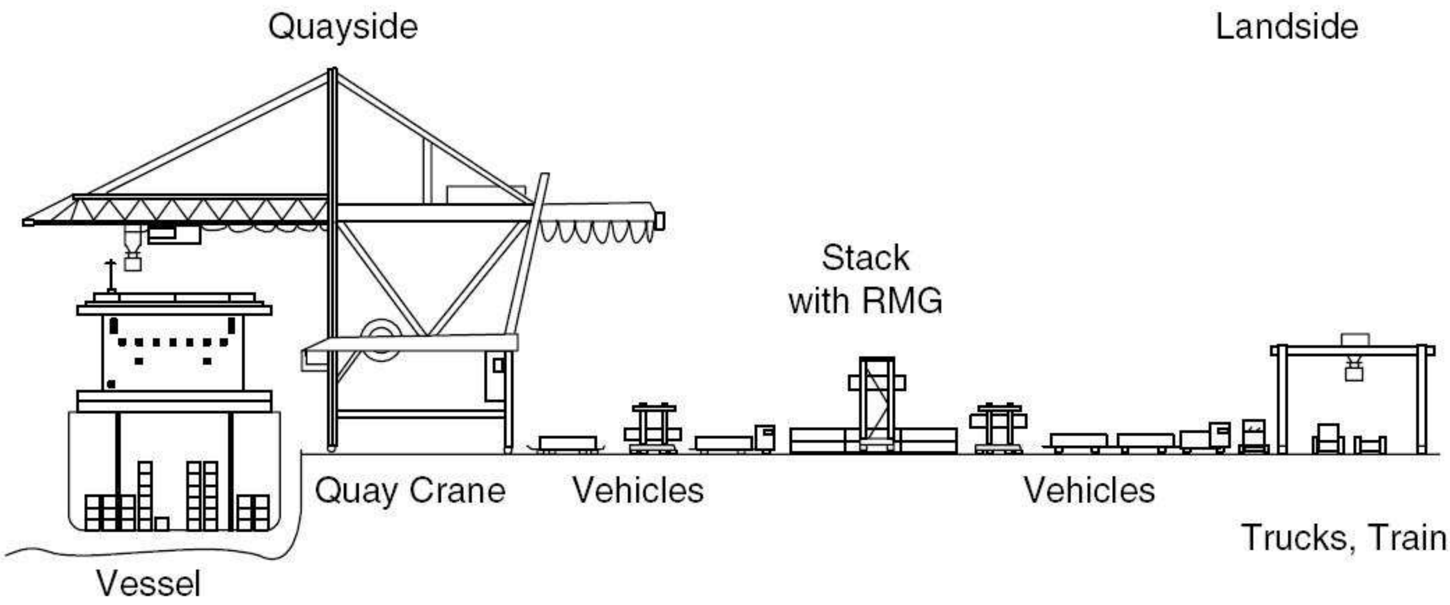
Motivation

Worldwide		2004	2006		2008	
1	Singapore	21’329	24’792	(+16%)	29’918	(+21%)
2	Shanghai	14’557	21’710	(+49%)	27’980	(+29%)
3	Hong Kong	21’984	23’539	(+07%)	24’248	(+03%)

Europe		2004	2006		2008	
1	Rotterdam	8’291	9’655	(+17%)	10’784	(+12%)
2	Hamburg	7’003	8’862	(+27%)	9’737	(+10%)
3	Antwerp	6’064	7’019	(+16%)	8’663	(+23%)

Table 1: Container traffic (in thousands TEUs).

Container terminals



Scheme of a container terminal system (Steenken et al., 2004).

Berth Allocation & Quay Crane Assignment

Berth Allocation Problem (BAP)

to assign and to schedule ships to berths over a time horizon, according to an *expected handling time*, time windows on the arrival time of ships and availability of berths.

Quay-Crane Assignment Problem (QCAP)

to assign quay cranes (QC) to ships scheduled by the given berth allocation plan, over a time horizon, taking into account the *QC capacity constraint* in terms of available quay cranes at the terminal.

Tactical Berth Allocation Problem (TBAP)

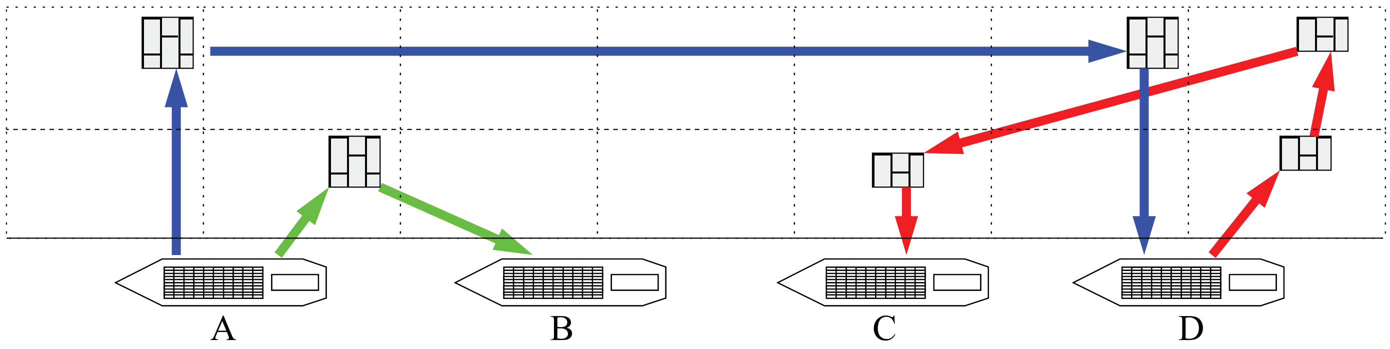
Integrated planning of BAP and QCAP

- *tactical decision level*: we analyze the problem from the terminal point of view, in order to provide decision support in the context of the negotiation between the terminal and shipping lines.
- *quay-crane profiles and handling time*: the handling time becomes a decision variable, dependent on the assigned quay crane profile (i.e. number of cranes per shift, ex. 332). Feasible profiles can vary in length (number of shifts dedicated to the ship) and in size (number of QCs dedicated to the ship in each active shift).

The concept of QC assignment profile

TIME	ws=1	ws=2	ws=3	ws=4	ws=5	ws=6	ws=7	ws=8
berth 1	ship 1	ship 1	ship 1		ship 2	ship 2	ship 2	ship 2
	3	2	2		4	4	5	5
berth 2		ship 3	ship 3			ship 4	ship 4	ship 4
		4	5			3	3	3
berth 3			ship 5	ship 5	ship 5	ship 5	ship 5	
			3	3	3	2	2	
QCs	3	6	10	3	7	9	10	8

Transshipment-related housekeeping yard costs



- Vessels A-B: no housekeeping, straddle carriers
- Vessels C-D: housekeeping, straddle carriers
- Vessels A-D: housekeeping, multi-trailers

Problem definition

Find

- a berth allocation;
- a schedule;
- a quay crane assignment;

Given

- time windows on availability of berths;
- time windows on arrival of ships;
- *handling times dependent on QC profiles*;
- values of QC profiles;

Objective

- maximize total value of QC assignment;
- minimize housekeeping costs of transshipment flows between ships.

Notation & data

N	set of **vessels**;
M	set of **berths**;
H	set of **time steps**;
P_i	set of **quay crane profiles** for the vessel $i \in N$;
t_i^p	**handling time** of ship $i \in N$ using QC profile $p \in P_i$;
v_i^p	monetary **value** associated to qc profile $p \in P_i,\ i \in N$;
q_i^{pu}	number of **quay cranes** used by profile $p \in P_i,\ i \in N$ at time position u;
Q^h	maximum number of quay cranes available at the time step $h \in H$;
f_{ij}	**flow of containers** exchanged between vessels $i, j \in N$;
g_{ij}	binary parameter equal to 1 if $f_{ij} > 0$ and 0 otherwise;
d_{kw}	**unit housekeeping cost** between yard slots corresponding to berths $k, w \in M$.

The Tactical Berth Allocation Problem (TBAP)

Decision variables

- berth assignment : $y_i^k \in \{0,1\}$;
- profiles' assignment : $\lambda_i^p \in \{0,1\}$;
- ship scheduling : $x_{ij}^k \in \{0,1\}\,,\ \ T_i^k \geq 0$.

Objective function maximize (qc profile value - housekeeping cost)

$$\max \sum_{i\in N}\sum_{p\in P_i} v_i^p \lambda_i^p - \frac{1}{2}\sum_{i\in N}\sum_{k\in M} y_i^k \sum_{j\in N}\sum_{w\in M} f_{ij} d_{kw} y_j^w$$

MIQP/MILP constraints

- ship covering constraints;
- arc-flow / precedence constraints;
- time windows;
- profile assignment;
- QC capacity constraint.

$$\max \sum_{i\in N}\sum_{p\in P_i} v_i^p \lambda_i^p - \frac{1}{2}\sum_{i\in N}\sum_{k\in M} y_i^k \sum_{j\in N}\sum_{w\in M} f_{ij} d_{kw} y_j^w \tag{1}$$

$$\text{s.t.} \quad \sum_{k\in M} y_i^k = 1 \qquad \forall i \in N, \tag{2}$$

$$\sum_{j\in N\cup\{d(k)\}} x_{o(k),j}^k = 1 \qquad \forall k \in M, \tag{3}$$

$$\sum_{i\in N\cup\{o(k)\}} x_{i,d(k)}^k = 1 \qquad \forall k \in M, \tag{4}$$

$$\sum_{j\in N\cup\{d(k)\}} x_{ij}^k - \sum_{j\in N\cup\{o(k)\}} x_{ji}^k = 0 \qquad \forall k \in M, \forall i \in N, \tag{5}$$

$$\sum_{j\in N\cup\{d(k)\}} x_{ij}^k = y_i^k \qquad \forall k \in M, \forall i \in N, \tag{6}$$

$$T_i^k + \sum_{p\in P_i} t_i^p \lambda_i^p - T_j^k \leq (1 - x_{ij}^k)M \qquad \forall k \in M, \forall i \in N, \forall j \in N \cup \{d(k)\}, \tag{7}$$

$$T_{o(k)}^k - T_j^k \leq (1 - x_{o(k),j}^k)M \qquad \forall k \in M, \forall j \in N, \tag{8}$$

$$a_i y_i^k \leq T_i^k \qquad \forall k \in M, \forall i \in N, \tag{9}$$

$$T_i^k \leq b_i y_i^k \qquad \forall k \in M, \forall i \in N, \tag{10}$$

$$a^k \leq T_{o(k)}^k \qquad \forall k \in M, \tag{11}$$

$$T_{d(k)}^k \leq b^k \qquad \forall k \in M, \tag{12}$$

$$\sum_{p\in P_i} \lambda_i^p = 1 \qquad \forall i \in N, \tag{13}$$

$$\sum_{h\in H^s} \gamma_i^h = \sum_{p\in P_i^s} \lambda_i^p \qquad \forall i \in N, \forall s \in S, \tag{14}$$

$$\sum_{k\in M} T_i^k - b^h \leq (1 - \gamma_i^h)M \qquad \forall h \in H, \forall i \in N, \tag{15}$$

$$a^h - \sum_{k\in M} T_i^k \leq (1 - \gamma_i^h)M \qquad \forall h \in H, \forall i \in N, \tag{16}$$

$$\rho_i^{ph} \geq \lambda_i^p + \gamma_i^h - 1 \qquad \forall h \in H, \forall i \in N, \forall p \in P_i, \tag{17}$$

$$\sum_{i\in N}\sum_{p\in P_i}\sum_{u=max\{h-t_i^p+1;1\}}^{h} \rho_i^{pu} q_i^{p(h-u+1)} \leq Q^h \qquad \forall h \in H^{\bar{s}}, \tag{18}$$

$$x_{ij}^k \in \{0,1\} \qquad \forall k \in M, \forall (i,j) \in A^k, \tag{19}$$

$$y_i^k \in \{0,1\} \qquad \forall k \in M, \forall i \in N, \tag{20}$$

$$\gamma_i^h \in \{0,1\} \qquad \forall h \in H, \forall i \in N, \tag{21}$$

$$\lambda_i^p \in \{0,1\} \qquad \forall p \in P_i, \forall i \in N, \tag{22}$$

$$\rho_i^{ph} \in \{0,1\} \qquad \forall p \in P_i, \forall h \in H, \forall i \in N, \tag{23}$$

$$T_i^k \geq 0 \qquad \forall k \in M, \forall i \in N \cup \{o(k), d(k)\}. \tag{24}$$

MIP formulation and heuristic algorithm

- MIQP and MILP formulations solved by CPLEX.
- Heuristic algorithm based on tabu search and mathematical programming.
- Real data provided by the port of Gioia Tauro, Italy.
- CPLEX fails, the problem is too complex.
- The heuristic is very fast and provides good solutions.

More details in (Giallombardo et al., 2010).

Column generation for TBAP

- We propose a Dantzig-Wolfe (DW) reformulation of the MILP by Giallombardo et al. (2010) and we solve it using column generation.
- A *column* represents the sequence of ships calling at a given berth.
- A quay crane profile is assigned to every ship in the sequence.
- The *master problem* selects sequences in order to provide a min-cost solution.
- Profitable columns are generated by the *pricing subproblem*.

Master problem

Additional notation

Ω^k	set of all feasible sequences for berth $k \in M$;
α_r^i	coefficient equal to 1 if ship i is operated in sequence r, 0 otherwise;
β_r^{ip}	coefficient equal to 1 if ship i is operated in sequence r with profile p, 0 otherwise;
q_r^h	number of quay cranes used by sequence r at time step h;
v_r	total value of sequence $r \in \Omega^k$ defined as $v_r = \sum_{i \in N} \sum_{p \in P_i} \beta_r^{ip} v_i^p$.

Decision variables

s_r	equal to 1 if sequence $r \in \Omega^k$ is chosen, 0 otherwise;
z_{ij}^{kw}	equal to 1 if ship $i \in N$ is assigned to berth $k \in M$ and ship $j \in N$ to berth $w \in M$, 0 otherwise.

Master problem

Objective function

$$\min \sum_{i \in N} \sum_{j \in N} \sum_{k \in M} \sum_{w \in M} f_{ij} d_{kw} z_{ij}^{kw} - \sum_{k \in M} \sum_{r \in \Omega^k} v_r s_r \tag{1}$$

Ship covering & berth assignment

$$\sum_{k \in M} \sum_{r \in \Omega^k} \alpha_r^i s_r = 1 \quad \forall i \in N, \tag{2}$$

$$\sum_{r \in \Omega^k} s_r \leq 1 \quad \forall k \in M, \tag{3}$$

Quay-cranes capacity

$$\sum_{k \in M} \sum_{r \in \Omega^k} q_r^h s_r \leq Q^h \quad \forall h \in H, \tag{4}$$

Master problem

Linearization constraints

$$\sum_{k\in M}\sum_{w\in M} z_{ij}^{kw} = g_{ij} \quad \forall i\in N, j\in N, \tag{5}$$

$$\sum_{r\in\Omega^k} a_r^i s_r - z_{ij}^{kw} \geq 0 \quad \forall i\in N, j\in N, k\in M, w\in M, \tag{6}$$

$$\sum_{r\in\Omega^w} a_r^j s_r - z_{ij}^{kw} \geq 0 \quad \forall i\in N, j\in N, k\in M, w\in M, \tag{7}$$

Variables' domain

$$z_{ij}^{kw} \geq 0 \quad \forall i\in N, j\in N, k\in M, w\in M, \tag{8}$$

$$s_r \geq 0 \quad \forall r\in\Omega^k, k\in M. \tag{9}$$

Pricing subproblem

Let $[\pi, \mu, \pi_0, \theta, \eta]$ be the dual vector associated to constraints (2), (3), (4), (6) and (7).

Reduced cost of sequence $r_k \in \Omega^k$

$$\tilde{c}_{r_k} = -v_{r_k} - \pi_0^k - \sum_{i \in N} \pi^i \alpha_r^i - \sum_{h \in H} \mu^h q_r^h - \sum_{i,j \in N} \sum_{w \in M} \theta_{ij}^{kw} a_r^i - \sum_{i,j \in N} \sum_{w \in M} \eta_{ij}^{kw} a_r^j$$

Multiple pricing

- at each iteration, we have $|M|$ subproblems, one for every berth;
- the subproblem identifies the column r_k^* with the minimum reduced cost.

Column generation

- if $\tilde{c}_{r_k^*} < 0$ for some k, we add column r_k^* and we iterate;
- otherwise the current master problem solution is proven to be optimal.

Pricing subproblem

- The pricing subproblem is an Elementary Shortest Path Problem with Resource Constraints (ESPP-RC).

- Generated sequences satisfy:
  - flow and precedence constraints (scheduling);
  - time windows constraints;
  - profile assignment constraints.

- The pricing problem is solved via dynamic programming.

Computational results

10 x 3	B&P		CPLEX (1h)		HEUR		
Instance	opt_sol	t(s)	best_sol	GAP	best_sol	GAP	t(s)
H1p10	790735	21	x	∞	786439	0.54%	7
H1p20	791011	25	x	∞	785460	0.70%	21
H1p30	791045	10	780722	1.30%	784658	0.81%	39
H2p10	733276	2	712669	2.81%	732101	0.16%	8
H2p20	735646	7	x	∞	729472	0.84%	20
H2p30	735682	9	723818	1.61%	727443	1.12%	33
L1p10	515902	7	515902	0.00%	513941	0.38%	7
L1p20	518049	5	515991	0.40%	513847	0.81%	18
L1p30	518084	27	513731	0.84%	509617	1.63%	37
L2p10	564831	9	564831	0.00%	560915	0.69%	8
L2p20	564867	7	561504	0.60%	559595	0.93%	18
L2p30	564903	8	559389	0.98%	556998	1.40%	36

Computational results

15 x 3	B&P (3h)		CPLEX (3h)		HEUR		
Instance	**opt_sol**	**t(s)**	**best_sol**	**GAP**	**best_sol**	**GAP**	**t(s)**
H1p10	1170783	3507	x	∞	1163063	2.26%	34
H2p10	1272247	3787	1250124	3.27%	1265782	2.06%	32
L1p10	1098411	1203	x	∞	x	∞	27
L2p10	890211	8975	x	∞	888112	1.31%	28

20 x 5	B&P (3h)		CPLEX (3h)		HEUR		
Instance	**best_sol**	**GAP**	**best_sol**	**GAP**	**best_sol**	**GAP**	**t(s)**
H1p104d	1293184	4.66%	x	∞	1305216	3.78%	65
H2p104d	1379208	4.49%	x	∞	x	∞	63
L1p104d	1224458	3.95%	x	∞	1230409	3.48%	66
L2p104d	1045778	3.61%	x	∞	1050171	3.20%	70

Pricing subproblem

Issues

- the underlying network has one node for every ship $i \in N$, for every quay crane profile $p \in P_i$ and for every time step $h \in H$;
- with a "standard" implementation, only small-size instances are solved in a reasonable time.

Accelerating strategies

- bi-directional dynamic programming;
- heuristic pricing, dual stabilization, primal heuristic;
- for every ship i, definition of the list of non-dominated (h, p) pairs;
- two-stage column generation.

Two-stage column generation

Context

- Dantzig-Wolfe reformulation of large-scale combinatorial problems;
- standard column generation techniques not efficient / successful;
- focus on problems where the large number of variables in the compact formulation directly affects the pricing problem;
- CG current issues: increasing size of problems and instability.

Objective

- to reduce the size of the problem to be solved while keeping optimality;

Main ingredients of the framework

- exploit the relationship between *compact* and *extensive* formulation;
- simultaneously generate “compact” and “extensive” columns.

Two-stage column generation

- we start with a subset of compact formulation variables *(partial compact formulation)*;
- the problem is reformulated via Dantzig-Wolfe and solved via standard column generation *(optimal partial master problem)*;
- at this point, profitable compact formulation variables are dynamically generated and added to the formulation *(optimal master problem)*.

Algorithm 1: Two-stage column generation

```
input  set X̄
repeat
    repeat
        CG1: generate extensive variables λ for partial master
        problem (PMP)
    until optimal partial master problem (PMP) ;
    CG2: generate compact variables x for partial compact
    formulation (PCF)
until optimal master problem (MP) ;
```

Two-stage column generation

Contribution of compact formulation variables

- we introduce the concept of *extensive reduced cost* that measures the contribution of compact formulation variables to the extensive formulation (master problem);
- we aim to add compact formulation variables that are profitable for the master problem, regardless of the optimal solution of the linear relaxation of the compact formulation;

Pricing problem with the integrality property

- extensive reduced cost can be computed using Walker's method (1969), that makes use of the optimal (linear) master and pricing solutions.

Pricing problem without the integrality property

- extensive reduced cost can be computed adapting Irnich's method (2010), that makes use of path reduced costs and dynamic programming.

Two-stage column generation for TBAP

- We refer to the MILP by Giallombardo et al. (2010) as *compact formulation*.
- The decision variables of the TBAP compact formulation are:

$y_i^k \in \{0, 1\}$	berth assignment;
$\lambda_i^p \in \{0, 1\}$	qc profile assignment;
$x_{ij}^k \in \{0, 1\}\,, \quad T_i^k \geq 0$	ship scheduling.

- We focus on variables λ_i^p since the number of profiles has impact on the size of the network in the pricing problem.

Basic idea

- start solving the problem only with a meaningful subset of qc profiles $\hat{P} \subset P$;
- *dynamically* add the profitable profiles $p \in P \setminus \hat{P}$ that are missing.

Two-stage column generation

Implementation and testing

- extensive reduced cost computation based on exact CG2 dynamic programming;
- we further propose a *relaxed* version of CG2 DP, in order to gain computational efficiency;
- different initialization strategies for the subset of compact formulation variables (*opt_basis* vs *opt_lp*);
- strategies for adding compact formulation columns; trade-off between:
  - limiting the number of CG2 iterations;
  - adding a few profitable columns per iteration;
- elimination of suboptimal variables;
- comparison with standard column generation in terms of number of generated columns and computational time.

Computational results

Rich VRP with $|N| = 50$ and compact-formulation variable similar to the TBAP profiles. Results for the root node.

		Stand.CG			Two-stage column generation					
					opt_master			opt_lp		
Class	**nr**	**sol**	**cols**	**t**	**sol**	**cols**	**t**	**sol**	**cols**	**t**
R_50_A_50	12	12	1789	37	12	721	12	12	1970	89
R_50_B_50	12	12	3435	647	12	1247	269	12	3099	1582
R_50_C_50	12	12	4801	1937	12	1738	2254	10	3917	6988
C_50_A_50	9	8	2910	501	8	1082	18	8	3991	102
C_50_B_50	9	8	5352	6200	8	1872	1614	6	5423	4467
C_50_C_50	9	5	6638	2335	5	2272	7450	1	7063	2479

Summing up

- The proposed methodology is applicable to column generation itself (and not necessarily to a branch-and-price algorithm).
- The size of the pricing underlying network is increased at every iteration, but (almost) never reaches the full size.
- It is worth for very complex problems where already the root node is difficult to solve.

Conclusion

Application

- Integrated planning of berth allocation and quay crane assignment
- Heuristic and exact solution algorithms for the specific problem

Methods

- Implementation of advanced column generation and branch-and-price codes
- Methodological contribution: two-stage column generation

Thanks for your attention!

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