The Tactical Berth Allocation Problem

Integrated optimization in container terminals

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Outline

- Container terminals
- Berth Allocation & Quay Crane Assignment
- TBAP models & algorithms
- Hierarchical vs integrated approach
- Conclusions
Container terminals
Container terminals
Berth Allocation & Quay Crane Assignment

Berth Allocation Problem (BAP)

to assign and to schedule ships to berths over a time horizon, according to an expected handling time, time windows on the arrival time of ships and availability of berths.

Quay-Crane Assignment Problem (QCAP)

to assign quay cranes (QC) to ships scheduled by the given berth allocation plan, over a time horizon, taking into account the QC capacity constraint in terms of available quay cranes at the terminal.
Tactical Berth Allocation Problem (TBAP)

Integration of BAP and QCAP

- *tactical decision level*: we analyze the problem from the terminal point of view, in order to provide decision support in the context of the negotiation between the terminal and shipping lines.

- *quay-crane profiles and handling time*: the handling time becomes a decision variable, dependent on the assigned quay crane profile (i.e. number of cranes per shift, ex. 332). Feasible profiles can vary in length (number of shifts dedicated to the ship) and in size (number of QCs dedicated to the ship in each active shift).

Housekeeping Yard Costs

- in the context of a *transshipment container terminal*, we take into account the cost generated by the exchange of containers between ships in terms of traveled distance quay-yard-quay.
The concept of QC assignment profile

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<td>3</td>
<td>7</td>
<td>9</td>
<td>10</td>
<td>8</td>
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Transshipment-related housekeeping yard costs

- Vessels A-B: no housekeeping, straddle carriers
- Vessels C-D: housekeeping, straddle carriers
- Vessels A-D: housekeeping, multi-trailers
TBAP model

Giallombardo, Moccia, Salani and Vacca (Transportation Research Part B, 2010).

Decision variables

- berth assignment : \(y^k_i \in \{0, 1\}\);
- profiles’ assignment : \(\lambda^p_i \in \{0, 1\}\);
- ship scheduling : \(x^k_{ij} \in \{0, 1\}, \ T^k_i \geq 0\).

Objective function : maximize total value of QC profile assignments & minimize the housekeeping yard cost of transshipment flows:

\[
\max \sum_{i \in N} \sum_{p \in P_i} v^p_i \lambda^p_i - \frac{1}{2} \sum_{i \in N} \sum_{k \in M} y^k_i \sum_{j \in N} \sum_{w \in M} f_{ij} d_{kw} y^w_j
\] (1)
TBAP model

Berth covering constraints

$$\sum_{k \in M} y_i^k = 1 \quad \forall i \in N,$$

(2)

Flow and linking constraints

$$\sum_{j \in N \cup \{d(k)\}} x^k_{o(k), j} = 1 \quad \forall k \in M,$$

(3)

$$\sum_{i \in N \cup \{o(k)\}} x^k_{i, d(k)} = 1 \quad \forall k \in M,$$

(4)

$$\sum_{j \in N \cup \{d(k)\}} x^k_{i,j} - \sum_{j \in N \cup \{o(k)\}} x^k_{j,i} = 0 \quad \forall k \in M, \forall i \in N,$$

(5)

$$\sum_{j \in N \cup \{d(k)\}} x^k_{i,j} = y_i^k \quad \forall k \in M, \forall i \in N,$$

(6)
TBAP model

Precedence constraints

\[ T_i^k + \sum_{p \in P_i} t_i^p \lambda_i^p - T_j^k \leq (1 - x_{ij}^k)M \quad \forall k \in M, \forall i \in N, \forall j \in N \cup d(k) \]  
\[ T_o^k - T_j^k \leq (1 - x_{o(k),j}^k)M \quad \forall k \in M, \forall j \in N, \]  

Ship and Berth time windows

\[ a_i y_i^k \leq T_i^k \quad \forall k \in M, \forall i \in N, \]  
\[ T_i^k \leq b_i y_i^k \quad \forall k \in M, \forall i \in N, \]  
\[ a^k \leq T_o^k \quad \forall k \in M, \]  
\[ T_d(k) \leq b^k \quad \forall k \in M, \]
TBAP model

Profile covering & linking constraints

\[\sum_{p \in P_i} \lambda_i^p = 1 \quad \forall i \in N,\]  \hspace{1cm} (13)

\[\sum_{h \in H^s} \gamma_i^h = \sum_{p \in \bar{P}_i^s} \lambda_i^p \quad \forall i \in N, \forall s \in S,\]  \hspace{1cm} (14)

\[\sum_{k \in M} T_i^k - b^h \leq (1 - \gamma_i^h)M \quad \forall h \in H, \forall i \in N,\]  \hspace{1cm} (15)

\[a^h - \sum_{k \in M} T_i^k \leq (1 - \gamma_i^h)M \quad \forall h \in H, \forall i \in N,\]  \hspace{1cm} (16)

\[\rho_i^{ph} \geq \lambda_i^p + \gamma_i^h - 1 \quad \forall h \in H, \forall i \in N, \forall p \in P_i,\]  \hspace{1cm} (17)

Quay crane and profile feasibility

\[\sum_{i \in N} \sum_{p \in P_i} \sum_{u = \max \{h - \tau_i^p + 1; 1\}}^{h} \rho_i^{pu} q_i^{p(h-u+1)} \leq Q^h \quad \forall h \in H^s\]  \hspace{1cm} (18)
Solving the TBAP

- Model implemented and validated using general-purpose solvers (CPLEX, GLPK).

- Test instances based on real data provided by MCT, Port of Gioia Tauro, Italy.

- Up to 30 ships over a time horizon of 1 week; up to 60 ships over a time horizon of 2 weeks. Up to 30 quay crane profiles per ship.

- Only small-size instances (10 ships) solved at optimality. Often, no feasible solution provided.

- Efficient heuristic for TBAP (based on tabu search and mathematical programming).
Nested tabu search for TBAP

Our heuristic algorithm for TBAP consists of 2 steps:

1. identify a QC profile assignment for the ships;
2. solve the resulting tactical berth allocation problem.

Algorithm 1: Nested tabu search

Initialization: Assign a QC profile to every ship.

repeat

1. solve BAP via tabu search;
2. update profiles using reduced cost arguments.

until stop criterion;

The BAP tabu search was adapted from Cordeau, Laporte, Legato and Moccia (2005).
## TBAP computational results

<table>
<thead>
<tr>
<th>Instance</th>
<th>10x3 CPLEX</th>
<th>10x3 HEUR</th>
<th>10x3 Time (sec)</th>
<th>20x5 CPLEX</th>
<th>20x5 HEUR</th>
<th>20x5 Time (sec)</th>
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### TBAP computational results

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<thead>
<tr>
<th>Instance</th>
<th>CPLEX</th>
<th>HEUR</th>
<th>Time (sec)</th>
<th>Instance</th>
<th>CPLEX</th>
<th>HEUR</th>
<th>Time (sec)</th>
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<td>340</td>
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<td>-</td>
<td>97.38</td>
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<td>L1_30</td>
<td>-</td>
<td>96.20</td>
<td>4862</td>
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</table>
A route represents the sequence of ships visiting a berth.

A quay crane profile is assigned to each ship belonging to the route.

Profitable routes are generated in the pricing subproblem.

The underlying network has one node for every ship, for every quay crane profile and for every time step.
Integrated vs hierarchical approach

The hierarchical approach consists of the following steps:

1. determine the expected handling time for every ship;

2. BAP: solve the classical berth allocation problem (no qc profile assignment, no capacity constraint);

3. QCAP: assign a qc profile to every ship, taking into account the capacity constraint and the provided bap schedule.
Integrated vs hierarchical approach

We consider 2 scenarios for the handling time:

- scenario A: longest feasible profile for every ship;
- scenario B: max-value profile for mother vessels, longest feasible profile for feeders.

Scenario A allows for comparison with TBAP, since all quay crane profiles are feasible for the QCAP.

Scenario B is more realistic, although it may lead to infeasibility of QCAP.
Integrated vs hierarchical approach

Scenario A, 10 ships, 3 berths, 1 week.

Improvement of the integrated approach w.r.t. hierarchical approach:

<table>
<thead>
<tr>
<th>instance (tbap gap)</th>
<th>objective</th>
<th>housekeeping cost</th>
<th>profiles’ value</th>
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</thead>
<tbody>
<tr>
<td>H1_10 (0.8%)</td>
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<td>2.8%</td>
<td>0.2%</td>
</tr>
<tr>
<td>H1_20 (1.7%)</td>
<td>-0.3%</td>
<td>0.0%</td>
<td>-0.3%</td>
</tr>
<tr>
<td>H1_30 (1.3%)</td>
<td>0.1%</td>
<td>0.4%</td>
<td>0.0%</td>
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<tr>
<td>H2_10 (0.0%)</td>
<td>0.7%</td>
<td>5.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td>H2_20 (0.0%)</td>
<td>0.9%</td>
<td>6.4%</td>
<td>0.0%</td>
</tr>
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</tr>
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<tr>
<td>L1_30 (0.0%)</td>
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<td>8.9%</td>
<td>0.0%</td>
</tr>
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</table>
BAP objective functions

We compare the following BAP models:

**Min-cost BAP**

\[
\min \sum_{i \in N} \sum_{k \in M} \sum_{j \in N} \sum_{w \in M} f_{ij} d_{kw} y_i^k y_j^w
\]

**Min-delay BAP**

\[
\min \max_{i \in N} (T_i - a_i)
\]

**Min-cost-bounded-delay BAP**

\[
\min \sum_{i \in N} \sum_{k \in M} \sum_{j \in N} \sum_{w \in M} f_{ij} d_{kw} y_i^k y_j^w
\]

s.t. \( T_i - a_i \leq (1 + \epsilon)T^* \) \( \forall i \in N \).
### Scenario A, 10 ships, 3 berths, 1 week

<table>
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<tr>
<th>instance</th>
<th>MIN-COST BAP</th>
<th>MIN-DELAY BAP</th>
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<tr>
<td></td>
<td>obj</td>
<td>gap</td>
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<tr>
<td>H1_10</td>
<td>232’794</td>
<td>8.93%</td>
</tr>
<tr>
<td>H2_10</td>
<td>145’770</td>
<td>5.13%</td>
</tr>
<tr>
<td>L1_10</td>
<td>162’061</td>
<td>6.10%</td>
</tr>
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</table>
### Scenario A, 10 ships, 3 berths, 1 week

<table>
<thead>
<tr>
<th>instance</th>
<th>MC-BD BAP ($\epsilon = 0.2$)</th>
<th>MC-BD BAP ($\epsilon = 0.5$)</th>
<th>MIN-COST</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>obj* t (sec)</td>
<td>obj* t (sec)</td>
<td>obj</td>
</tr>
<tr>
<td>H1_10</td>
<td>252'324 (+8%) 1471</td>
<td>239'808 (+3%) 1401</td>
<td>232'794</td>
</tr>
<tr>
<td>H2_10</td>
<td>165'300 (+13%) 75</td>
<td>159'672 (+9%) 167</td>
<td>145'770</td>
</tr>
<tr>
<td>L1_10</td>
<td>178'105 (+10%) 90</td>
<td>178'105 (+10%) 92</td>
<td>162'061</td>
</tr>
</tbody>
</table>
Conclusions

- Integration of BAP and QCAP
- Model and algorithms for TBAP
- Hierarchical vs integrated approach
- Next step: exact method (improve solution and/or bounds)
Thanks for your attention!
References

