The Tactical Berth Allocation Problem with QC Assignment and Transshipment Costs

Models and Heuristics

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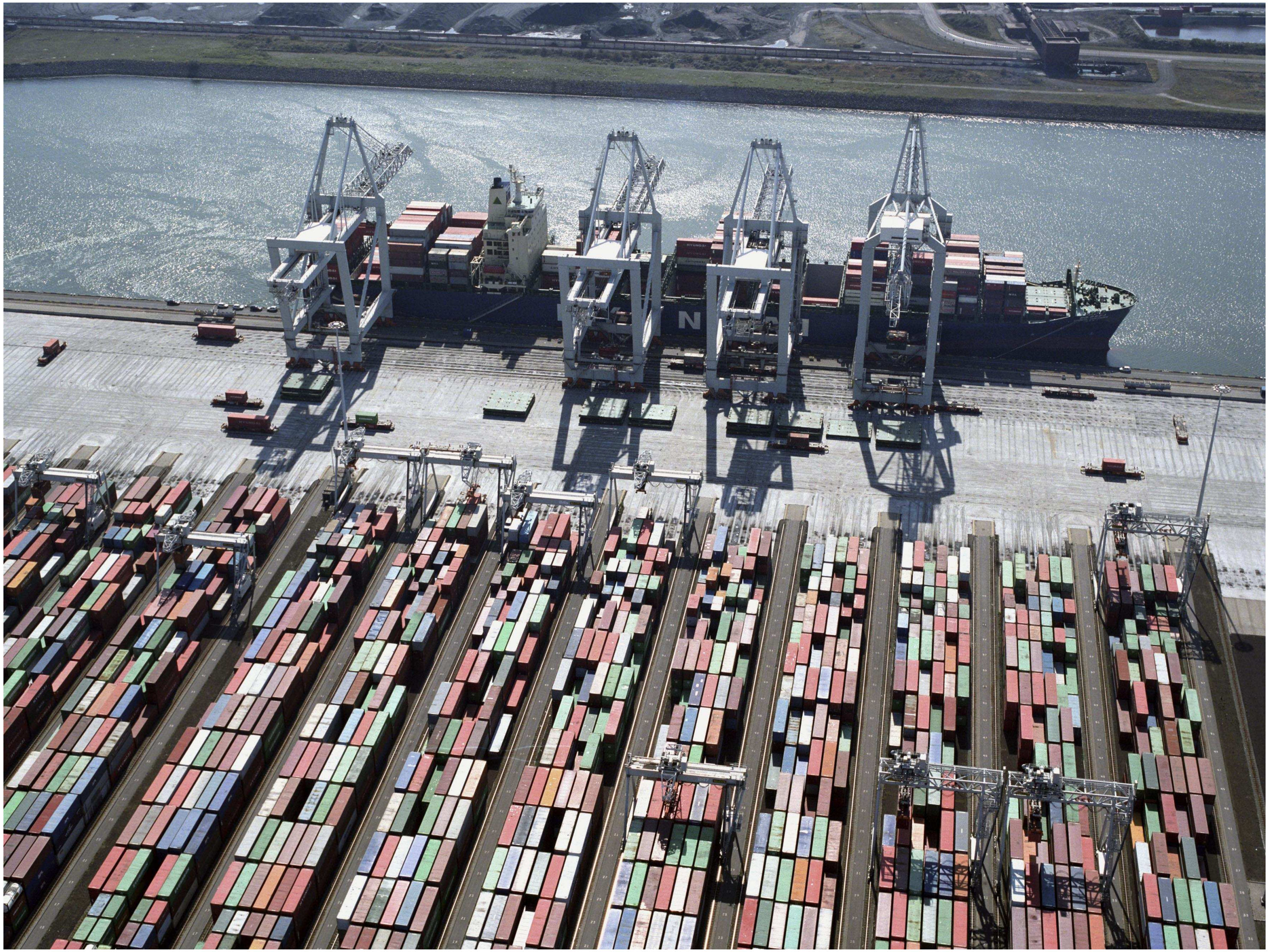
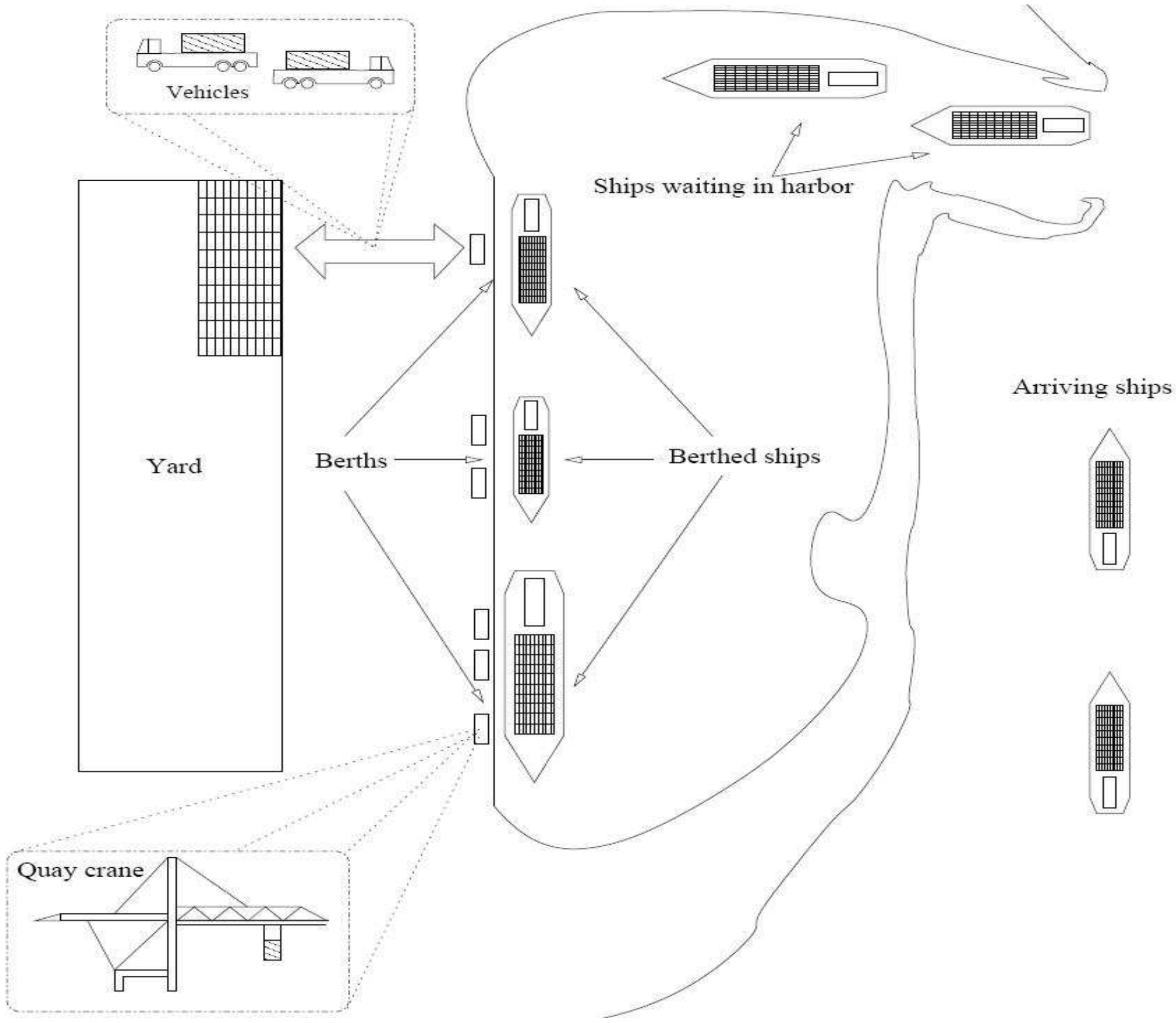
Outline

- Container terminals
- Tactical Berth Allocation Problem (TBAP) with Quay Crane Assignment
- MILP and MIQP models
- Heuristics for TBAP: Tabu Search & Math Programming
- Computational results
- Conclusions

Context: container terminals



Container terminal operations



Tactical Berth Allocation with QCs Assignment

Giallombardo, Moccia, Salani and Vacca (2009)

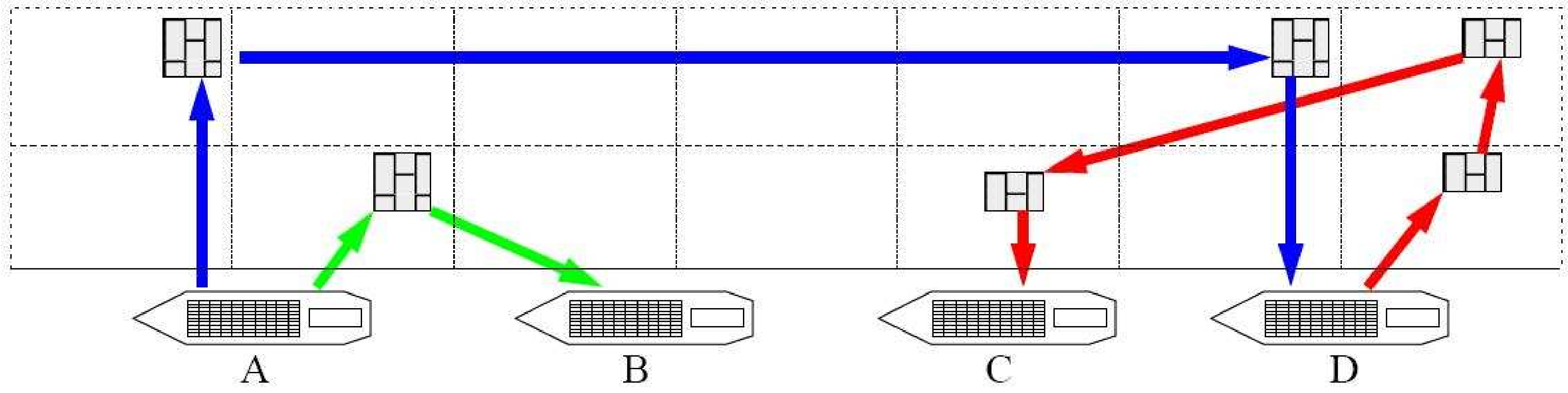
Problem description

- *Tactical Berth Allocation Problem* (TBAP): assignment and scheduling of ships to berths, according to time windows for both berths and ships; tactical decision level, w.r.t. negotiation between terminal and shipping lines;
- *Quay-Cranes Assignment Problem* (QCAP): a quay crane (QC) profile (number of cranes per shift, ex. 332) is assigned to each ship;
- *Housekeeping Quadratic Yard Costs*: take into account the exchange of containers between ships, in the context of transshipment container terminals.

The concept of QC assignment profile

<table>
<tr><th>TIME</th><th>ws=1</th><th>ws=2</th><th>ws=3</th><th>ws=4</th><th>ws=5</th><th>ws=6</th><th>ws=7</th><th>ws=8</th></tr>
<tr><td rowspan="2">berth 1</td><td colspan="3">ship 1</td><td rowspan="2"></td><td colspan="4">ship 2</td></tr>
<tr><td>3</td><td>2</td><td>2</td><td>4</td><td>4</td><td>5</td><td>5</td></tr>
<tr><td rowspan="2">berth 2</td><td rowspan="2"></td><td colspan="2">ship 3</td><td rowspan="2"></td><td rowspan="2"></td><td colspan="3">ship 4</td></tr>
<tr><td>4</td><td>5</td><td>3</td><td>3</td><td>3</td></tr>
<tr><td rowspan="2">berth 3</td><td rowspan="2"></td><td rowspan="2"></td><td colspan="5">ship 5</td><td rowspan="2"></td></tr>
<tr><td>3</td><td>3</td><td>3</td><td>2</td><td>2</td></tr>
<tr><td>QCs</td><td>3</td><td>6</td><td>10</td><td>3</td><td>7</td><td>9</td><td>10</td><td>8</td></tr>
</table>

Transshipment-related housekeeping yard costs



- Vessels A-B: no housekeeping, straddle carriers
- Vessels C-D: housekeeping, straddle carriers
- Vessels A-D: housekeeping, multi-trailers

Tactical Berth Allocation with QCs Assignment

Issues

- the chosen profile determines the ship’s handling time and thus impacts on the scheduling;
- feasible profiles can vary in length (number of shifts dedicated to the ship) and in size (number of QCs dedicated to the ship in each active shift).

Tactical Berth Allocation with QCs Assignment

Find

- a berth allocation
- a schedule
- a quay crane assignment

Given

- time windows on availability of berths
- time windows on arrival of ships
- *handling times dependent on QC profiles*
- values of QC profiles

Aiming to

- maximize total value of QC assignment
- minimize housekeeping costs of transshipment flows between ships

TBAP with QCs assignment: the model

- N = set of **vessels**;
- M = set of **berths**;
- H = set of **time steps** (each time step $h \in H$ is submultiple of the work shift length);
- S = set of the time step indexes $\{1, ..., \bar{s}\}$ relative to a work shift; ($\bar{s}$ represents the number of time steps in a work shift);
- H^s = subset of H which contains all the time steps corresponding to the same time step $s \in S$ within a work shift;
- P_i^s = set of feasible QC assignment profiles for the vessel $i \in N$ when vessel arrives at a time step with index $s \in S$ within a work shift;
- P_i = set of **quay crane assignment profiles** for the vessel $i \in N$, where $P_i = \cup_{s \in S} P_i^s$;

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TBAP with QCs assignment: the model

- t_i^p = **handling time** of ship $i \in N$ under the QC profile $p \in P_i$ expressed as multiple of the time step length;
- v_i^p = the **value** of serving the ship $i \in N$ by the quay crane profile $p \in P_i$;
- q_i^{pu} = number of **quay cranes** assigned to the vessel $i \in N$ under the profile $p \in P_i$ at the time step $u \in (1, ..., t_i^p)$, where $u = 1$ corresponds to the ship arrival time;
- Q^h = maximum number of quay cranes available at the time step $h \in H$;
- f_{ij} = **flow of containers** exchanged between vessels $i, j \in N$;
- d_{kw} = **unit housekeeping cost** between yard slots corresponding to berths $k, w \in M$;
- $[a_i, b_i]$ = [earliest, latest] feasible arrival time of ship $i \in N$;
- $[a^k, b^k]$ = [start, end] of availability time of berth $k \in M$;
- $[a^h, b^h]$ = [start, end] of the time step $h \in H$.

TBAP with QCs assignment: the model

Consider a graph $G^k = (V^k,\ A^k)\ \ \forall k \in M$, where $V^k = N \cup \{o(k),\ d(k)\}$, with $o(k)$ and $d(k)$ additional vertices representing berth k, and $A^k \subseteq V^k \times V^k$.

Decision variables:

- $x_{ij}^k \in \{0,1\}\ \ \forall k \in M,\ \forall (i,j) \in A^k$, 1 if ship j is scheduled after ship i at berth k;
- $y_i^k \in \{0,1\}\ \ \forall k \in M,\ \forall i \in N$, set to 1 if ship i is assigned to berth k;
- $\lambda_i^p \in \{0,1\}\ \ \forall p \in P_i, \forall i \in N$, set to 1 if ship i is served by the profile p;
- $T_i^k \geq 0\ \ \forall k \in M,\ \forall i \in N$, representing the berthing time of ship i at the berth k i.e. the time when the ship moors.

Linking variables:

- $\gamma_i^h \in \{0,1\}\ \ \forall h \in H, \forall i \in N$, set to 1 if ship i arrives at time step h;
- $\rho_i^{ph} \in \{0,1\}\ \ \forall p \in P_i, \forall h \in H, \forall i \in N$, set to 1 if ship i is served by profile p and arrives at time step h;
- $T_{o(k)}^k, T_{d(k)}^k \geq 0\ \ \forall k \in M$, representing the starting and ending operation time of berth k respectively.

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TBAP with QCs assignment: the MIQP model

Objective function

Maximize total value of QC profile assignments + Minimize the (quadratic) housekeeping yard cost of transshipment flows between ships:

$$\max \sum_{i \in N} \sum_{p \in P_i} v_i^p \lambda_i^p - \frac{1}{2} \sum_{i \in N} \sum_{k \in M} y_i^k \sum_{j \in N} \sum_{w \in M} f_{ij} d_{kw} y_j^w \qquad (1)$$

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TBAP with QCs assignment: the MIQP model

Berth covering constraints

$$\sum_{k \in M} y_i^k = 1 \qquad \forall i \in N, \tag{2}$$

Flow and linking constraints

$$\sum_{j \in N \cup \{d(k)\}} x_{o(k),j}^k = 1 \qquad \forall k \in M, \tag{3}$$

$$\sum_{i \in N \cup \{o(k)\}} x_{i,d(k)}^k = 1 \qquad \forall k \in M, \tag{4}$$

$$\sum_{j \in N \cup \{d(k)\}} x_{ij}^k - \sum_{j \in N \cup \{o(k)\}} x_{ji}^k = 0 \qquad \forall k \in M, \forall i \in N, \tag{5}$$

$$\sum_{j \in N \cup \{d(k)\}} x_{ij}^k = y_i^k \qquad \forall k \in M, \forall i \in N, \tag{6}$$

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TBAP with QCs assignment: the MIQP model

Precedence constraints

$$T_i^k + \sum_{p \in P_i} t_i^p \lambda_i^p - T_j^k \leq (1 - x_{ij}^k) M \qquad \forall k \in M, \forall i \in N, \forall j \in N \cup d(k) \quad (7)$$

$$T_{o(k)}^k - T_j^k \leq (1 - x_{o(k),j}^k) M \qquad \forall k \in M, \forall j \in N, \quad (8)$$

Ship and Berth time windows

$$a_i y_i^k \leq T_i^k \qquad \forall k \in M, \forall i \in N, \quad (9)$$

$$T_i^k \leq b_i y_i^k \qquad \forall k \in M, \forall i \in N, \quad (10)$$

$$a^k \leq T_{o(k)}^k \qquad \forall k \in M, \quad (11)$$

$$T_{d(k)}^k \leq b^k \qquad \forall k \in M, \quad (12)$$

TBAP with QCs assignment: the MIQP model

Profile covering & linking constraints

$$\sum_{p \in P_i} \lambda_i^p = 1 \qquad \forall i \in N, \tag{13}$$

$$\sum_{h \in H^s} \gamma_i^h = \sum_{p \in P_i^s} \lambda_i^p \qquad \forall i \in N, \forall s \in S, \tag{14}$$

$$\sum_{k \in M} T_i^k - b^h \leq (1 - \gamma_i^h) M \qquad \forall h \in H, \ \forall i \in N, \tag{15}$$

$$a^h - \sum_{k \in M} T_i^k \leq (1 - \gamma_i^h) M \qquad \forall h \in H, \ \forall i \in N, \tag{16}$$

$$\rho_i^{ph} \geq \lambda_i^p + \gamma_i^h - 1 \qquad \forall h \in H, \ \forall i \in N, \ \forall p \in P_i, \tag{17}$$

Quay crane and profile feasibility

$$\sum_{i \in N} \sum_{p \in P_i} \sum_{u = max\{h - t_i^p + 1; 1\}}^{h} \rho_i^{pu} q_i^{p(h-u+1)} \leq Q^h \qquad \forall h \in H^{\bar{s}} \tag{18}$$

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TBAP with QCs assignment: the MILP model

Additional decision variable

$z_{ij}^{kw} \in \{0,1\}\ \forall i,j \in N,\ \forall k,w \in M$, set to 1 if $y_i^k = y_j^w = 1$ and 0 otherwise.

Linearized objective function

$$\max \quad \sum_{i\in N}\sum_{p\in P_i} v_i^p \lambda_i^p - \frac{1}{2}\sum_{i\in N}\sum_{j\in N}\sum_{k\in M}\sum_{w\in M} f_{ij} d_{kw} z_{ij}^{kw} \tag{19}$$

Additional constraints

$$\sum_{k\in K}\sum_{w\in K} z_{ij}^{kw} = g_{ij} \qquad \forall i,j \in N, \tag{20}$$

$$z_{ij}^{kw} \leq y_i^k \qquad \forall i,j \in N, \forall k,w \in M \tag{21}$$

$$z_{ij}^{kw} \leq y_j^w \qquad \forall i,j \in N, \forall k,w \in M \tag{22}$$

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Generation of test instances

- Based on real data provided by MCT, Port of Gioia Tauro, Italy:
  - container flows
  - housekeeping yard costs
  - vessel's arrival times

- Crane productivity of 24 containers per hours

- Set of feasible profiles synthetically generated, according to ranges given by practitioners:

Class	min QC	max QC	min HT	max HT	volume (min,max)
Mother	3	5	3	6	(1296, 4320)
Feeder	1	3	2	4	(288, 1728)

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Generation of test instances

- 6 classes of instances:
  - 10 ships and 3 berths, 1 week, 8 quay cranes;
  - 20 ships and 5 berths, 1 week, 13 quay cranes;
  - 30 ships and 5 berths, 1 week, 13 quay cranes;
  - 40 ships and 5 berths, 2 weeks, 13 quay cranes;
  - 50 ships and 8 berths, 2 weeks, 13 quay cranes;
  - 60 ships and 13 berths, 2 weeks, 13 quay cranes.

- 12 scenarios for each class, with high (H) and low (L) traffic volumes;

- each scenario is tested with a set of $\bar{p} = 10, 20, 30$ feasible profiles for each ship;

- CPLEX 10.2 solver for MILP and MIQP formulations.

CPLEX results

10x3			10x3		
Instance	MILP	MIQP	Instance	MILP	MIQP
H1_10	99.17	98.90	L1_10	97.68	100.00
H1_20	97.91	97.96	L1_20	100.00	99.76
H1_30	97.98	98.76	L1_30	98.64	99.99
H2_10	98.87	99.26	L2_10	98.82	99.63
H2_20	96.97	96.91	L2_20	99.42	99.06
H2_30	96.79	-	L2_30	99.08	100.00

20x5			40x5		
Instance	MILP	MIQP	Instance	MILP	MIQP
H1_10	94.33	-	L1_10	94.92	-
H1_20	93.74	-	L1_20	94.47	-
H2_10	93.52	96.66	L2_20	94.93	-
L2_10	93.87	96.74	L2_30	94.61	-

CPLEX results

- Time limits:
  - 1 hour for class 10x3;
  - 2 hours for classes 20x5 and 30x5;
  - 3 hours for classes 40x5, 50x8 and 60x13.

- The objective function value is scaled to 100 with respect to the upper bound:

$$\text{scaled obj} = \frac{\text{obj} * 100}{\text{UB}}$$

  A value of 100 means that the solution is certified to be optimal.

- No feasible solution was found for classes 30x5, 50x8 and 60x13;

- However, an upper bound is always provided (although very bad).

A New Heuristics for TBAP

- Our heuristic algorithm is organized in two stages:
  1. identify a QC profile assignment for the ships;
  2. solve the resulting berth allocation problem for the given QC assignment.
- The procedure is iterated over several sets of QC profiles;
- New profiles are chosen via reduced costs arguments (MILP formulation).

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A New Heuristics for TBAP

Algorithm 1: TBAP Bi-level Heuristics

Initialization : Assign a QC profile to each ship

repeat

1. solve BAP
2. update profiles

until *stop criterion* ;

TBAP Bi-level Heuristics:

1. BAP solution via Tabu Search
2. Profiles' updating via Math Programming

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1. Tabu Search for BAP

Adapted from Cordeau, Laporte, Legato and Moccia (2005).

- New objective function: minimization of yard-related transshipment quadratic costs
- New constraints: QCs availability
- Each solution $s \in S$ is represented by a set of m berth sequences such that every ship belongs to exactly one sequence.
- Penalized cost function:

$$f(s) = c(s) + \alpha_1 w_1(s) + \alpha_2 w_2(s) + \alpha_3 w_3(s)$$

  where $w_1(s)$ is the total violation of ships' TWs, $w_2(s)$ is the total violation of berths' TWs and $w_3(s)$ is the total violation of QCs availability.
- "Move": ship i is removed from sequence k and inserted in sequence $k' \neq k$. The new position in k' is such that $f(s)$ is minimized.
- Initial solution: randomly built assigning ships to berths and relaxing the QCs availability constraint.

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2. Profiles' Updating via Math Programming

Basic idea: use information of reduced costs to update the vector of assigned QC profiles in a "smart" way.

- Let $\bar{X} = [\bar{x}, \bar{y}, \bar{T}]$ be the BAP solution found by the Tabu Search for a given QC profile assignment $\bar{\lambda}$.
- We solve the linear relaxation of the TBAP MILP formulation, with the additional constraints:

$$\bar{X} - \epsilon \leq X \leq \bar{X} + \epsilon \quad (23)$$

$$\bar{\lambda} - \epsilon \leq \lambda \leq \bar{\lambda} + \epsilon \quad (24)$$

- As suggested by Desrosiers and Lübbecke (2005), the shadow prices of these constraints are the reduced costs of original variables X and λ.
- We identify the λ_i^p variable with the maximum reduced cost and we assign this new profile p to ship i.
- If all reduced costs are ≤ 0, we stop.

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Computational results

- The heuristic has been implemented in C++ using GLPK 4.31.
- Stopping criteria:
  - $n \times \bar{p}$ iterations;
  - time limit of 1 hour for classes 10x3, 20x5 and 30x5;
  - time limit of 2 hours for classes 40x5, 50x8 and 60x13.
- Results are compared to the best solution found by CPLEX for either the MILP or MIQP formulation.

Computational results

10x3				20x5			
Instance	CPLEX	HEUR	Time (sec)	Instance	CPLEX	HEUR	Time (sec)
H1_10	99.17	98.52	7	H1_10	-	97.26	81
H1_20	97.96	98.36	15	H1_20	94.33	97.19	172
H1_30	98.76	98.33	27	H1_30	93.74	97.37	259
H2_10	99.26	98.92	7	H2_10	-	97.27	82
H2_20	96.97	98.48	16	H2_20	96.66	97.38	173
H2_30	96.79	98.17	28	H2_30	-	97.26	274
L1_10	100.00	99.12	6	L1_10	-	97.30	74
L1_20	100.00	99.01	15	L1_20	-	97.25	158
L1_30	99.99	98.29	26	L1_30	-	97.06	254
L2_10	99.63	98.92	6	L2_10	-	97.55	80
L2_20	99.42	98.68	15	L2_20	96.74	97.39	170
L2_30	100.00	98.22	27	L2_30	-	97.25	295

Computational results

30x5				40x5			
Instance	CPLEX	HEUR	Time (sec)	Instance	CPLEX	HEUR	Time (sec)
H1_10	-	95.67	340	H1_10	-	97.38	1104
H1_20	-	95.31	677	H1_20	-	97.38	2234
H1_30	-	95.54	1009	H1_30	-	97.25	3387
H2_10	-	95.88	316	H2_10	-	97.40	1095
H2_20	-	95.81	684	H2_20	-	97.33	2198
H2_30	-	95.30	969	H2_30	-	97.27	3296
L1_10	-	96.55	324	L1_10	94.92	97.41	1421
L1_20	-	96.43	652	L1_20	94.47	97.14	2996
L1_30	-	96.18	966	L1_30	-	96.20	4862
L2_10	-	95.68	308	L2_10	-	97.41	1382
L2_20	-	95.12	614	L2_20	94.93	97.34	3144
L2_30	-	-	920	L2_30	94.61	96.60	4352

Computational results

	50x8				60x13		
Instance	CPLEX	HEUR	Time (sec)	Instance	CPLEX	HEUR	Time (sec)
H1_10	-	96.52	3291	H1_10	-	95.40	6332
H1_20	-	96.37	6020	H1_20	-	95.07	10809
H1_30	-	96.21	9432	H1_30	-	94.76	10807
H2_10	-	96.03	3066	H2_10	-	95.54	6397
H2_20	-	95.64	6180	H2_20	-	94.11	10803
H2_30	-	95.16	9501	H2_30	-	-	10806
L1_10	-	95.97	2752	L1_10	-	95.67	5807
L1_20	-	96.04	6467	L1_20	-	95.40	10803
L1_30	-	95.80	9119	L1_30	-	94.45	10806
L2_10	-	96.18	3157	L2_10	-	95.63	5986
L2_20	-	95.96	5857	L2_20	-	95.64	10809
L2_30	-	96.27	8783	L2_30	-	95.34	10804

Conclusions

- The heuristics is able to find feasible solutions in 70 out of 72 instances, whereas CPLEX succeeds at that only on 20 instances, the smaller ones.
- Our algorithm is up to 2 order of magnitude faster than CPLEX, especially on small instances.
- The heuristics performs very well also on the instances of bigger size, where CPLEX generally fails.
- Next step: improve upper bounds using decomposition techniques.

Thanks for your attention!

Paper reference:

Giallombardo, G., Moccia, L., Salani, M., and Vacca, I. (2009).
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Transportation Research Part B: Methodological
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