The Tactical Berth Allocation Problem with Quay Crane Assignment and Transshipment-related Quadratic Yard Costs

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European Transport Conference (ETC)
October 6, 2008
Outline

• Introduction

• Container Terminal Operations

• Berth Allocation Problem: Tactical vs Operational

• Tactical Berth Allocation with Quay Crane Assignment: MIQP / MILP models

• Computational preliminary results

• Final remarks
Introduction

- Crucial role of *maritime transport* in the exchange of goods
- Growth of *container traffic* worldwide

<table>
<thead>
<tr>
<th>Worldwide</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Singapore</td>
<td>23,190,000</td>
<td>24,800,000 (+6.94%)</td>
<td>27,932,000 (+12.63%)</td>
</tr>
<tr>
<td>2 Shanghai</td>
<td>18,084,000</td>
<td>21,700,000 (+20.00%)</td>
<td>26,150,000 (+20.51%)</td>
</tr>
<tr>
<td>3 Hong Kong</td>
<td>22,602,000</td>
<td>23,230,000 (+2.78%)</td>
<td>23,881,000 (+2.80%)</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Europe</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Rotterdam</td>
<td>9,287,000</td>
<td>9,690,000 (+4.34%)</td>
<td>10,790,000 (+11.35%)</td>
</tr>
<tr>
<td>2 Hamburg</td>
<td>8,087,550</td>
<td>8,861,804 (+9.57%)</td>
<td>9,900,000 (+11.72%)</td>
</tr>
<tr>
<td>3 Antwerp</td>
<td>6,482,030</td>
<td>7,018,799 (+8.28%)</td>
<td>8,176,614 (+16.50%)</td>
</tr>
</tbody>
</table>
Container terminal overview
Container terminal operations
Motivation

Focus on Transshipment

- Collaboration with Medcenter Container Terminal (MTC), port of Gioia Tauro, Italy.

Context

- Hub-and-spoke
- Mother vessels and feeders
- Terminal operations
  - Berth Allocation Problem (BAP)
  - Quay Crane Assignment Problem (QCAP)

Approach

- Tactical viewpoint: support the terminal in the negotiation with shipping lines.
The Berth Allocation Problem (BAP)

Aim

- Assign and schedule incoming ships to berthing positions

Constraints

- Depth of the water (allowable draft)
- Distance from the most favorable location
- Time windows on completion time
- Handling times depend on berthing point and on the number of QCs assigned

Standard scenario

- QCAP solved before BAP

We remark that this approach is not efficient, because terminal resources are not taken into account in an integrated way.
Operational vs Tactical BAP

Operational BAP

- The objective is to comply with a predetermined plan (in terms of expected handling times and favourite berths) as much as possible.

Tactical BAP

- The template used at the operational level is determined at the tactical decision level.
- In addition to favourite berthing positions, the concept of quay cranes assignment profile, i.e. the number of QCs per shift assigned to a vessel, is used to determine expected handling times.
- Service levels are negotiated with shipping lines at this stage.
BAP & QCAP: literature review

Operational BAP + QCAP

- Park & Kim (2003)
- Imai et al. (2008)

Tactical BAP (with no QCAP)

- Moorthy & Teo (2006)
- Cordeau et al. (2007)
## Berth Allocation Plan

<table>
<thead>
<tr>
<th></th>
<th>h=1</th>
<th>h=2</th>
<th>h=3</th>
<th>h=4</th>
<th>h=5</th>
<th>h=6</th>
<th>h=7</th>
<th>h=8</th>
</tr>
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<tbody>
<tr>
<td><strong>Ship 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>berth 1</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td></td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>berth 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>berth 3</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td><strong>Ship 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>berth 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>berth 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>berth 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>QC Total</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>3</td>
<td>7</td>
<td>9</td>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>

*Ship 3* and *Ship 4* have no assignments in the specified period.

*Ship 5* is not assigned in the specified period.
TBAP with QCs assignment

Combination of 2 decision problems

- Berth Allocation Problem (BAP)
- Quay-Cranes Assignment Problem (QCAP)

Tactical decision level

- the amount of quay crane hours is negotiated months in advance with shipping lines

Issues

- the chosen profile determines the ship's handling time and thus impacts on the scheduling;
- feasible profiles can vary in length (number of shifts dedicated to the ship) and in size (number of QCs dedicated to the ship in each active shift).
TBAP with QCs assignment

Find

- A berth allocation
- A schedule
- A quay crane assignment

Given

- Time windows on availability of berths
- Time windows on arrival of ships
- Handling times dependent on QC profiles
- Values of QC profiles

Aiming to

- Maximize total value of QC assignment
- Minimize housekeeping costs of transshipment flows between ships
Housekeeping yard costs

- the analysis refers to the Medcenter Container Terminal
- transshipment context
- the cost function depends on the distance between the incoming and the outgoing berths
Housekeeping yard costs

Piecewise linear function depending on the distance and on the type of carrier used:

- < 600m : no housekeeping, straddle carriers
- 600 - 1100 m : housekeeping, straddle carriers
- > 1100 m : housekeeping, multi-trailer
TBAP with QCs assignment: the model

- $N$ = set of vessels;
- $M$ = set of berths;
- $H$ = set of time steps (each time step $h \in H$ is submultiple of the work shift length);
- $S$ = set of the time step indexes $\{1, \ldots, \bar{s}\}$ relative to a work shift; ($\bar{s}$ represents the number of time steps in a work shift);
- $H^s$ = subset of $H$ which contains all the time steps corresponding to the same time step $s \in S$ within a work shift;
- $P_i^s$ = set of feasible QC assignment profiles for the vessel $i \in N$ when vessel arrives at a time step with index $s \in S$ within a work shift;
- $P_i$ = set of quay crane assignment profiles for the vessel $i \in N$, where $P_i = \bigcup_{s \in S} P_i^s$;
TBAP with QCs assignment: the model

- $t^p_i$ = handling time of ship $i \in N$ under the QC profile $p \in P_i$ expressed as multiple of the time step length;
- $v^p_i$ = the value of serving the ship $i \in N$ by the quay crane profile $p \in P_i$;
- $q^{pu}_i$ = number of quay cranes assigned to the vessel $i \in N$ under the profile $p \in P_i$ at the time step $u \in (1, \ldots, t^p_i)$, where $u = 1$ corresponds to the ship arrival time;
- $Q^h$ = maximum number of quay cranes available at the time step $h \in H$;
- $f_{ij}$ = flow of containers exchanged between vessels $i, j \in N$;
- $d_{kw}$ = unit housekeeping cost between yard slots corresponding to berths $k, w \in M$;
- $[a_i, b_i]$ = [earliest, latest] feasible arrival time of ship $i \in N$;
- $[a^k, b^k]$ = [start, end] of availability time of berth $k \in M$;
- $[a^h, b^h]$ = [start, end] of the time step $h \in H$. 

Ilaria Vacca (EPFL) - The Tactical Berth Allocation Problem (TBAP) with Quay Crane Assignment – p.17/27
TBAP with QCs assignment: the model

Consider a graph \( G^k = (V^k, A^k) \) \( \forall k \in M \), where \( V^k = N \cup \{o(k), d(k)\} \), with \( o(k) \) and \( d(k) \) additional vertices representing berth \( k \), and \( A^k \subseteq V^k \times V^k \).

- \( x_{ij}^k \in \{0, 1\} \) \( \forall k \in M, \forall (i, j) \in A^k \), set to 1 if ship \( j \) is scheduled after ship \( i \) at berth \( k \);
- \( y_i^k \in \{0, 1\} \) \( \forall k \in M, \forall i \in N \), set to 1 if ship \( i \) is assigned to berth \( k \);
- \( \gamma_i^h \in \{0, 1\} \) \( \forall h \in H, \forall i \in N \), set to 1 if ship \( i \) arrives at time step \( h \);
- \( \lambda_i^p \in \{0, 1\} \) \( \forall p \in P_i, \forall i \in N \), set to 1 if ship \( i \) is served by the profile \( p \);
- \( \rho_{ih}^{ph} \in \{0, 1\} \) \( \forall p \in P_i, \forall h \in H, \forall i \in N \), set to 1 if ship \( i \) is served by profile \( p \) and arrives at time step \( h \);
- \( T_{ki}^k \geq 0 \) \( \forall k \in M, \forall i \in N \), representing the berthing time of ship \( i \) at the berth \( k \) i.e. the time when the ship moors;
- \( T_{o(k)}^k \geq 0 \) \( \forall k \in M \), representing the starting operation time of berth \( k \) i.e. the time when the first ship moors at the berth;
- \( T_{d(k)}^k \geq 0 \) \( \forall k \in M \), representing the ending operation time of berth \( k \) i.e. the time when the last ship departs from the berth.
TBAP with QCs assignment: the MIQP model

Objective function

Maximize total value of QC profile assignments + Minimize the (quadratic) housekeeping yard cost of transshipment flows between ships:

$$\max \sum_{i \in N} \sum_{p \in P_i} v^p_i \lambda^p_i - \frac{1}{2} \sum_{i \in N} \sum_{k \in M} y^k_i \sum_{j \in N} \sum_{w \in M} f_{ij} d_{k,w} y^w_j$$  (1)
TBAP with QCs assignment: the MIQP model

Berth covering constraints

\[ \sum_{k \in M} y_i^k = 1 \quad \forall i \in N, \quad (2) \]

Flow and linking constraints

\[ \sum_{j \in N \cup \{d(k)\}} x_{o(k),j}^k = 1 \quad \forall k \in M, \quad (3) \]

\[ \sum_{i \in N \cup \{o(k)\}} x_{i,d(k)}^k = 1 \quad \forall k \in M, \quad (4) \]

\[ \sum_{j \in N \cup \{d(k)\}} x_{ij}^k - \sum_{j \in N \cup \{o(k)\}} x_{ji}^k = 0 \quad \forall k \in M, \forall i \in N, \quad (5) \]

\[ \sum_{j \in N \cup \{d(k)\}} x_{ij}^k = y_i^k \quad \forall k \in M, \forall i \in N, \quad (6) \]
TBAP with QCs assignment: the MIQP model

Precedence constraints

\[ T^k_i + \sum_{p \in P_i} t^P_i \lambda^P_i - T^k_j \leq (1 - x^k_{ij}) M \quad \forall k \in M, \forall i \in N, \forall j \in N \cup d(k) \quad (7) \]

\[ T^k_o(k) - T^k_j \leq (1 - x^k_{o(k), j}) M \quad \forall k \in M, \forall j \in N, \quad (8) \]

Ship and Berth time windows

\[ a^k_i y^k_i \leq T^k_i \quad \forall k \in M, \forall i \in N, \quad (9) \]

\[ T^k_i \leq b^k_i y^k_i \quad \forall k \in M, \forall i \in N, \quad (10) \]

\[ a^k \leq T^k_o(k) \quad \forall k \in M, \quad (11) \]

\[ T^k_d(k) \leq b^k \quad \forall k \in M, \quad (12) \]
TBAP with QCs assignment: the MIQP model

Profile covering & linking constraints

\[ \sum_{p \in P_i} \lambda_i^p = 1 \quad \forall i \in N, \]  
(13)

\[ \sum_{h \in H^s} \gamma_i^h = \sum_{p \in P_i^s} \lambda_i^p \quad \forall i \in N, \forall s \in S, \]  
(14)

\[ \sum_{k \in M} T_i^k - b^h \leq (1 - \gamma_i^h)M \quad \forall h \in H, \forall i \in N, \]  
(15)

\[ a^h - \sum_{k \in M} T_i^k \leq (1 - \gamma_i^h)M \quad \forall h \in H, \forall i \in N, \]  
(16)

\[ \rho_i^p \geq \lambda_i^p + \gamma_i^h - 1 \quad \forall h \in H, \forall i \in N, \forall p \in P_i, \]  
(17)

Quay crane and profile feasibility

\[ \sum_{i \in N} \sum_{p \in P_i} \sum_{u = \max\{h - t_i^p + 1; 1\}}^{h} \rho_i^p q_i^{p(h-u+1)} \leq Q^h \quad \forall h \in H^s \]  
(18)
TBAP with QCs assignment: the MILP model

Additional decision variable

\[ z_{ij}^{kw} \in \{0, 1\} \quad \forall i, j \in N, \forall k, w \in M, \text{ set to 1 if } y_i^k = y_j^w = 1 \text{ and 0 otherwise.} \]

Linearized objective function

\[
\text{max} \quad \sum_{i \in N} \sum_{p \in P_i} v_p^i \lambda_p^i - \frac{1}{2} \sum_{i \in N} \sum_{j \in N} \sum_{k \in M} \sum_{w \in M} f_{ij} d_{kw} z_{ij}^{kw} \]

Additional constraints

\[
\sum_{k \in K} \sum_{w \in K} z_{ij}^{kw} = g_{ij} \quad \forall i, j \in N, \quad (20)
\]

\[
z_{ij}^{kw} \leq y_i^k \quad \forall i, j \in N, \forall k, w \in M \quad (21)
\]

\[
z_{ij}^{kw} \leq y_j^w \quad \forall i, j \in N, \forall k, w \in M \quad (22)
\]
Generation of test instances

- Based on real data provided by MCT
  - container flows
  - housekeeping yard costs
  - vessel's arrival times

- Crane productivity of 24 containers per hours

- Set of feasible profiles synthetically generated:

<table>
<thead>
<tr>
<th>Class</th>
<th>min QC</th>
<th>max QC</th>
<th>min HT</th>
<th>max HT</th>
<th>volume (min,max)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mother</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>6</td>
<td>(1296, 4320)</td>
</tr>
<tr>
<td>Feeder</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>(288, 1728)</td>
</tr>
</tbody>
</table>
Generation of test instances

- 24 instances organized in 3 classes: E (easy), M (medium) and D (difficult)
  - Class E: 9 instances, 10 ships, 3 berths, 8 QCs
  - Class M: 9 instances, 20 ships, 5 berths, 13 QCs
  - Class D: 6 instances, 30 ships, 5 berths, 13 QCs

- Different traffic volumes in scenarios A, B, C

- Each scenario is tested with a set of $\bar{p} = 10, 20, 30$ feasible profiles for each ship

MIQP and MILP formulations tested with CPLEX 10.2 on an Intel 3GHz workstation
Numerical results

- Class E: always solved at optimality (MILP 8/9, MIQP 4/9) or near-optimality
- Class M and D: even a feasible solution is hardly found (MILP finds one feasible solution for class M)
- As expected:
  - the quadratic term in the objective function adds complexity (comparison with MaxTotalValue formulation)
  - the higher the number of feasible profiles, the higher complexity
- Interesting findings:
  - MILP provides better bounds than MIQP
  - MIQP seems to be independent from time granularity
  - Symmetry in the problem
Conclusions and future work

Contribution

- Tactical viewpoint: Integration between BAP and QCAP
- QC profiles
- Analysis of yard costs
- MIQP/MILP models
- Preliminary numerical results

Forthcoming

- Heuristics
- Decomposition methods
- Analysis of value functions for QC profiles