Modelling competition in demand-based optimization models

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Outline

Competition in transportation

- Competition is often present in the form of oligopolies (regulations, limited capacity of the infrastructure, barriers to entry, etc.).

- Deregulation has generally led to oligopolistic markets.
    - Airlines: U.S. Deregulation Act (1978), then similar laws in Europe.
    - Railways: directives 91/440/EC and 2012/34/EU give open access to railway lines in the EU to companies other than those that own the infrastructure.
    - Buses: many countries recently opened the market of long-distance buses.

Example

Operators connecting Milan and Rome:

High-speed train operators

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Other transport operators

Modelling demand

- Each customer chooses the alternative that maximizes his/her utility.

- Customers have different tastes and socioeconomic characteristics that influence their choice.

Modelling supply

- Operators take decisions that optimize their objective function (e.g. revenue maximization).

- Decisions can be related to pricing, capacity, frequency, availability, and other variables.

- Decisions are influenced by:
  - The preferences of the customers
  - The decisions of the competitors

Modelling competition

- We consider non-cooperative games.

- We aim at understanding the Nash equilibrium solutions of such games, i.e. stationary states of the system in which no competitor has an incentive to change its decisions.

The framework

Three-level framework: customers, operators and market.

Customer level: discrete choice models take into account preference heterogeneity and model individual decisions. These can be integrated in a MILP by relying on simulation to draw from the distribution of the error term of the utility function.

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Customer level: discrete choice models take into account preference heterogeneity and model individual decisions. These can be integrated in a MILP by relying on simulation to draw from the distribution of the error term of the utility function.

Operator level: a mixed integer linear program can maximize any relevant objective function.

Market level : Nash equilibrium solutions are found by enforcing best response constraints.

Description

Sequential algorithm to nd Nash
equilibrium solutions of a two-players
game:

Initialization: de nition of the rst
optimizing operator and of an initial
feasible strategy of the competitor.

Iterative phase: operators take turns
and each plays its best response pure
strategy to the last strategy played by
the competitor.

Termination criterion: either a Nash
equilibrium or a cyclic equilibrium is
reached.

Discussion

The algorithm reproduces the behavior of two or more operators that do not know the competitors' objective function.

It can be used with both nite and in nite strategy sets.

Di erent initial strategies can lead to di erent equilibria.

There is no guarantee that a pure strategy Nash equilibrium exists or that it is unique.

Description

Each operator $k \in K$ can choose a pure strategy from its finite set of strategies S_k.

Number of pure strategy solutions of the game is $|S| = \prod_{k \in K} S_k$.

For each solution $s \in S$ we can derive a payoff function V_{ks} for each operator $k \in K$.

If $s \in S$ includes only best response strategies for all operators, then it is a pure strategy Nash equilibrium for the finite game.

Customer level

Customer constraints:

$$\sum_{i \in I} w_{inrs} = 1 \qquad \qquad \forall n \in N, \forall r \in R, \forall s \in S \qquad (1)$$

$$w_{inrs} \leq y_{inrs} \qquad \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \qquad (2)$$

$$y_{inrs} \leq y_{ins} \qquad \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \qquad (3)$$

$$y_{ins} = 0 \qquad \qquad \forall i \in I, \forall n \in N : i \notin C_n, \forall s \in S \qquad (4)$$

$$\sum_{n \in N} w_{inrs} \geq C_i \qquad \qquad \forall i \in I \cap F_{0g}, \forall r \in R, \forall s \in S \qquad (5)$$

$$C_i(y_{ins} - y_{inrs}) \leq \sum_{m \in N : L_{im} < L_{in}} w_{imrs} \qquad \qquad \forall i \in I \cap F_{0g}, \forall n \in N, \forall r \in R, \forall s \in S \qquad (6)$$

$$\sum_{m \in N : L_{im} < L_{in}} w_{imrs} \leq (C_i - 1)y_{inrs} + (n-1)(1 - y_{inrs}) \qquad \forall i \in I \cap F_{0g}, \forall n \in N, \forall r \in R, \forall s \in S \qquad (7)$$

$$U_{inrs} = \beta_{in} p_{ins} + q_{in}^d + \xi_{inr} \qquad \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \qquad (8)$$

$$lbU_{nr} \leq z_{inrs} - lbU_{nr} + M_{U_{nr}} y_{inrs} \qquad \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \qquad (9)$$

$$U_{inrs} - M_{U_{nr}}(1 - y_{inrs}) \leq z_{inrs} \leq U_{inrs} \qquad \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \qquad (10)$$

$$z_{inrs} \leq U_{nr} \qquad \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \qquad (11)$$

$$U_{nr} \geq z_{inrs} + M_{U_{nr}}(1 - w_{inrs}) \qquad \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \qquad (12)$$

Operator level

Operator constraints:

$$V_{ks} = \frac{1}{R} \sum_{i \in C_k} \sum_{n \in N} \sum_{r \in R} p_{ins} w_{inrs} \qquad\qquad \forall k \in K; \forall s \in S \qquad (13)$$

$$V_{ks} \leq V_{kt}^{max} \qquad\qquad \forall k \in K; \forall s \in S_k; \forall t \in S_k^C \qquad (14)$$

$$V_{kt}^{max} \leq V_{ks} + M_r(1 - x_{ks}) \qquad\qquad \forall k \in K; \forall s \in S_k; \forall t \in S_k^C \qquad (15)$$

$$\sum_{s \in S} x_{ks} = S_k^C \qquad\qquad \forall k \in K \qquad (16)$$

Market level

Find $s \in S$ such that $e_s = 1$

$s.t.$

Equilibrium constraints:

$$e_s \geq \sum_{k \in K} x_{ks} - (|K| - 1) \qquad \forall s \in S \qquad (17)$$

$$e_s \leq x_{ks} \qquad \forall k \in K, \forall s \in S \qquad (18)$$

Numerical examples

Payoff matrix of player 1

S1 \ S2	0,70	0,73	0,76	0,79	0,82	0,85
0,50	10,00	10,00	10,00	10,00	10,00	10,00
0,53	10,49	**10,60**	**10,60**	10,60	10,60	10,60
0,56	**10,53**	10,42	10,53	10,80	**11,20**	11,20
0,59	10,27	10,03	9,80	9,91	10,62	**11,45**
0,62	10,04	9,80	9,42	9,42	9,42	9,92
0,65	9,62	9,36	8,84	8,45	8,71	8,58

Payoff matrix of player 1

S1 \ S2	0,75	0,77	0,79	0,81	0,83	0,85
0,50	10,00	10,00	10,00	10,00	10,00	10,00
0,52	10,40	10,40	10,40	10,40	10,40	10,40
0,54	**10,80**	**10,80**	10,80	10,80	10,80	10,80
0,56	10,42	10,53	**10,86**	**11,09**	**11,20**	11,20
0,58	9,74	9,86	10,09	10,44	10,67	**11,37**
0,60	9,60	9,60	9,72	10,08	10,44	10,68

Payoff matrix of player 2

S1 \ S2	0,70	0,73	0,76	0,79	0,82	0,85
0,50	14,00	14,45	14,74	14,69	**14,76**	14,62
0,53	14,00	14,45	14,74	**15,01**	14,60	14,45
0,56	14,00	14,60	14,74	**14,85**	14,76	14,28
0,59	14,00	14,60	15,05	**15,48**	15,09	14,45
0,62	14,00	14,60	15,20	15,48	**15,91**	15,81
0,65	14,00	14,60	15,20	15,80	15,91	**16,32**

Payoff matrix of player 2

S1 \ S2	0,75	0,77	0,79	0,81	0,83	0,85
0,50	14,70	**14,78**	14,69	14,74	14,28	14,62
0,52	14,70	**15,09**	14,85	14,58	14,61	14,45
0,54	14,85	14,94	**15,17**	14,74	14,44	14,45
0,56	14,85	**14,94**	14,85	14,90	14,61	14,28
0,58	15,00	15,09	**15,17**	15,07	15,11	14,45
0,60	15,00	15,25	**15,48**	15,39	15,27	14,30

(a) Game with 1 pure strategy Nash equilibrium   (b) Game with no pure strategy Nash equilibrium

Payoff matrices for two games with different support strategies. Best response payoffs are in bold. Equilibrium payoffs are in blue.

Discussion

- The model requires finite strategy sets (enumeration).

- All pure strategy Nash equilibria of the game can be found, however the problem is solvable with small solution spaces only.

- The assumption of a finite game requires price discretization.

Summary

- We are analyzing oligopolistic markets from three integrated perspectives:
  - Customer level, by using discrete choice models
  - Operator level, by solving a mixed integer program
  - Market level, by including equilibrium constraints

- We presented two different approaches to find Nash equilibrium solutions for the resulting non-cooperative multi-leader-follower game.

Open questions

- Extension of the current MILP to include mixed strategy games (Nash's existence theorem).

- Efficient search for equilibria in the solution space to avoid enumeration.

- Investigation of the concept of Nash equilibrium region for real-life applications.

Questions?
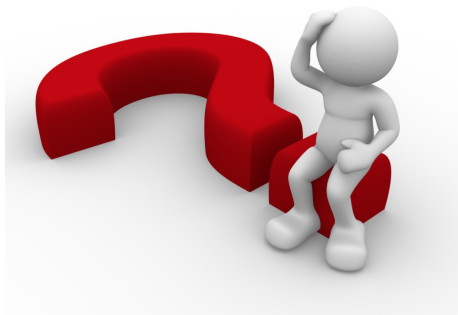
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