Modelling competition in demand-based optimization models

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18th Swiss Transport Research Conference
Ascona, 17 May 2018
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3 Conclusions
Competition in transportation

- Competition is often present in the form of oligopolies (regulations, limited capacity of the infrastructure, barriers to entry, etc.).

- Deregulation has generally led to oligopolistic markets.
  - Railways: directives 91/440/EC and 2012/34/EU give open access to railway lines in the EU to companies other than those that own the infrastructure.
  - Buses: many countries recently opened the market of long-distance buses.
Operators connecting Milan and Rome:

High-speed train operators

![Trenitalia](image1.jpg) ![Italo](image2.jpg)
Operators connecting Milan and Rome:

High-speed train operators

Other transport operators

Motivation

Example

SB, VL, MB
Modelling demand

- Each customer chooses the alternative that maximizes his/her utility.

- Customers have different tastes and socioeconomic characteristics that influence their choice.
Modelling supply

- Operators take decisions that optimize their objective function (e.g. revenue maximization).

- Decisions can be related to pricing, capacity, frequency, availability, and other variables.

- Decisions are influenced by:
  - The preferences of the customers
  - The decisions of the competitors
Motivation

Modelling competition

- We consider non-cooperative games.
- We aim at understanding the Nash equilibrium solutions of such games, i.e. stationary states of the system in which no competitor has an incentive to change its decisions.
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The framework

Three-level framework: customers, operators and market.

1. **Customer level**: discrete choice models take into account preference heterogeneity and model individual decisions. These can be integrated in a MILP by relying on simulation to draw from the distribution of the error term of the utility function.
The framework

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2. **Operator level**: a mixed integer linear program can maximize any relevant objective function.
The framework

Three-level framework: customers, operators and market.

1. **Customer level**: discrete choice models take into account preference heterogeneity and model individual decisions. These can be integrated in a MILP by relying on simulation to draw from the distribution of the error term of the utility function.

2. **Operator level**: a mixed integer linear program can maximize any relevant objective function.

3. **Market level**: Nash equilibrium solutions are found by enforcing best response constraints.
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Modelling the problem

A fixed-point iteration method

Description

Sequential algorithm to find Nash equilibrium solutions of a two-players game:

- Initialization: definition of the first optimizing operator and of an initial feasible strategy of the competitor.

- Iterative phase: operators take turns and each plays its best response pure strategy to the last strategy played by the competitor.

- Termination criterion: either a Nash equilibrium or a cyclic equilibrium is reached.
Discussion

- The algorithm reproduces the behavior of two or more operators that do not know the competitors’ objective function.

- It can be used with both finite and infinite strategy sets.

- Different initial strategies can lead to different equilibria.

- There is no guarantee that a pure strategy Nash equilibrium exists or that it is unique.
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Description

- Each operator $k \in K$ can choose a pure strategy from its finite set of strategies $S_k$.

- Number of pure strategy solutions of the game: $|S| = \prod_{k \in K} S_k$.

- For each solution $s \in S$ we can derive a payoff function $V_{ks}$ for each operator $k \in K$.

- If $s \in S$ includes only best response strategies for all operators, then it is a pure strategy Nash equilibrium for the finite game.
Customer level

Customer constraints:

\[ \sum_{i \in I} w_{inrs} = 1 \]  \quad \forall n \in N, \forall r \in R, \forall s \in S \tag{1} \\
\[ w_{inrs} \leq y_{inrs} \]  \quad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \tag{2} \\
\[ y_{inrs} \leq y_{ins} \]  \quad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \tag{3} \\
\[ y_{ins} = 0 \]  \quad \forall i \in I, \forall n \in N : i \not\in C_n, \forall s \in S \tag{4} \\
\[ \sum_{n \in N} w_{inrs} \leq C_i \]  \quad \forall i \in I \setminus \{0\}, \forall r \in R, \forall s \in S \tag{5} \\
\[ C_i (y_{ins} - y_{inrs}) \leq \sum_{m \in N : L_{im} < L_{in}} w_{imrs} \]  \quad \forall i \in I \setminus \{0\}, \forall n \in N, \forall r \in R, \forall s \in S \tag{6} \\
\[ \sum_{m \in N : L_{im} < L_{in}} w_{imrs} \leq (C_i - 1)y_{inrs} + (n - 1)(1 - y_{inrs}) \]  \quad \forall i \in I \setminus \{0\}, \forall n \in N, \forall r \in R, \forall s \in S \tag{7} \\
\[ U_{inrs} = \beta_{inpins} + q_{in}^{d} + \xi_{inr} \]  \quad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \tag{8} \\
\[ lb_{nr} \leq z_{inrs} \leq lb_{nr} + M_{nr} y_{inrs} \]  \quad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \tag{9} \\
\[ U_{inrs} - M_{nr} (1 - y_{inrs}) \leq z_{inrs} \leq U_{inrs} \]  \quad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \tag{10} \\
\[ z_{inrs} \leq U_{nr} \]  \quad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \tag{11} \\
\[ U_{nr} \leq z_{inrs} + M_{nr} (1 - w_{inrs}) \]  \quad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \tag{12}
Operator level

Operator constraints:

\[ V_{ks} = \frac{1}{R} \sum_{i \in C_k} \sum_{n \in N} \sum_{r \in R} p_{ins} w_{inrs} \quad \forall k \in K, \forall s \in S \tag{13} \]

\[ V_{ks} \leq V_{kt}^{\text{max}} \quad \forall k \in K, \forall s \in S_k, \forall t \in S_k^C \tag{14} \]

\[ V_{kt}^{\text{max}} \leq V_{ks} + M_r(1 - x_{ks}) \quad \forall k \in K, \forall s \in S_k, \forall t \in S_k^C \tag{15} \]

\[ \sum_{s \in S} x_{ks} = \left| S_k^C \right| \quad \forall k \in K \tag{16} \]
Market level

Find $s \in S$ such that $e_s = 1$

s.t.

Equilibrium constraints:

$e_s \geq \sum_{k \in K} x_{ks} - (|K| - 1) \quad \forall s \in S \quad (17)$

$e_s \leq x_{ks} \quad \forall k \in K, \forall s \in S \quad (18)$
Numerical examples

(a) Game with 1 pure strategy Nash equilibrium

(b) Game with no pure strategy Nash equilibrium

Payoff matrices for two games with different support strategies. Best response payoffs are in bold. Equilibrium payoffs are in blue.
Discussion

- The model requires finite strategy sets (enumeration).
- All pure strategy Nash equilibria of the game can be found, however the problem is solvable with small solution spaces only.
- The assumption of a finite game requires price discretization.
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Conclusions
Summary

- We are analyzing oligopolistic markets from three integrated perspectives:
  - Customer level, by using discrete choice models
  - Operator level, by solving a mixed integer program
  - Market level, by including equilibrium constraints

- We presented two different approaches to find Nash equilibrium solutions for the resulting non-cooperative multi-leader-follower game.
Open questions

- Extension of the current MILP to include mixed strategy games (Nash’s existence theorem).

- Efficient search for equilibria in the solution space to avoid enumeration.

- Investigation of the concept of Nash equilibrium region for real-life applications.
Questions?

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