Modelling competition in demand-based optimization models

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Outline

Competition

In many markets competition is present in the form of oligopolies (regulations, barriers to entry, mergers, acquisitions, alliances).

In transportation, deregulation often led to oligopolistic markets.

- Airlines
- Railways
- Buses
- Multi-modal networks

How to study competitive transport markets?

Modelling demand

Modelling supply

Modelling competition

Demand

Each customer chooses the alternative that maximizes his/her utility.

Customers have different tastes and socioeconomic characteristics that influence their choice.

Supply

Operators take decisions that optimize their objective function (e.g. revenue maximization).

Decisions can be related to pricing, capacity, frequency, availability ...

Decisions are influenced by:

- The preferences of the customers
- The decisions of the competitors

Competition

We consider non-cooperative games.

We aim at understanding the Nash equilibrium solutions of such games, i.e. stationary states of the system in which no competitor has an incentive to change its decisions.

The framework

Three elements to be modelled: customers, operators and market.

Customers: discrete choice models take into account preference heterogeneity and model individual decisions.

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Customers: discrete choice models take into account preference heterogeneity and model individual decisions.

Operators: a mixed integer program can maximize any relevant objective function.

Market: Nash equilibrium solutions are found by enforcing best response constraints.

The framework: customer level

Non-linear formulation :

The probability of customer $n \in N$ choosing alternative $i \in I$ depends on the discrete choice model specification.

For the logit model, there exists a closed-form expression:

$$P_{in} = \frac{\exp(V_{in})}{\sum_{j \in I} \exp(V_{jn})}$$

For other discrete choice models, there is no closed-form expression.

The framework: customer level

Linear formulation :

A linear formulation can be obtained by relying on simulation to draw from the distribution of the error term of the utility function.

For all customers and all alternatives, R draws of are extracted from the error term distribution. Each $_{inr}$ corresponds to a different behavioral scenario.

$$U_{inr} = \; _{in} p_{in} + q_{in} + \; _{inr}$$

In each scenario, customers choose the alternative with the highest utility:

$$w_{inr} = 1 \text{ if } U_{inr} = \max_{j \in I} U_{jnr}, \text{ and } w_{inr} = 0 \text{ otherwise}$$

Over multiple scenarios, the probability of customer n choosing alternative i is given by

$$P_{in} = \frac{\sum_{r \in R} w_{inr}}{R}.$$

The framework: operators level

- We assume that an operator $k \in K$ can decide on the price p_{in} of each alternative $i \in I$ for all customers $n \in N$.
- Stackelberg game: the operator (the leader) knows the best response of t customers ("collective" follower) to all strategies.
- Objective function to be maximized by operator k:

$$V_k = \sum_{i \in I} \sum_{n \in N} p_{in} P_{in}$$

Non-linear optimization model for a single operator

$$
\begin{aligned}
\max \quad & V = \sum_{i \in I} \sum_{n \in N} p_{in} P_{in} & \\
s.t.: \quad & P_{in} = \frac{\exp(U_{in})}{\sum_{j \in I} \exp(U_{jn})} & \forall i \in I; \forall n \in N \\
& U_{in} = \beta_{in} p_{in} + q_{in} & \forall i \in I; \forall n \in N
\end{aligned}
$$

Linear optimization model for a single operator

$$\max \quad V = \sum_{i \in I} \sum_{n \in N} p_{in} P_{in}$$

$$\text{s.t.:} \quad P_{in} = \frac{\sum_{r \in R} w_{inr}}{R} \qquad \forall i \in I; \forall n \in N$$

$$U_{inr} = \quad {}_{in} p_{in} + q_{in} + \quad {}_{inr} \qquad \forall i \in I; \forall n \in N; \forall r \in R$$

$$U_{inr} \quad U_{nr} \qquad \forall i \in I; \forall n \in N; \forall r \in R$$

$$U_{nr} \quad U_{inr} + M_{U_{nr}} (1 \quad w_{inr}) \qquad \forall i \in I; \forall n \in N; \forall r \in R$$

$$\sum_{i \in I} w_{inr} = 1 \qquad \forall n \in N; \forall r \in R$$

$$w_{inr} \in \{0; 1\} \qquad \forall i \in I; \forall n \in N; \forall r \in R$$

The framework: market level

- The payo of an operator also depends on the strategies of the competitors

- Let's de ne as X_k the set of strategies that can be played by operator $k \in K$.

- Condition for Nash equilibrium (best response constraints):

$$V_k = V_k = \max_{x_k \in X_k} V_k(x_k; x_{K \setminus \{k\}}) \qquad \forall k \in K$$

- Nash (1951): nite games have at least one mixed strategy equilibrium solution.

- Finite/in nite strategy sets; pure/mixed strategies; continuous/discrete payo function.

A xed-point iteration method

Sequential algorithm to nd Nash equilibrium solutions of a k-player game:

- Initialization: players select an initial feasible strategy.
- Iterative phase: players take turns and each plays its best response p strategy to the current solution.
- Termination criterion: either a Nash equilibrium or a cyclic equilibrium is reached.

A mixed integer model for the xed-point problem

We can write a model that minimizes the distance between two consecutive xed-point iterations.

A solution for a two-operator problem: $(\hat{x}_1, \hat{x}_2)$

Optimization problems for the operators:

$$x_1 = \arg\max_{x_1 \in X_1} V_1(x_1, \hat{x}_2)$$

$$x_2 = \arg\max_{x_2 \in X_2} V_2(\hat{x}_1, x_2)$$

Fixed-point problem:

$$\min_{x_1, x_2, \hat{x}_1, \hat{x}_2} \|x_1 - \hat{x}_1\| + \|x_2 - \hat{x}_2\|$$

Initial con guration

No optimization at operator level: any feasible strategy could be selected.

Constraints:

- Customer choice
  - Continuous (MINLP), or
  - Binary (MILP)
- Customer utility maximization

$$\sum_{i \in I} w^b_{inr} = 1 \qquad \forall n \in N; \forall r \in R$$

$$U^b_{inr} = {}_{in} p^b_{in} + q_{in} + {}_{inr} \qquad \forall i \in I; \forall n \in N; \forall r \in R$$

$$U^b_{inr} \quad U^b_{nr} \qquad \forall i \in I; \forall n \in N; \forall r \in R$$

$$U^b_{nr} \quad z^b_{inr} + M(1 \quad w^b_{inr}) \qquad \forall i \in I; \forall n \in N; \forall r \in R$$

Best response con gurations

Each operator solves an optimization problem having the following constraints:

- Customer choice
  - Continuous (MINLP), or
  - Binary (MILP)
- Customer utility maximization
- Best response

Best response con gurations

Best response constraints:

$$V_{ks} = \frac{1}{R} \sum_{i \in I_k} \sum_{n \in N} \sum_{r \in R} p_{ins} w^a_{inrs} \qquad \forall k \in K; \forall s \in S_k$$

$$V_{ks} \quad V^{max}_{ks} \qquad \forall s \in S_k$$

$$V^{max}_{ks} \quad V_{ks} + M(1 \quad x_{ks}) \qquad \forall s \in S_k$$

$$\sum_{s \in S} x_{ks} = 1$$

Customer constraints:

$$U^a_{inrs} = \quad {}_{in} p_{ins} + q_{in} + \quad {}_{inr} \qquad \forall i \in I_k; \forall n \in N; \forall r \in R; \forall s \in S_k$$

$$U^a_{inrs} = U^b_{inr} \qquad \forall i \in I \setminus I_k; \forall n \in N; \forall r \in R; \forall s \in S_k$$

$$\sum_{i \in I} w^a_{inrs} = 1 \qquad \forall n \in N; \forall r \in R; \forall s \in S_k$$

$$U^a_{inrs} \quad U^a_{nrs} \qquad \forall i \in I; \forall n \in N; \forall r \in R; \forall s \in S_k$$

$$U^a_{nrs} \quad z^a_{inrs} + M(1 \quad w^a_{inrs}) \qquad \forall i \in I; \forall n \in N; \forall r \in R; \forall s \in S_k$$

Objective function

Minimization problem:

$$z \;=\; \min_{x_1;x_2;x_1\;;x_2} kx_1 \quad x_1^b k + kx_2 \quad x_2^b k$$

If $z\;=0$, we have an equilibrium. What can we say about this equilibrium?

If $z\;>\;0$, can we conclude something? Are we in an equilibrium region?

Numerical experiments

Case study: 3 parking choices. 2 owned by 2 di erent operators and opt-out option. Parameter estimation available in the literature.

Tests: non-linear and linear formulations with logit and mixed logit speci cations.

	Non-linear	Linear
Logit	-	
Mixed logit		,

Figure: Random draws needed in the di erent sets of experiments

Numerical experiments

Model speci cation: drawing from the error term distribution ($R = 50; 100; 200$) gives good approximation of the choice probabilities found with the logit formula.

Preliminary results: equilibrium solutions are found for all tested instances with the MILP formulation.

Computational times:

- Logit:
  - The non-linear model is faster (no need for simulation).
  - The time required by the linear model increases with the number of draws.
- Mixed logit:
  - The non-linear model (highly non convex) does not converge for larger instances.
  - The linear model outperforms the non-linear model on larger instances.

Open questions and future work

The current model uses nite strategy sets (i.e. price discretization). Is it possible to reformulate the problem with the help of complementarity?

How can the structure of the problem be exploited to e ciently search for equilibria in the solution space?

Is it possible to compare di erent equilibrium solution or to prove uniqueness within a region of the solution space?