Integrated schedule planning with supply-demand interactions

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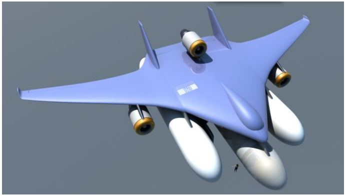
May 13, 2011

Motivation

- Increased air travel demand
- Demand responsiveness
  - Flexible supply capacity
  - Improved demand management
- Sustainability

Clip-Air concept

- Flexible capacity with modular-detachable capsules
- Carrier and capsule separation: security, maintenance, storage and crew costs
- Multi-modal transportation for both passenger and cargo
- Sustainable transportation
  - Gas emissions
  - Noise
  - Accident rates



Objectives

- Development of integrated schedule design and fleet assignment model
  - maximize *revenue - operating costs*
  - itinerary-based demand
  - integration of supply-demand interactions
    - logit demand model $\Rightarrow$ pricing
    - spill and recapture effects
  - Fare-class segmentation
    - demand model for each segment
    - seat allocation for business and economy
- Solution techniques for the resulting mixed integer nonlinear problem
- Comparative analysis between standard fleet and Clip-Air

Demand model for itinerary choice

- Utility of itinerary i, class h:

$$V_i^h = \beta_{fare}^h p_i^h + \beta_{time}^h time_i + \beta_{stops}^h nonstop_i$$

  - p_i^h is the price of itinerary i for class h.
  - $time_i$, binary variable, 1 if departure time is between 07:00-11:00.
  - $nonstop_i$, binary variable, 1 if it is a non-stop itinerary.

- Demand for class h for each itinerary i in market segment s:

$$\tilde{d}_i^h = D_s^h \frac{\exp(V_i^h)}{\sum_{j \in I_s} \exp(V_j^h)}$$

  - D_s^h is the total expected demand for class h and segment s.
  - $\tilde{d}_i^h$ serves as an upper bound for the actual demand.

Spill and recapture effects

Example

- In case of capacity shortage some passengers may not fly on their desired itineraries
- They may accept to fly on other available itineraries in the same market segment
- Recapture ratio is given by:

$$b_{i,j}^{h} = \frac{\exp(V_j^h)}{\sum_{k \in I_s \setminus i} \exp(V_k^h)}$$

- *No-purchase* represented by the subset $I_s' \in I_s$ for segment s.

Integrated model - Supply part

$$Max \sum_{s\in S}\sum_{h\in H}\sum_{i\in(I_s\setminus I_s^{'})}(d_i^h - \sum_{\substack{j\in I_s\\ i\neq j}} t_{i,j}^h + \sum_{\substack{j\in(I_s\setminus I_s^{'})\\ i\neq j}} t_{j,i}^h b_{j,i}^h)p_i^h - \sum_{\substack{k\in K\\ f\in F}} C_{k,f}x_{k,f} : \text{ revenue - cost} \quad (1)$$

$$s.t. \sum_{k\in K} x_{k,f} = 1 : \text{ mandatory flights} \qquad \forall f \in F^M \quad (2)$$

$$\sum_{k\in K} x_{k,f} \leq 1 : \text{ optional flights} \qquad \forall f \in F^O \quad (3)$$

$$y_{k,a,t^-} + \sum_{f\in In(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in Out(k,a,t)} x_{k,f} : \text{ flow conservation} \qquad \forall [k,a,t] \in N \quad (4)$$

$$\sum_{a\in A} y_{k,a,t_n} + \sum_{f\in CT} x_{k,f} \leq R_k : \text{ fleet availability} \qquad \forall k \in K \quad (5)$$

$$y_{k,a,minE_a^-} = y_{k,a,maxE_a^+} : \text{ cyclic schedule} \qquad \forall k \in K, a \in A \quad (6)$$

$$\sum_{s\in S}\sum_{i\in(I_s\setminus I_s^{'})} \delta_{i,f} d_i^h - \sum_{\substack{j\in I_s\\ i\neq j}} \delta_{i,f} t_{i,j}^h + \sum_{\substack{j\in(I_s\setminus I_s^{'})\\ i\neq j}} \delta_{i,f} t_{j,i}^h b_{j,i}^h \leq \sum_{k\in K} \pi_{k,f}^h : \text{ seat allocation} \qquad \forall h \in H, f \in F \quad (7)$$

$$\sum_{h\in H} \pi_{k,f}^h = Q_k x_{k,f} : \text{ seat capacity} \qquad \forall f \in F, k \in K \quad (8)$$

$$x_{k,f} \in \{0,1\} \qquad \forall k \in K, f \in F \quad (9)$$

$$y_{k,a,t} \geq 0 \qquad \forall [k,a,t] \in N \quad (10)$$

$$\pi_{k,f}^h \geq 0 \qquad \forall h \in H, k \in K, f \in F \quad (11)$$

Integrated model - Demand part

▸ H

$$\sum_{\substack{j \in I_s \\ i \neq j}} t^h_{i,j} \leq d^h_i: \textit{ total spill} \qquad \forall s \in S, h \in H, i \in (I_s \setminus I^{'}_s) \qquad (12)$$

$$\tilde{d}^h_i = D^h_s \upsilon^h_s \exp(V^h_i): \textit{ logit demand} \qquad \forall s \in S, h \in H, i \in I_s \qquad (13)$$

$$\sum_{i \in I_s} \upsilon^h_s \exp(V^h_i) = 1: \textit{ choice probability} \qquad \forall s \in S, h \in H \qquad (14)$$

$$b^h_{i,j} = \frac{\exp(V^h_j)}{\frac{1}{\upsilon^h_s} - \exp(V^h_i)}: \textit{ recapture ratio} \qquad \forall s \in S, h \in H, i \in (I_s \setminus I^{'}_s), j \in I_s \qquad (15)$$

$$d^h_i \leq \tilde{d}^h_i \leq D^h_i: \textit{ realized demand} \qquad \forall h \in H, i \in I \qquad (16)$$

$$0 \leq p^h_i \leq UB^h_i: \textit{ upper bound on price} \qquad \forall h \in H, i \in I \qquad (17)$$

$$t^h_{i,j} \geq 0 \qquad \forall s \in S, h \in H, i \in (I_s \setminus I^{'}_s), j \in I_s \qquad (18)$$

$$b^h_{i,j} \geq 0 \qquad \forall s \in S, h \in H, i \in (I_s \setminus I^{'}_s), j \in I_s \qquad (19)$$

$$\upsilon^h_s \geq 0 \qquad \forall s \in S, h \in H \qquad (20)$$

Model extension for Clip-Air

- Decision variables for the assignment of wing and capsules:
  $x_f^w \in \{0,1\}$
  $x_{k,f} \in \{0,1\}$ for $k \in \{1,2,3\}$
- Operating cost:

$$\sum_{f \in F} C_f^w x_f^w + \sum_{k \in K} C_{k,f} x_{k,f}$$

- Constraints:

$$\sum_{k \in K} x_{k,f} = 1 \qquad \forall f \in F^M: \textit{ mandatory flights}$$

$$\sum_{k \in K} x_{k,f} \leq x_f^w \qquad \forall f \in F: \textit{ capsule - wing}$$

Results

- Dataset from a major European airline
- Other inputs:
  - Cost figures for Clip-Air
    - Weight differences => adjustment of fuel cost and airport and air navigation charges
    - Capsule wing separation => adjustment of crew cost
  - Parameters of the demand model
- Model is implemented in AMPL and solved with BONMIN
- Results provide the schedule design, fleet assignment, seat allocation for fare classes and pricing.

Demand model parameters

- Estimation of logit model parameters by maximum likelihood estimation using BIOGEME
- Booking data does not have the non-chosen alternatives ⇒ lack of variability
- Adjusted parameters to have enough elasticity

	Business demand	**Economy demand**
β_{fare}	-0.025	-0.050
β_{time}	0.323	0.139
$\beta_{nonstop}$	1.150	0.900

Small data instance

Airports	3 (ORY, LYS, NCE)
Flights	9
Passengers	800
Capsule capacity	50
Standard fleet types	A318 (123), ERJ145 (50)
Total fleet size (seats)	400
Fare classes	Business, economy

	origin	destination	expected demand	nonstop	time
1	ORY	LYS	132	1	1
2	ORY	LYS	133	1	0
3	ORY	NCE	68	1	1
4	NCE	ORY	56	1	1
5	ORY	NCE	79	1	0
6	NCE	ORY	63	1	0
7	ORY	NCE	80	1	0
8	LYS	ORY	108	1	1
9	LYS	ORY	81	1	0

Impact of demand model

Competing itineraries with close utility values, high price elasticity

	Fixed model	Integrated model
Operating cost	72,482	65,635
Revenue	104,142	102,497
Profit	**31,660**	**36,862**
Transported pax.	**580**	**532**
Flight count	6	8
Average pax/flight	**96**	**66**
Total Flight Hours (min)	425	590
Used fleet	2 A318	2 A318
	1 ERJ145	3 ERJ145
Used capacity (seats)	296	396

- Integrated model increases the prices
- Fixed demand model accumulates the demand

			Fixed demand model		**Integrated demand model**		**outside**
	O	D	realized demand	fixed price	realized demand	realized price	price
1	ORY	LYS	50	162	123	179	185
2	ORY	LYS	123	162	50	194	185
3	ORY	NCE	123	200	50	220	250
4	NCE	ORY	111	212	50	230	250
5	ORY	NCE	0	200	50	218	250
6	NCE	ORY	0	212	50	228	250
7	ORY	NCE	0	200	0	214	250
8	LYS	ORY	123	162	109	159	185
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Different scenarios

Cheaper competing itineraries				
	High price elasticity		**Low price elasticity**	
	Fixed demand model	Integrated model	Fixed demand model	Integrated model
Profit	30,966	23,141	31,250	17,159
Transported pax.	541	400	543	499
Flight count	8	8	8	8
Comparable competing itineraries				
	High price elasticity		**Low price elasticity**	
	Fixed demand model	Integrated model	Fixed demand model	Integrated model
Profit	31,660	36,862	31,617	36,484
Transported pax.	579	531	546	400
Flight count	6	8	8	8
More expensive competing itineraries				
	High price elasticity		**Low price elasticity**	
	Fixed demand model	Integrated model	Fixed demand model	Integrated model
Profit	32,849	41,657	31,645	40,487
Transported pax.	585	535	579	400
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- When competing itineraries are cheaper, integrated model keeps the prices low to attract passengers.
- When elasticity is lower, integrated model results with higher prices and less transported passengers.

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Standard planes vs Clip-Air - Small data instance

	Standard Fleet	**Clip-Air**
Operating cost	65,635	52,924
Revenue	118,494	143,193
Profit	**52,859**	**81,269**
Transported pax.	**532**	**621**
	124 B, 408 E	132 B, 489 E
Flight count	8	8
Average pax/flight	**66**	**78**
Total Flight Hours (min)	590	590
Used fleet	2 A318	4 wings
	3 ERJ145	7 capsules
Used capacity (seats)	396	350
Running time(min)	0.5	3.5

Standard planes vs Clip-Air - Large data instance

An instance with more fleet types, 18 flights, 1096 passengers for the same OD pairs. Running time considerably increases.

	Standard Fleet	**Clip-Air**
Operating cost	128,080	89,512
Revenue	188,405	198,905
Profit	**60,325**	**109,393**
Transported pax.	**828**	**909**
	183 B, 645 E	191 B, 718 E
Flight count	16	16
Average pax/flight	**52**	**57**
Total Flight Hours (min)	1200	1200
Used fleet	2 A318, 2 A319	5 wings
	1 ERJ135, 3 ERJ145	8 capsules
Used capacity (seats)	**591**	**400**
Running time (min)	2090	1470
Optimality gap	3.2%	1.5%

Heuristic method

▸ Model

- The resulting mixed integer nonlinear problem is highly complex.
- We propose a heuristic method based on Lagrangian relaxation, sub-gradient optimization and a Lagrangian heuristic.
- Seat allocation constraint is relaxed.
- Problem is decomposed into 2 subproblems: revenue maximization and fleet assignment:

$$z_{REV}(\lambda) = Max \sum_{h \in H} \sum_{f \in F} \sum_{s \in S} \sum_{i \in (I_s \setminus I_s')} \delta_{i,f}(p_i^h - \lambda_f^h) \left(d_i^h - \sum_{\substack{j \in I_s \\ i \neq j}} t_{i,j}^h + \sum_{\substack{j \in (I_s \setminus I_s') \\ i \neq j}} t_{j,i}^h b_{j,i}^h \right)$$

$$z_{FAM}(\lambda) = Min \sum_{k \in K} \sum_{f \in F} \left(C_{k,f} x_{k,f} - \sum_{h \in H} \lambda_f^h \pi_{k,f}^h \right)$$

λ_f^h represent the Lagrangian multipliers for each fare class h and flight f.

Conclusions and future work

- Clip-Air
  - Potential increase in transportation capacity and profit
  - Update of the cost figures of Clip-Air
- Integrated scheduling model
  - Further investigation of the effects of the demand model
- Heuristic method
  - Finalization of the implementation
  - Results on the performance of the heuristic

Thank you for your attention !

Spill and recapture effects - Illustration ▸ Back

Information regarding the itineraries in segment ORY-NCE:

OD	fare	nonstop	time
ORY-NCE$_1$	220	1	1
ORY-NCE$_2$	218	1	0
ORY-NCE$_3$	214	1	0
ORY-NCE$'$	250	1	1

Resulting recapture ratios:

	ORY-NCE$_1$	ORY-NCE$_2$	ORY-NCE$_3$	ORY-NCE$'$
ORY-NCE$_1$	0	0.401	0.503	0.096
ORY-NCE$_2$	0.417	0	0.490	0.093
ORY-NCE$_3$	0.463	0.434	0	0.103

Price elasticity of demand

- Price elasticity of logit:

$$(1 - P^h(i))p_i^h \, \beta_{fare}^h$$

- When β_{fare} is -0.05 and -0.025 is for economy and business demand, the elasticities are around -3 and -2.
- When we decrease them to -0.03 and -0.015 elasticity values become -2 and -1.3