Demand based airline scheduling

for a new generation of aircraft

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3ème cycle romand de Recherche Opérationnelle

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Clip-Air concept

- Flexible capacity
- Modular-detachable capsules
- Wing and capsule separation
- Multi-modality
- Passenger and cargo
- Sustainability
  - Gas emissions
  - Noise
  - Accident rates
Objectives

- Analyze the potential performance of Clip-Air by developing appropriate models
- Introduce demand notion in optimization models through appropriate demand models
- Develop solution methodologies for the integrated model
- Investigate multi-modality aspect of Clip-Air
Analysis on the potential performance of Clip-Air

- Compared to a fleet composed of standard planes...
  - Clip-Air carries on the average 5-10% more passengers by using 20-30% less capacity,
  - Clip-Air copes better with the insufficient transportation capacity,
  - Uses less aircraft/wing $\Rightarrow$ less crew, simplified operations.

- The scheduling decisions are robust to the estimated cost figures of Clip-Air. Sensitivity analysis showed that Clip-Air is always better in the number of transported passengers and generates higher profit in 89% of the instances.

- A preliminary analysis is carried out on the multi-modality aspect of Clip-Air where the empty capsules are allowed to be repositioned via railways. The results show a potential increase in profit and transportation capacity.
Integration of demand model

**Motivation**: Demand responsive transportation systems
- Supply $\Rightarrow$ Flexibility provided by Clip-Air
- Demand $\Rightarrow$ integration of appropriate demand models

**Demand model**
- Simple models (e.g. linear, exp.) fail to represent the reality
- Integrated model becomes very sensitive to demand model parameters
- Appropriate models need to be developed
Itinerary choice model

- Itinerary choice among the set of alternatives for each O-D pair
- Utility of each alternative is defined with the relevant variables
- Choice probability is defined by logit
- Estimation of the coefficients of the variables are done with maximum likelihood estimation

**Revealed preferences (RP) data:** Booking data from a major European airline
- Lack of variability
- Price inelastic demand
Introduction  Demand model  Integrated model  Results  Conclusions

Itinerary choice model

- RP data is combined with a **stated preferences (SP)** data
- For each itinerary \( i \) and class \( h \) the utility is defined as follows:

\[
V^h_i = ASC_i + \beta^h_{fare} \cdot \ln(fare^h_i) + \beta^h_{time} \cdot time^h_i + \beta^h_{morning} \cdot morning^h_i
\]

- \( ASC_i \) : alternative specific constant
- \( fare \) and \( time \) are interacted with non-stop/stop
- \( morning \) is a dummy variable and 1 if the itinerary is a morning itinerary

- Time, cost and morning parameters are fixed to be the same for the two datasets.

- A scale parameter is introduced for SP to have equal variances.
Itinerary choice model

- Demand for class $h$ for each itinerary $i$ in market segment $s$:

$$
\hat{d}_i^h = D_s^h \frac{\exp(V_i^h)}{\sum_{j \in I_s} \exp(V_j^h)}
$$

- $D_s^h$ is the total expected demand for class $h$ and segment $s$.

- **Spill and recapture effects**: Capacity shortage $\Rightarrow$ passengers may be recaptured by other itineraries (instead of their desired itineraries)

- Recapture ratio is given by: $b_{i,j}^h = \frac{\exp(V_j^h)}{\sum_{k \in I_s \setminus i} \exp(V_k^h)}$

- **No-revenue** represented by the subset $I_s' \in I_s$ for segment $s$. 
Estimation results for the demand model

<table>
<thead>
<tr>
<th></th>
<th>( \beta_{\text{fare}} )</th>
<th></th>
<th>( \beta_{\text{time}} )</th>
<th></th>
<th>( \beta_{\text{morning}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>non-stop</td>
<td>one-stop</td>
<td>non-stop</td>
<td>one-stop</td>
<td></td>
</tr>
<tr>
<td>economy</td>
<td>-2.23</td>
<td>-2.17</td>
<td>-0.102</td>
<td>-0.0762</td>
<td>0.0283</td>
</tr>
<tr>
<td>business</td>
<td>-1.97</td>
<td>-1.97</td>
<td>-0.104</td>
<td>-0.0821</td>
<td>0.079</td>
</tr>
</tbody>
</table>

**Price and time elasticity of demand:**

\[
E^{P_i}_{\text{price}_i} = \frac{\partial P_i}{\partial \text{price}_i} \cdot \frac{\text{price}_i}{P_i}
\]

For a selected OD pair...
- maximum price elasticity among economy itineraries is \(-2.16\), and \(-1.95\) for business.
- maximum time elasticity among economy itineraries is \(-0.33\), and \(-0.36\) for business.
Estimation results for the demand model

- **Value of time (VOT):**

\[
VOT_i = \frac{\partial V_i / \partial \text{time}_i}{\partial V_i / \partial \text{cost}_i} = \frac{\beta_{\text{time}} \cdot \text{cost}_i}{\beta_{\text{cost}}}
\]

For the same OD pair...

- VOT for economy, non-stop: 8 €/hour
- VOT for economy, one-stop: 19.8, 11, 9.2 €/hour
- VOT for business, non-stop: 21.7 €/hour
Integrated model - Supply part

\[
\begin{align*}
\text{Max} & \quad \sum_{s \in S} \sum_{h \in H} \sum_{i \in (I_S \setminus I_S')} (d_i^h - \sum_{j \in I_S \setminus I_S'} t_{i,j}^h + \sum_{j \in I_S \setminus I_S'} t_{j,i}^h) p_i^h - \sum_{k \in K} \sum_{f \in F} C_{k,f} x_{k,f}: \text{revenue - cost} \\
\text{s.t.} & \quad \sum_{k \in K} x_{k,f} = 1: \text{mandatory flights} \quad \forall f \in F^M \\
& \quad \sum_{k \in K} x_{k,f} \leq 1: \text{optional flights} \quad \forall f \in F^O \\
& \quad y_{k,a,t^-} + \sum_{f \in \text{In}(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f \in \text{Out}(k,a,t)} x_{k,f}: \text{flow conservation} \quad \forall [k,a,t] \in N \\
& \quad \sum_{a \in A} y_{k,a,t_n} + \sum_{f \in CT} x_{k,f} \leq R_k: \text{fleet availability} \quad \forall k \in K \\
& \quad y_{k,a,\min E_a^-} = y_{k,a,\max E_a^+}: \text{cyclic schedule} \quad \forall k \in K, a \in A \\
& \quad \sum_{s \in S} \sum_{i \in (I_S \setminus I_S')} \delta_{i,f} t_{i,j}^h \leq \sum_{f \in F} \sum_{h \in H} \pi_{h,f}^k: \text{capacity} \quad \forall h \in H, f \in F \\
& \quad \pi_{h,f}^k = Q_k x_{k,f}: \text{seat capacity} \quad \forall f \in F, k \in K \\
x_{k,f} \in \{0,1\} \quad \forall k \in K, f \in F \\
y_{k,a,t} \geq 0 \quad \forall [k,a,t] \in N \\
\pi_{h,f}^k \geq 0 \quad \forall h \in H, k \in K, f \in F
\end{align*}
\]
Integrated model - Demand part

\[ \sum_{j \in I_s, i \neq j} t_{i,j}^h \leq d_i^h : \text{total spill} \quad \forall s \in S, h \in H, i \in (I_s \setminus I'_s) \] (12)

\[ \tilde{d}_i^h = D_s^h \frac{\exp(V_i^h)}{\sum_{j \in I_s} \exp(V_j^h)} : \text{logit demand} \quad \forall s \in S, h \in H, i \in I_s \] (13)

\[ b_{i,j}^h = \frac{\exp(V_j^h)}{\sum_{k \in I_s \setminus i} \exp(V_k^h)} : \text{recapture ratio} \quad \forall s \in S, h \in H, i \in (I_s \setminus I'_s), j \in I_s \] (14)

\[ d_i^h \leq \tilde{d}_i^h : \text{realized demand} \quad \forall h \in H, i \in I \] (15)

\[ 0 \leq p_{i,j}^h \leq UB_i^h : \text{upper bound on price} \quad \forall h \in H, i \in I \] (16)

\[ t_{i,j}^h \geq 0 \quad \forall s \in S, h \in H, i \in (I_s \setminus I'_s), j \in I_s \] (17)

\[ b_{i,j}^h \geq 0 \quad \forall s \in S, h \in H, i \in (I_s \setminus I'_s), j \in I_s \] (18)
Model extension for Clip-Air

- Decision variables for the assignment of wing and capsules:
  \[ x_{fw} \in \{0, 1\} \]
  \[ x_{kf} \in \{0, 1\} \text{ for } k \in \{1, 2, 3\} \]

- Operating cost:
  \[
  \sum_{f \in F} C_{fw} x_{fw} + \sum_{k \in K} C_{kf} x_{kf}
  \]

- Constraints:
  \[
  \sum_{k \in K} x_{kf} = 1 \quad \forall f \in F^M: \text{mandatory flights}
  \]
  \[
  \sum_{k \in K} x_{kf} \leq x_{fw} \quad \forall f \in F: \text{capsule - wing}
  \]
Example data instance

Data from a major European airline: set of flights, itineraries, aircraft, airports

<table>
<thead>
<tr>
<th></th>
<th>Fixed demand-price</th>
<th>Integrated model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airports</td>
<td>3 (CDG, MRS, LYS)</td>
<td></td>
</tr>
<tr>
<td>Flights</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>Passengers</td>
<td>1459 (1352 economy, 107 business)</td>
<td></td>
</tr>
<tr>
<td>Capsule capacity</td>
<td>31 seats</td>
<td></td>
</tr>
<tr>
<td>Standard fleet types</td>
<td>ERJ135 (37), ERJ145 (50), A319 (79), BAE300(100), A318 (123)</td>
<td></td>
</tr>
<tr>
<td>Operating cost</td>
<td>160,653</td>
<td>159,774</td>
</tr>
<tr>
<td>Revenue</td>
<td>216,149</td>
<td>233,355</td>
</tr>
<tr>
<td>Profit</td>
<td>55,496</td>
<td>73,581</td>
</tr>
<tr>
<td>Transported pax.</td>
<td>997 (936 E, 61 B)</td>
<td>1006 (948 E, 58 B)</td>
</tr>
<tr>
<td>Flight count</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td>Average pax/flight</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>Total Flight Hours</td>
<td>28h50</td>
<td>28h50</td>
</tr>
<tr>
<td>Used fleet</td>
<td>1 ERJ135, 4 ERJ145, 1 A319</td>
<td>1 ERJ135, 3 ERJ145, 1 A319</td>
</tr>
<tr>
<td>Used capacity</td>
<td>316 seats</td>
<td>266 seats</td>
</tr>
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</table>
Impact of the demand model integration

CDG-LYS pair, total demand 253 (226 business, 27 economy), all non-stop.

<table>
<thead>
<tr>
<th></th>
<th>class</th>
<th>expected demand</th>
<th>morning</th>
<th>Fixed demand-price demand</th>
<th>fixed price</th>
<th>Integrated model demand</th>
<th>realized price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>E</td>
<td>33</td>
<td>0</td>
<td>34</td>
<td>175</td>
<td>37</td>
<td>183</td>
</tr>
<tr>
<td>2</td>
<td>E</td>
<td>33</td>
<td>0</td>
<td>0</td>
<td>175</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>E</td>
<td>34</td>
<td>1</td>
<td>37</td>
<td>175</td>
<td>50</td>
<td>175</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>7</td>
<td>1</td>
<td>10</td>
<td>409.5</td>
<td>7</td>
<td>450</td>
</tr>
<tr>
<td>5</td>
<td>E</td>
<td>34</td>
<td>1</td>
<td>40</td>
<td>175</td>
<td>30</td>
<td>200</td>
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<tr>
<td>6</td>
<td>B</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>409.5</td>
<td>0</td>
<td>450</td>
</tr>
<tr>
<td>7</td>
<td>E</td>
<td>33</td>
<td>0</td>
<td>0</td>
<td>175</td>
<td>0</td>
<td>200</td>
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<tr>
<td>8</td>
<td>B</td>
<td>7</td>
<td>0</td>
<td>7</td>
<td>409.5</td>
<td>7</td>
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<td>E</td>
<td>28</td>
<td>0</td>
<td>29</td>
<td>185</td>
<td>17</td>
<td>185</td>
</tr>
<tr>
<td>11</td>
<td>B</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>425</td>
<td>4</td>
<td>425</td>
</tr>
</tbody>
</table>
## Clip-Air vs Standard planes

<table>
<thead>
<tr>
<th></th>
<th>Standard fleet</th>
<th>Clip-Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating cost</td>
<td>159,774</td>
<td>178,921</td>
</tr>
<tr>
<td>Revenue</td>
<td>233,355</td>
<td>267,957</td>
</tr>
<tr>
<td>Profit</td>
<td>73,581</td>
<td>89,036</td>
</tr>
<tr>
<td>Transported pax.</td>
<td>1006 (948 E, 58 B)</td>
<td>1136 (1069 E, 67 B)</td>
</tr>
<tr>
<td>Flight count</td>
<td>22</td>
<td>26</td>
</tr>
<tr>
<td>Average pax/flight</td>
<td>46</td>
<td>44</td>
</tr>
<tr>
<td>Total Flight Hours</td>
<td>28h50</td>
<td>33h30</td>
</tr>
<tr>
<td>Used fleet</td>
<td>1 ERJ135, 3 ERJ145, 1 A319</td>
<td>9 capsules</td>
</tr>
<tr>
<td></td>
<td>5 aircrafts</td>
<td>6 wings</td>
</tr>
<tr>
<td>Used capacity</td>
<td>266 seats</td>
<td>279 seats</td>
</tr>
<tr>
<td>Running time</td>
<td>2h45</td>
<td>0h19</td>
</tr>
</tbody>
</table>
Conclusions and future work

- Demand model: Challenging with the available data!
- Integration of demand notion: further analysis of the impacts
- Solution methods for the resulting mixed integer nonlinear problem
  - A Lagrangian relaxation based heuristic is already developed
  - Performance of the method should be tested
- Clip-Air
  - There is a clear potential
  - Multi-modality aspect will be further investigated
Thank you for your attention!
Discrete choice analysis

- Finite and discrete set of alternatives
  - Choice of transportation mode: car, bus, etc.
  - Choice of brand: Leonidas, Lindt, Suchard, Toblerone, etc.
  - Choice of flight: GVA-NCE 10:00, GVA-NCE 06:30, etc.
- Individual $n$ associates a utility to alternative $i$
- Represented by a random function

\[ U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + \varepsilon_{in} \]
Discrete choice analysis

Individual \( n \) chooses alternative \( i \) if \( U_{in} \geq U_{jn} \), for all \( j \).

Utility is random, so we have a probabilistic model

\[
P_n(i|C_n) = Pr(U_{in} \geq U_{jn}) = Pr(V_{in} + \varepsilon_{in} \geq V_{jn} + \varepsilon_{jn})
\]

Concrete models require
- specification of \( V_{in} \)
- assumptions about \( \varepsilon_{in} \)
- estimation of the parameters from data