Integrated airline schedule planning with supply-demand interactions

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3ème cycle romand de Recherche Opérationnelle

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Outline

1. Introduction
2. Integrated Airline Scheduling
3. Results
4. Demand model
5. Future Work
Clip-Air concept

- Flexible capacity with modular-detachable capsules
- Carrier and capsule separation: security, maintenance, storage and crew costs
- Multi-modal transportation for both passenger and cargo
- Sustainable transportation
  - Gas emissions
  - Noise
  - Accident rates
Objectives

- Comparative analysis between standard fleet and Clip-Air
  - profit
  - transported passengers
- Integrated schedule design and fleet assignment models
  - maximize revenue - operating costs
  - itinerary-based demand
  - integration of supply and demand interactions
Integrated Airline Scheduling

Considered literature:

- Itinerary based fleet assignment
- Integration of demand modeling
Integrated schedule design and fleet assignment model

- Schedule Design: Set of mandatory and optional flights
- Schedule is represented by time-space network
- Cyclic schedule with a period of 1 day
- Single airline
- Supply-demand interaction: demand function as a function of price
  - linear
  - exponential
  - piecewise linear
Model

Max \( \sum_{i \in I} d_i(p) p_i - \sum_{k \in K, f \in F} C_{k,f} x_{k,f} \)

s.t. \( \sum_{k \in K} x_{k,f} = 1 \quad \forall f \in F^M \)

\( \sum_{k \in K} x_{k,f} \leq 1 \quad \forall f \in F^O \)

\( y_{k,a,t^-} + \sum_{f \in I(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f \in O(k,a,t)} x_{k,f} \quad \forall [k,a,t] \in N \)

\( \sum_{a \in A} y_{k,a,t^-} + \sum_{f \in CT} x_{k,f} \leq R_k \quad \forall k \in K \)

\( y_{k,a,minE^-_a} = y_{k,a,maxE^+_a} \quad \forall k \in K, a \in A \)

\( \sum_{i \in I} \delta_{f_i} d_i(p) \leq \sum_{k \in K} s_k x_{k,f} \quad \forall f \in F \)

\( x_{k,f} \in \{0, 1\} \quad \forall k \in K, f \in F \)

\( y_{k,a,t} \geq 0 \quad \forall (k,a,t) \in N \)

\( d_i(p) \geq 0 \quad p_i \geq 0 \quad \forall i \in I \)
Model

\[
\begin{align*}
\text{Max} & \sum_{i \in I} d_i(p) p_i - \sum_{k \in K, f \in F} C_{k,f} x_{k,f} : \text{revenue - operating cost} \\
\text{s.t.} & \sum_{k \in K} x_{k,f} = 1: \text{mandatory flights} \quad \forall f \in F^M \\
& \sum_{k \in K} x_{k,f} \leq 1: \text{optional flights} \quad \forall f \in F^O \\
& y_{k,a,t^{-}} + \sum_{f \in I(k,a,t)} x_{k,f} = y_{k,a,t^{+}} + \sum_{f \in O(k,a,t)} x_{k,f} : \text{flow conservation} \quad \forall [k,a,t] \in N \\
& \sum_{a \in A} y_{k,a,t_{n}} + \sum_{f \in CT} x_{k,f} \leq R_k : \text{fleet availability} \quad \forall k \in K \\
& y_{k,a,minE_{a}^{-}} = y_{k,a,maxE_{a}^{+}} : \text{cyclic schedule} \quad \forall k \in K, a \in A \\
& \sum_{i \in I} \delta_i^f d_i(p) \leq \sum_{k \in K} s_k x_{k,f} : \text{fleet capacity} \quad \forall f \in F \\
& x_{k,f} \in \{0, 1\} \quad \forall k \in K, f \in F \\
& y_{k,a,t} \geq 0 \quad \forall (k, a, t) \in N \\
& d_i(p) \geq 0 \quad p_i \geq 0 \quad \forall i \in I
\end{align*}
\]
Model - Clip-Air extension

\[ x_f^w \in \{0,1\} \quad \text{: allocation of wing} \]
\[ x_{k,f} \in \{0,1\} \quad \forall k \in \{1,2,3\} : \text{allocation of capsules} \]
\[ x_f^w = 1 \quad \forall f \in F^M : \text{mandatory coverage} \]
\[ \sum_k x_{k,f} \leq x_f^w \quad \forall f \in F : \text{wing-capsule relation} \]
Realized demand

- Realized demand is limited by both...
  - demand modeling
  - supply decisions
- Embedding the demand model directly into the supply model is not feasible.
- Definition of an additional variable, \textit{realized demand}, is needed which represents the actual number of passengers traveling. Therefore:
  - Demand modeling imposes a demand \( d_i \) which is an upper bound for the realized demand.
  - Scheduling model deals with realized demand \( \tilde{d}_i \) which is \( \leq d_i \)
Linear demand function

Objective function becomes quadratic

\[ d_i(p) = a_i + b_i p_i \quad \forall i \in I \]

Parameters of the demand function are estimated by simple linear regression for the origin-destination pairs.

As another explanatory variable, departure time of the itinerary is used.

\[ d_i(p) = a_i + b_i p_i + c_i t_i \quad \forall i \in I \]
Exponential demand function

\[ d_i(p) = \exp(a_i + b_i \cdot p_i) \quad \forall i \in I \]

Piecewise linear approximation:

\[ d_i(p) = a_i + b_i \min(p_i, p_i^r) + c_i \max(p_i - p_i^r, 0) \quad \forall i \in I, \]

Linearization:

\[ d_i \leq a_i + b_i \cdot p_i + M \cdot \lambda_i, \]
\[ d_i \leq a_i + p_i^r \cdot (b_i - c_i) + c_i \cdot p_i + M \cdot (1 - \lambda_i). \]

where \( \lambda \) is a binary variable determining which line segment is active.
Results

- Input: data from a major European airline company (ROADEF Challenge)
  - set of optional and mandatory flights
  - set of airports
  - set of itinerary demands and fares
  - set of aircrafts for the standard fleet
- Problem resolution with AMPL+BONMIN solver
- Output: an optimized schedule design, fleet assignment and pricing for the given instances
Small data instance

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Airports</td>
<td>2</td>
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<tr>
<td>Flights</td>
<td>4</td>
</tr>
<tr>
<td>Itineraries</td>
<td>4</td>
</tr>
<tr>
<td>Capsule capacity</td>
<td>56</td>
</tr>
<tr>
<td>Passengers</td>
<td>454</td>
</tr>
<tr>
<td>Total fleet size (seats)</td>
<td>392</td>
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</table>
Approximations
## Results - small data instance

<table>
<thead>
<tr>
<th></th>
<th>Base Model</th>
<th>Linear demand fct.</th>
<th>Linear demand - time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Std. Fleet</td>
<td>Clip-Air</td>
<td>Std. Fleet</td>
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<tr>
<td>Operating cost</td>
<td>24,756</td>
<td>47,372</td>
<td>24,756</td>
</tr>
<tr>
<td>Revenue</td>
<td>36,288</td>
<td>66,906</td>
<td>47,854</td>
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<tr>
<td>Profit</td>
<td>11,532</td>
<td>19,534</td>
<td>23,098</td>
</tr>
<tr>
<td>Total pax</td>
<td>224</td>
<td>413</td>
<td>224</td>
</tr>
<tr>
<td>Avg. pax/flight</td>
<td>56</td>
<td>103</td>
<td>56</td>
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<table>
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<tr>
<th></th>
<th>Exp. demand fct.</th>
<th>Piecewise linear</th>
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<tr>
<td>Operating cost</td>
<td>24,756</td>
<td>47,372</td>
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<tr>
<td>Revenue</td>
<td>40,808</td>
<td>63,702</td>
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<tr>
<td>Profit</td>
<td>16,052</td>
<td>16,330</td>
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<tr>
<td>Total pax</td>
<td>224</td>
<td>413</td>
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Evaluation on results

- Results are very sensitive to the assumptions regarding the demand model
- There is a need for a more reliable demand modeling
- Inclusion of other explanatory variables
Demand model specification

- **Variables**
  - fare
  - time of day
  - number of stops

- **Type of model**
  - linear
  - logit

\[ V_{i}^{OD} = \beta_{OD} + \beta_{\text{fare}} fare_i + \beta_{\text{time of day}} + \beta_{\text{stops}} stops_i \]
Identification issues

- Aggregate data
- Lack of variability
Use of published models

- The results from models in literature can be used
  - Willingness to pay for time of day, number of stops can be taken
  - Coefficient for fare is estimated with the data
- Validation should be done through sensitivity analysis
Future work

- More reliable demand model
- Time of the itinerary can be a decision variable in the context of re-timing in a time-windows.
- Spill and recapture
  - When there is not enough capacity on the desired itinerary of passengers, they can be redirected to alternative itineraries.
  - The portion of redirected passengers which actually accepted the offer needs to be estimated.
  - To what extend spill is effective
Thanks

Any question?
Results - large data instance

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<td>Base Model</td>
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<td>45</td>
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<td>3511</td>
<td>858</td>
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<tr>
<td>Linear demand fct.</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std. Fleet</td>
<td>357,725</td>
<td>367,621</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std. Fleet</td>
<td>345,341</td>
<td>503,174</td>
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<td>157,833</td>
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<tr>
<td>Clip-Air</td>
<td>412,248</td>
<td>580,487</td>
<td></td>
<td>168,239</td>
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<td>357,725</td>
<td>367,621</td>
<td>299,621</td>
</tr>
<tr>
<td>Revenue</td>
<td>532,189</td>
<td>558,322</td>
<td>532,799</td>
</tr>
<tr>
<td>Profit</td>
<td>174,464</td>
<td>190,701</td>
<td>233,178</td>
</tr>
<tr>
<td>Total pax</td>
<td>2,954</td>
<td>3,122</td>
<td>2,226</td>
</tr>
<tr>
<td>Avg. pax/flight</td>
<td>74</td>
<td>78</td>
<td>52</td>
</tr>
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Clip-Air: linear and exponential functions take 75 and 19 hours respectively.
Standard fleet: optimality gap is set to 2% and running time is around 10 hours.