Integrated airline schedule planning with supply-demand interactions

for a new generation of aircrafts

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Motivation

- Increased air travel demand
- Demand responsiveness
  - Flexible supply capacity
  - Improved demand management
- Sustainability

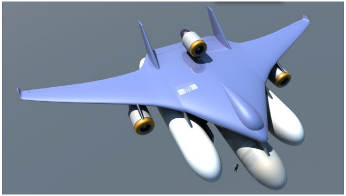
Clip-Air concept

Flexibility in transportation...

- Modular capacity with detachable capsules
  - security, maintenance, storage and crew costs
- Multi-modality for passenger and cargo
- Robustness
- Demand management

Sustainable transportation

- Gas emissions, noise, accident rates



- Exists in a simulated environment

Objectives

- Comparative analysis between standard fleet and Clip-Air
- Development of integrated schedule design and fleet assignment model
  - integration of supply-demand interactions
    - logit demand model $\Rightarrow$ pricing
    - spill and recapture effects
  - Fare-class segmentation
    - demand model for each segment
    - seat allocation for business and economy
- Solution techniques for the resulting mixed integer nonlinear problem

Demand model for itinerary choice

- Utility of itinerary i, class h:

$$V_i^h = \beta_{fare}^h p_i^h + \beta_{time}^h time_i + \beta_{stops}^h nonstop_i$$

  - p_i^h is the price of itinerary i for class h.
  - $time_i$, binary variable, 1 if departure time is between 07:00-11:00.
  - $nonstop_i$, binary variable, 1 if it is a non-stop itinerary.

- Demand for class h for each itinerary i in market segment s:

$$\tilde{d}_i^h = D_s^h \frac{\exp(V_i^h)}{\sum_{j \in I_s} \exp(V_j^h)}$$

  - D_s^h is the total expected demand for class h and segment s.
  - $\tilde{d}_i^h$ serves as an upper bound for the actual demand.

Spill and recapture effects

Example

- In case of capacity shortage some passengers may not fly on their desired itineraries
- They may accept to fly on other available itineraries in the same market segment
- Recapture ratio is given by:

$$b_{i,j}^{h} = \frac{\exp(V_j^h)}{\sum_{k \in I_s \setminus i} \exp(V_k^h)}$$

- *No-revenue* represented by the subset $I_s^{'} \in I_s$ for segment s.

Integrated model - Supply part ▸ H

$$Max \sum_{s\in S}\sum_{h\in H}\sum_{i\in(I_s\setminus I_s^{'})}(d_i^h - \sum_{\substack{j\in I_s\\ i\neq j}} t_{i,j}^h + \sum_{\substack{j\in(I_s\setminus I_s^{'})\\ i\neq j}} t_{j,i}^h b_{j,i}^h)p_i^h - \sum_{\substack{k\in K\\ f\in F}} C_{k,f}x_{k,f}: \text{ revenue - cost} \quad (1)$$

$$s.t. \sum_{k\in K} x_{k,f} = 1: \text{ mandatory flights} \quad \forall f\in F^M \quad (2)$$

$$\sum_{k\in K} x_{k,f} \leq 1: \text{ optional flights} \quad \forall f\in F^O \quad (3)$$

$$y_{k,a,t^-} + \sum_{f\in In(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in Out(k,a,t)} x_{k,f}: \text{ flow conservation} \quad \forall [k,a,t]\in N \quad (4)$$

$$\sum_{a\in A} y_{k,a,t_n} + \sum_{f\in CT} x_{k,f} \leq R_k: \text{ fleet availability} \quad \forall k\in K \quad (5)$$

$$y_{k,a,minE_a^-} = y_{k,a,maxE_a^+}: \text{ cyclic schedule} \quad \forall k\in K, a\in A \quad (6)$$

$$\sum_{s\in S}\sum_{i\in(I_s\setminus I_s^{'})} \delta_{i,f} d_i^h - \sum_{\substack{j\in I_s\\ i\neq j}} \delta_{i,f} t_{i,j}^h + \sum_{\substack{j\in(I_s\setminus I_s^{'})\\ i\neq j}} \delta_{i,f} t_{j,i}^h b_{j,i}^h \leq \sum_{k\in K} \pi_{k,f}^h: \text{ capacity} \quad \forall h\in H, f\in F \quad (7)$$

$$\sum_{h\in H} \pi_{k,f}^h = Q_k x_{k,f}: \text{ seat capacity} \quad \forall f\in F, k\in K \quad (8)$$

$$x_{k,f} \in \{0,1\} \quad \forall k\in K, f\in F \quad (9)$$

$$y_{k,a,t} \geq 0 \quad \forall [k,a,t]\in N \quad (10)$$

$$\pi_{k,f}^h \geq 0 \quad \forall h\in H, k\in K, f\in F \quad (11)$$

Integrated model - Demand part ▸ H

$$\sum_{\substack{j \in I_s \\ i \neq j}} t_{i,j}^h \leq d_i^h : \textit{ total spill} \qquad \forall s \in S, h \in H, i \in (I_s \setminus I_s^{'}) \qquad (12)$$

$$\tilde{d}_i^h = D_s^h \frac{\exp(V_i^h)}{\sum_{j \in I_s} \exp(V_j^h)} : \textit{ logit demand} \qquad \forall s \in S, h \in H, i \in I_s \qquad (13)$$

$$b_{i,j}^h = \frac{\exp(V_j^h)}{\sum_{k \in I_s \setminus i} \exp(V_k^h)} : \textit{ recapture ratio} \qquad \forall s \in S, h \in H, i \in (I_s \setminus I_s^{'}), j \in I_s \qquad (14)$$

$$d_i^h \leq \tilde{d}_i^h \leq D_i^h : \textit{ realized demand} \qquad \forall h \in H, i \in I \qquad (15)$$

$$0 \leq p_i^h \leq UB_i^h : \textit{ upper bound on price} \qquad \forall h \in H, i \in I \qquad (16)$$

$$t_{i,j}^h \geq 0 \qquad \forall s \in S, h \in H, i \in (I_s \setminus I_s^{'}), j \in I_s \qquad (17)$$

$$b_{i,j}^h \geq 0 \qquad \forall s \in S, h \in H, i \in (I_s \setminus I_s^{'}), j \in I_s \qquad (18)$$

Model extension for Clip-Air

- Decision variables for the assignment of wing and capsules:
  $x_f^w \in \{0,1\}$
  $x_{k,f} \in \{0,1\}$ for $k \in \{1,2,3\}$
- Operating cost:

$$\sum_{f \in F} C_f^w x_f^w + \sum_{k \in K} C_{k,f} x_{k,f}$$

- Constraints:

$$\sum_{k \in K} x_{k,f} = 1 \qquad \forall f \in F^M\text{: } \textit{mandatory flights}$$

$$\sum_{k \in K} x_{k,f} \leq x_f^w \qquad \forall f \in F\text{: } \textit{capsule - wing}$$

Results

- Dataset from a major European airline
- Other inputs:
  - Cost figures for Clip-Air
    - Weight differences => adjustment of fuel cost and airport and air navigation charges
    - Capsule wing separation => adjustment of crew cost
  - Parameters of the demand model
- Model is implemented in AMPL and solved with BONMIN
- Results provide the schedule design, fleet assignment, seat allocation and pricing.

Demand model parameters

- Estimation of logit model parameters by maximum likelihood estimation using BIOGEME
- Booking data does not have the non-chosen alternatives ⇒ lack of variability
- Adjusted parameters to have enough elasticity

	Business demand	**Economy demand**
β_{fare}	*-0.025*	*-0.050*
β_{time}	0.323	0.139
$\beta_{nonstop}$	1.150	0.900

Standard planes vs Clip-Air

An instance with 18 flights and 1096 passengers:

	Standard Fleet	**Clip-Air**
Operating cost	107,560	89,512
Revenue	185,835	200,199
Profit	**78,275**	**110,687**
Transported pax.	**817**	**909**
	184 B, 633 E	192 B, 717 E
Flight count	16	16
Average pax/flight	**51**	**57**
Total Flight Hours (min)	1200	1200
Used fleet	2 A319, 1 ERJ135	5 wings
	3 ERJ145	8 capsules
Used aircraft	**6**	**5**
Used capacity (seats)	345	400
Running time (min)	33.89	31.72

- More passengers
- Less aircraft ⇒ less flight crew

Impacts of the demand model - Different scenarios

	High price elasticity		Low price elasticity	
Cheaper competing itineraries				
	Fixed demand model	Integrated model	Fixed demand model	Integrated model
Profit	30,966	23,141	31,250	17,159
Transported pax.	541	400	543	499
Flight count	8	8	8	8
Comparable competing itineraries				
	High price elasticity		**Low price elasticity**	
	Fixed demand model	Integrated model	Fixed demand model	Integrated model
Profit	31,660	36,862	31,617	36,484
Transported pax.	579	531	546	400
Flight count	6	8	8	8
More expensive competing itineraries				
	High price elasticity		**Low price elasticity**	
	Fixed demand model	Integrated model	Fixed demand model	Integrated model
Profit	32,849	41,657	31,645	40,487
Transported pax.	585	535	579	400
Flight count	6	8	6	8

- When competing itineraries are cheaper, integrated model keeps the prices low to attract passengers.
- When elasticity is lower, integrated model results with higher prices and less transported passengers.

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Heuristic method

Model

- The resulting mixed integer nonlinear problem is highly complex.
- We propose a heuristic method based on Lagrangian relaxation, sub-gradient optimization and a Lagrangian heuristic.
- *Capacity* constraint is relaxed.
- Problem is decomposed into 2 subproblems: revenue maximization and fleet assignment:

$$z_{REV}(\lambda) = Max \sum_{h \in H} \sum_{f \in F} \sum_{s \in S} \sum_{i \in (I_s \setminus I'_s)} \delta_{i,f}(p_i^h - \lambda_f^h) \left(d_i^h - \sum_{\substack{j \in I_s \\ i \neq j}} t_{i,j}^h + \sum_{\substack{j \in (I_s \setminus I'_s) \\ i \neq j}} t_{j,i}^h b_{j,i}^h \right)$$

$$z_{FAM}(\lambda) = Min \sum_{k \in K} \sum_{f \in F} \left(C_{k,f} x_{k,f} - \sum_{h \in H} \lambda_f^h \pi_{k,f}^h \right)$$

Lagrangian procedure

Require: z_{LB}, $\bar{k}$, $\bar{j}$, ε

$\lambda^0 := 0$, $k := 0$, $z_{UB} := \infty$

repeat

$\{\bar{d},\bar{t},\bar{b}\} :=$ solve $z_{REV}(\lambda^k)$, $\{\bar{x},\bar{y},\bar{\pi}\} :=$ solve $z_{FAM}(\lambda^k)$

$z_{UB}(\lambda^k) := z_{REV}(\lambda^k) - z_{FAM}(\lambda^k)$

$z_{UB} := \min(z_{UB}, z_{UB}(\lambda^k))$

loop

$\{\bar{x},\bar{\pi}\} :=$ Local search($\{\bar{x},\bar{\pi}\}$)

$lb :=$ Lagrangian heuristic ($\{\bar{x},\bar{\pi}\}$)

end loop

$z_{LB} := \max(z_{LB}, lb)$

$G :=$ compute sub-gradient($z_{UB}, z_{LB}, \{\bar{d},\bar{t},\bar{b},\bar{x},\bar{y},\bar{\pi}\}$)

$T :=$ compute step($z_{UB}, z_{LB}, \{\bar{d},\bar{t},\bar{b},\bar{x},\bar{y},\bar{\pi}\}$)

$\lambda^{k+1} := \max(0, \lambda^k - TG)$

$k := k+1$

until $||TG||^2 \leq \varepsilon$ **or** $k \geq \bar{k}$

Lagrangian procedure

Require: z_{LB}, $\bar{k}$, $\bar{j}$, ε

$\lambda^0 := 0$, $k := 0$, $z_{UB} := \infty$

repeat

$\{\bar{d},\bar{t},\bar{b}\}$:= solve $z_{REV}(\lambda^k)$, $\{\bar{x},\bar{y},\bar{\pi}\}$:= solve $z_{FAM}(\lambda^k)$

$z_{UB}(\lambda^k) := z_{REV}(\lambda^k) - z_{FAM}(\lambda^k)$

$z_{UB} := \min(z_{UB}, z_{UB}(\lambda^k))$ **update UPPER BOUND**

loop

$\{\bar{x},\bar{\pi}\}$:= Local search($\{\bar{x},\bar{\pi}\}$)

lb := Lagrangian heuristic ($\{\bar{x},\bar{\pi}\}$)

end loop

$z_{LB} := \max(z_{LB}, lb)$

G := compute sub-gradient($z_{UB}, z_{LB}, \{\bar{d},\bar{t},\bar{b},\bar{x},\bar{y},\bar{\pi}\}$)

T := compute step($z_{UB}, z_{LB}, \{\bar{d},\bar{t},\bar{b},\bar{x},\bar{y},\bar{\pi}\}$)

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$z_{UB} := \min(z_{UB}, z_{UB}(\lambda^k))$

loop

$\{\bar{x},\bar{\pi}\} :=$ Local search$(\{\bar{x},\bar{\pi}\})$ **based on λ's under a Tabu mechanism**

$lb :=$ Lagrangian heuristic $(\{\bar{x},\bar{\pi}\})$

end loop

$z_{LB} := \max(z_{LB}, lb)$

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until $||TG||^2 \leq \varepsilon$ **or** $k \geq \bar{k}$

TRANSP-OR

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

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$z_{UB}(\lambda^k) := z_{REV}(\lambda^k) - z_{FAM}(\lambda^k)$

$z_{UB} := \min(z_{UB}, z_{UB}(\lambda^k))$

loop

$\{\bar{x}, \bar{\pi}\} :=$ Local search$(\{\bar{x}, \bar{\pi}\})$

$lb :=$ Lagrangian heuristic $(\{\bar{x}, \bar{\pi}\})$ **a feasible solution**

end loop

$z_{LB} := \max(z_{LB}, lb)$

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loop

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$lb :=$ Lagrangian heuristic ($\{\bar{x},\bar{\pi}\}$)

end loop

$z_{LB} := \max(z_{LB}, lb)$ **update LOWER BOUND**

$G :=$ compute sub-gradient($z_{UB}, z_{LB}, \{\bar{d},\bar{t},\bar{b},\bar{x},\bar{y},\bar{\pi}\}$)

$T :=$ compute step($z_{UB}, z_{LB}, \{\bar{d},\bar{t},\bar{b},\bar{x},\bar{y},\bar{\pi}\}$)

$\lambda^{k+1} := \max(0, \lambda^k - TG)$

$k := k+1$

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TRANSP-OR

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$\lambda^{k+1} := \max(0, \lambda^k - TG)$ **update λ's**

$k := k+1$

until $||TG||^2 \leq \varepsilon$ **or** $k \geq \bar{k}$

Performance of the heuristic

	BONMIN solver		Heuristic		
Instances	opt solution	time(min)	best solution	GAP	time(min)
9 flights. 800 pax.	52,876	0.24	52,876	0%	0.07
18 flights 1096 pax.	78,275	41.04	77,126	1.47%	20.49
26 flights 2329 pax.	176,995	204.56	169,913	4.00%	39.27

	BONMIN solver			Heuristic		
Instances	best solution	GAP	time(h)	best solution	GAP	time(h)
41 flights 3430 pax.	300,949	3.33%	15.01	278,375	10.48%	5.51

Conclusions and future work

- Clip-Air
  - Potential increase in transportation capacity and profit
  - A system level consideration
    - Repositioning of Clip-Air capsules

- Integrated scheduling model
  - Further investigation of the effects of the demand model
- Heuristic method
  - Improvement of the solutions
  - Test of the heuristic on a comprehensive test set

Thank you for your attention !

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Spill and recapture effects - Illustration ▸ Back

Information regarding the itineraries in segment ORY-NCE:

OD	fare	nonstop	time
$ORY\text{-}NCE_1$	220	1	1
$ORY\text{-}NCE_2$	218	1	0
$ORY\text{-}NCE_3$	214	1	0
$ORY\text{-}NCE'$	250	1	1

Resulting recapture ratios:

	$ORY\text{-}NCE_1$	$ORY\text{-}NCE_2$	$ORY\text{-}NCE_3$	$ORY\text{-}NCE'$
$ORY\text{-}NCE_1$	0	0.401	0.503	0.096
$ORY\text{-}NCE_2$	0.417	0	0.490	0.093
$ORY\text{-}NCE_3$	0.463	0.434	0	0.103

Price elasticity of demand

- Price elasticity of logit:

$$(1 - P^h(i))p_i^h\ \beta_{fare}^h$$

- When β_{fare} is -0.05 and -0.025 is for economy and business demand, the elasticities are around -3 and -2.
- When we decrease them to -0.03 and -0.015 elasticity values become -2 and -1.3