An integrated fleet assignment model with supply-demand interactions

Bilge Atasoy
Michel Bierlaire
Matteo Salani

25th European Conference on Operational Research

July 09, 2012
Motivation

- Demand responsive transportation systems
  - Better representation of demand ⇒ Appropriate demand models
  - Flexibility in supply ⇒ New concept: Clip-Air
  - Integration of supply-demand interactions in transportation models
Itinerary choice model

- Market segments, $s$, defined by the class and each OD pair
- Itinerary choice among the set of alternatives, $I_s$, for each segment $s$
- For each itinerary $i \in I_s$ the utility is defined by:

$$V_i = ASC_i + \beta_p \cdot \ln(p_i) + \beta_{time} \cdot time_i + \beta_{morning} \cdot morning_i$$

$$V_i = V_i(p_i, z_i, \beta)$$

- $ASC_i$: alternative specific constant
- $p$ is a policy variable and included as log
- $p$ and $time$ are interacted with non-stop/stop
- morning is 1 if the itinerary is a morning itinerary

- No-revenue represented by the subset $I'_s \in I_s$ for segment $s$.  
Itinerary choice model

- Demand for class $h$ for each itinerary $i$ in market segment $s$:

\[
\tilde{d}_i = D_s \frac{\exp(V_i(p_i, z_i, \beta))}{\sum_{j \in I_s} \exp(V_j(p_j, z_j, \beta))}
\]

- $D_s$ is the total expected demand for market segment $s$.

- **Spill and recapture effects**: Capacity shortage $\Rightarrow$ passengers may be recaptured by other itineraries (instead of their desired itineraries)

- Recapture ratio is given by:

\[
b_{i,j} = \frac{\exp(V_j(p_j, z_j, \beta))}{\sum_{k \in I_s \setminus \{i\}} \exp(V_k(p_k, z_k, \beta))}
\]
Estimation

- **Revealed preferences (RP) data**: Booking data from a major European airline
  - Lack of variability
  - Price inelastic demand

- RP data is combined with a **stated preferences (SP) data**

- Time, cost and morning parameters are **fixed** to be the same for the two datasets.

- A **scale** parameter is introduced for SP to capture the differences in variance.
Estimation results

<table>
<thead>
<tr>
<th></th>
<th>$\beta_{fare}$</th>
<th></th>
<th>$\beta_{time}$</th>
<th></th>
<th>$\beta_{morning}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>non-stop</td>
<td>one-stop</td>
<td>non-stop</td>
<td>one-stop</td>
<td></td>
</tr>
<tr>
<td>economy</td>
<td>-2.23</td>
<td>-2.17</td>
<td>-0.102</td>
<td>-0.0762</td>
<td>0.0283</td>
</tr>
<tr>
<td>business</td>
<td>-1.97</td>
<td>-1.97</td>
<td>-0.104</td>
<td>-0.0821</td>
<td>0.079</td>
</tr>
</tbody>
</table>

- **Price elasticity** of demand:

  $$E_{price_i}^P = \frac{\partial P_i}{\partial price_i} \cdot \frac{price_i}{P_i}$$

An example
- for a non-stop itinerary
  - price elasticity for economy is $-2.03$ and $-1.86$ for business
- for a one-stop itinerary
  - price elasticity for economy is $-2.14$ and $-1.95$ for business
Integrated schedule planning and revenue management

Schedule planning
- Schedule design
  - Mandatory flights
  - Optional flights

Fleet assignment

Revenue management

Pricing-demand
- Spill-recapture

Capacity allocation
- Business seats
- Economy seats
Integrated model - Schedule planning

\[ \text{Max } \sum_{h \in H} \sum_{s \in S^h} \sum_{i \in (I_s \setminus I'_s)} (d_i - \sum_{j \in I_s} t_{i,j} + \sum_{j \in (I_s \setminus I'_s)} t_{j,i} b_{j,i}) p_i - \sum_{k \in K} C_{k,f} x_{k,f} : \text{revenue - cost} \]  

(1)

\[ \sum_{k \in K} x_{k,f} = 1: \text{mandatory flights} \]  

(2)

\[ \sum_{k \in K} x_{k,f} \leq 1: \text{optional flights} \]  

(3)

\[ y_{k,a,t^-} + \sum_{f \in \text{In}(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f \in \text{Out}(k,a,t)} x_{k,f} : \text{flow conservation} \]  

(4)

\[ \sum_{a \in A} y_{k,a,minE^-_a} + \sum_{f \in \text{CT}} x_{k,f} \leq R_k: \text{fleet availability} \]  

(5)

\[ y_{k,a,minE^-_a} = y_{k,a,maxE^+_a} : \text{cyclic schedule} \]  

(6)

\[ \sum_{h \in H} \pi^h_{k,f} = Q_k x_{k,f} : \text{seat capacity} \]  

(7)

\[ x_{k,f} \in \{0,1\} \]  

(8)

\[ y_{k,a,t} \geq 0 \]  

(9)
Integrated model - Revenue management

$$\sum_{s \in S^h} \sum_{i \in (I_s \setminus I_s')} \delta_{i,f} d_i - \sum_{j \in I_s} \delta_{i,f} t_{i,j} + \sum_{j \in (I_s \setminus I_s')} \delta_{i,f} t_{j,i} b_{j,i} \leq \sum_{k \in K} \pi_{k,f} : \text{capacity}$$ \quad \forall h \in H, f \in F \quad (10)

$$\sum_{j \in I_s \setminus i \neq j} t_{i,j} \leq d_i : \text{total spill}$$ \quad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s') \quad (11)

$$\tilde{d}_i = D_s \frac{\exp(V_i(p_i,z_i,\beta))}{\sum_{j \in I_s} \exp(V_j(p_j,z_j,\beta))} : \text{logit demand}$$ \quad \forall h \in H, s \in S^h, i \in I_s \quad (12)

$$b_{i,j} = \frac{\exp(V_j(p_j,z_j,\beta))}{\sum_{k \in I_s \setminus \{i\}} \exp(V_k(p_k,z_k,\beta))} : \text{recapture ratio}$$ \quad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s'), j \in I_s \quad (13)

$$d_i \leq \tilde{d}_i : \text{realized demand}$$ \quad \forall h \in H, s \in S^h, i \in I_s \quad (14)

$$0 \leq p_i \leq UB_i : \text{upper bound on price}$$ \quad \forall h \in H, s \in S^h, i \in I_s \quad (15)

$$t_{i,j} \geq 0$$ \quad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s'), j \in I_s \quad (16)

$$b_{i,j} \geq 0$$ \quad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s'), j \in I_s \quad (17)

$$\pi^h_{k,f} \geq 0$$ \quad \forall h \in H, k \in K, f \in F \quad (18)
Integrated model

- We consider reference models to evaluate the integrated model
  - **Price-inelastic schedule planning**: M. Lohatepanont and C. Barnhart (2004)
  - **Sequential approach**: Revenue management considers fixed supply capacity

- The resulting model is a mixed integer nonlinear problem
- Nonlinearity is due to the explicit supply-demand interactions
- The model is implemented in AMPL and BONMIN solver is used
- BONMIN does not guarantee optimality
Illustration

Change of the market share

![Graph showing the change of market share for different itineraries. The x-axis represents the price of itinerary 1, ranging from 0 to 1500, and the y-axis represents the market share of each itinerary, ranging from 0 to 1.4. The graph includes four lines representing different itineraries: itinerary 1 (blue), itinerary 2 (red), itinerary 3 (green), and itinerary 4 (purple). Each itinerary line shows a different trend in market share change as the price increases.](image-url)
Illustration

Change of the revenue

Revenue

Thousands

price of itinerary 1

itinerary 1
itinerary 2
itinerary 3
itinerary 4
total
Sequential versus integrated

<table>
<thead>
<tr>
<th></th>
<th>Sequential approach</th>
<th>Integrated model - % Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Profit</td>
<td>Pax.</td>
</tr>
<tr>
<td>1</td>
<td>15,091</td>
<td>284</td>
</tr>
<tr>
<td>2</td>
<td>35,372</td>
<td>400</td>
</tr>
<tr>
<td>3</td>
<td>50,149</td>
<td>859</td>
</tr>
<tr>
<td>4</td>
<td>43,990</td>
<td>882</td>
</tr>
<tr>
<td>5</td>
<td>69,901</td>
<td>931</td>
</tr>
<tr>
<td>6</td>
<td>82,311</td>
<td>1,145</td>
</tr>
<tr>
<td>7</td>
<td>84,186</td>
<td>1,131</td>
</tr>
<tr>
<td>8</td>
<td>904,054</td>
<td>1,448</td>
</tr>
<tr>
<td>9</td>
<td>135,656</td>
<td>1,814</td>
</tr>
<tr>
<td>10</td>
<td>115,983</td>
<td>2,236</td>
</tr>
<tr>
<td>11</td>
<td>93,920</td>
<td>2,270</td>
</tr>
<tr>
<td>12</td>
<td>854,902</td>
<td>1,270</td>
</tr>
<tr>
<td>13</td>
<td>27,076</td>
<td>448</td>
</tr>
<tr>
<td>14</td>
<td>52,369</td>
<td>599</td>
</tr>
<tr>
<td>15</td>
<td>51,160</td>
<td>793</td>
</tr>
<tr>
<td>16</td>
<td>37,100</td>
<td>1,067</td>
</tr>
<tr>
<td>17</td>
<td>137,428</td>
<td>1,517</td>
</tr>
<tr>
<td>18</td>
<td>93,347</td>
<td>1,144</td>
</tr>
<tr>
<td>19</td>
<td>83,251</td>
<td>1,104</td>
</tr>
</tbody>
</table>
Heuristic method

- We are limited in terms of the computational time
- A heuristic based on two simplified versions of the model:
  - \( \text{FAM}^{LS} \): price-inelastic schedule planning model \( \Rightarrow \) MILP
    - Explores new fleet assignment solutions based on a local search
    - Price sampling
    - Variable neighborhood search
  - \( \text{REV}^{LS} \): Revenue management with fixed capacity \( \Rightarrow \) NLP
    - Optimizes the revenue for the explored fleet assignment solution
Heuristic method

Require: $\bar{x}_0, \bar{y}_0, \bar{d}_0, \bar{p}_0, \bar{t}_0, \bar{b}_0, \bar{\pi}_0, z^*, z_{opt}, k_{max}, \varepsilon, n_{min}, n_{max}$

$k := 0, n_{fixed} := n_{min}$

repeat

$\bar{p}_k := \text{Price sampling}$

$\{\bar{d}_k, \bar{b}_k\} := \text{Demand model}(\bar{p}_k)$

$\{\bar{x}_k, \bar{y}_k, \bar{\pi}_k, \bar{t}_k\} := \text{solve} \ z_{FAMLS}(\bar{d}_k, \bar{b}_k, n_{fixed})$

$\{\bar{p}_k, \bar{d}_k, \bar{b}_k, \bar{\pi}_k, \bar{t}_k\} := \text{solve} \ z_{REVLS}(\bar{x}_k, \bar{y}_k)$

if improvement($z_{REVLS}$) then

Update $z^*$

Intensification: $n_{fixed} := n_{fixed} + 1$ when $n_{fixed} < n_{max}$

else

Diversification: $n_{fixed} := n_{fixed} - 1$ when $n_{fixed} > n_{min}$

end if

$k := k + 1$

until $||z_{opt} - z^*||^2 \leq \varepsilon$ or $k \geq k_{max}$
Performance of the heuristic

The omitted instances are the ones where the sequential approach has the same solution as the integrated model.

<table>
<thead>
<tr>
<th>Flights</th>
<th>Sequential approach</th>
<th>Best solution reported by Bonmin</th>
<th>Heuristic results Average over 5 replications</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>flights</td>
<td>profit</td>
<td>% dev.</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>35,372</td>
<td>5.26%</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>43,990</td>
<td>4.45%</td>
</tr>
<tr>
<td>5</td>
<td>26</td>
<td>69,901</td>
<td>1.41%</td>
</tr>
<tr>
<td>7</td>
<td>19</td>
<td>84,186</td>
<td>3.47%</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td>904,054</td>
<td>0.30%</td>
</tr>
<tr>
<td>11</td>
<td>32</td>
<td>93,920</td>
<td>0.30%</td>
</tr>
<tr>
<td>12</td>
<td>11</td>
<td>854,902</td>
<td>0.42%</td>
</tr>
<tr>
<td>13</td>
<td>39</td>
<td>137,428</td>
<td>0.83%</td>
</tr>
<tr>
<td>14</td>
<td>23</td>
<td>93,347</td>
<td>3.25%</td>
</tr>
<tr>
<td>16</td>
<td>19</td>
<td>37,100</td>
<td>2.89%</td>
</tr>
<tr>
<td>18</td>
<td>14</td>
<td>52,369</td>
<td>1.43%</td>
</tr>
<tr>
<td>20</td>
<td>33</td>
<td>146,464</td>
<td>0.00%</td>
</tr>
<tr>
<td>21</td>
<td>77</td>
<td>208,561</td>
<td>-7.18%</td>
</tr>
<tr>
<td>22</td>
<td>61</td>
<td>226,615</td>
<td>0.33%</td>
</tr>
<tr>
<td>23</td>
<td>48</td>
<td>163,114</td>
<td>-6.06%</td>
</tr>
</tbody>
</table>

max 43200 max 3600
Conclusions and future work

- Integrated schedule planning and revenue management
  - More efficient schedule planning with the information on supply-demand interactions
- Heuristic
  - Inclusion of larger instances to test the limits of the heuristic
- Further solution methods for the resulting mixed integer nonlinear problem
  - Convex approximation of the nonlinearity
  - Decomposition methods ⇒ FAM and REV models
Thank you for your attention!
Discrete choice analysis

- Finite and discrete set of alternatives
  - Choice of transportation mode: car, bus, etc.
  - Choice of brand: Leonidas, Lindt, Suchard, Toblerone, etc.
  - Choice of flight: GVA-NCE 10:00, GVA-NCE 06:30, etc.
- Individual $n$ associates a utility to alternative $i$
- Represented by a random function

$$U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + \varepsilon_{in}$$
Individual $n$ chooses alternative $i$ if $U_{in} \geq U_{jn}$, for all $j$.

Utility is random, so we have a probabilistic model

$$P_n(i|C_n) = Pr(U_{in} \geq U_{jn}) = Pr(V_{in} + \varepsilon_{in} \geq V_{jn} + \varepsilon_{jn})$$

Concrete models require

- specification of $V_{in}$
- assumptions about $\varepsilon_{in}$
- estimation of the parameters from data