Optimization and Discrete Choice Models

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Outline

Outline

Microeconomics
in a nutshell

Outline

Microeconomics in a nutshell $\longrightarrow$ Logit and MEV models

Outline

Microeconomics in a nutshell → Logit and MEV models → Profit maximization, facility location

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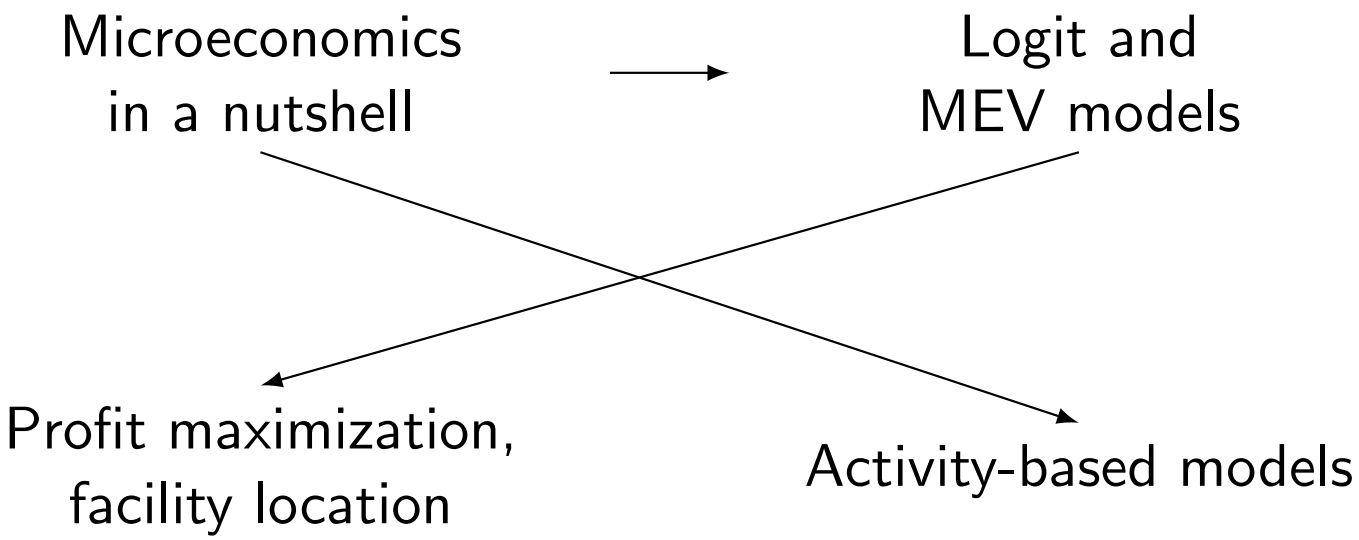
Microeconomics in a nutshell
Logit and MEV models
Profit maximization, facility location
Activity-based models

Outline

Optimization

Microeconomics in a nutshell → Logit and MEV models

Profit maximization, facility location

Activity-based models

Outline

Optimization

Microeconomics in a nutshell

Discrete choice

Logit and MEV models

Profit maximization, facility location

Activity-based models

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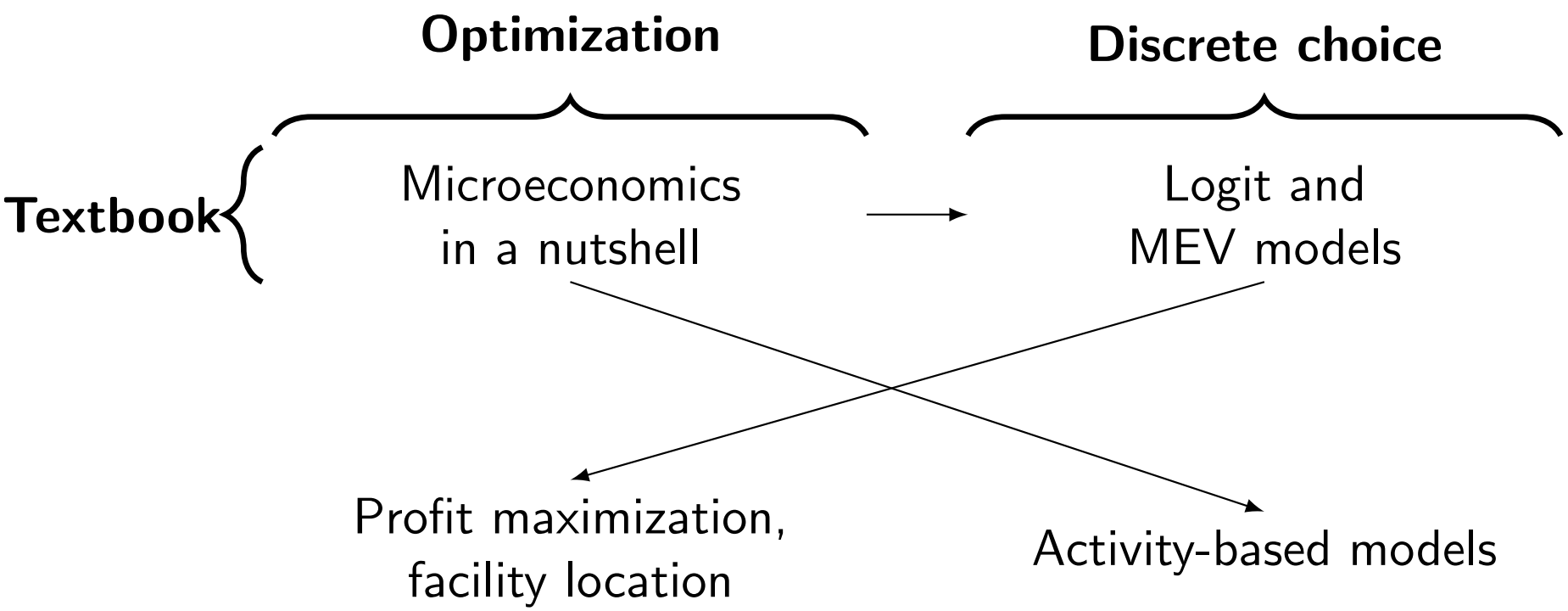
Optimization
Discrete choice
Textbook
Microeconomics in a nutshell
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Outline

	Optimization	**Discrete choice**
Textbook	Microeconomics in a nutshell	Logit and MEV models
Research	Profit maximization, facility location	Activity-based models

Outline

Decision rule

Homo economicus

Rational and narrowly self-interested economic actor who is optimizing her outcome

Utility

$$U_n : \mathcal{C}_n \longrightarrow \mathbb{R} : a \rightsquigarrow U_n(a)$$

- captures the attractiveness of an alternative
- measure that the decision maker wants to optimize

Behavioral assumption

- the decision maker associates a utility with each alternative
- the decision maker is a perfect optimizer
- the alternative with the highest utility is chosen

Microeconomic consumer theory

Continuous choice set

- Consumption bundle

$$Q = \begin{pmatrix} q_1 \\ \vdots \\ q_L \end{pmatrix}; p = \begin{pmatrix} p_1 \\ \vdots \\ p_L \end{pmatrix}$$

- Budget constraint

$$p^T Q = \sum_{\ell=1}^{L} p_\ell q_\ell \leq I.$$

- No attributes, just quantities

Optimization

Optimization problem

$$\max_{Q} \widetilde{U}(Q;\theta)$$

subject to

$$p^T Q \leq I, \; Q \geq 0.$$

Demand function

- Solution of the optimization problem
- Quantity as a function of prices and budget

$$Q^* = f(I, p; \theta)$$

Example

Optimization problem

$$\max_{q_1,q_2} \widetilde{U}(q_1, q_2; \theta_0, \theta_1, \theta_2) = \theta_0 q_1^{\theta_1} q_2^{\theta_2}$$

subject to

$$p_1 q_1 + p_2 q_2 = I.$$

Lagrangian of the problem:

$$L(q_1, q_2, \lambda) = \theta_0 q_1^{\theta_1} q_2^{\theta_2} + \lambda(I - p_1 q_1 - p_2 q_2).$$

Necessary optimality condition

$$\nabla L(q_1, q_2, \lambda) = 0$$

Demand functions

Product 1

$$q_1^* = \frac{I}{p_1} \frac{\theta_1}{\theta_1 + \theta_2}$$

Product 2

$$q_2^* = \frac{I}{p_2} \frac{\theta_2}{\theta_1 + \theta_2}$$

Comments

- Demand decreases with price
- Demand increases with budget
- Demand independent of θ_0, which does not affect the ranking
- Property of Cobb Douglas: the demand for a good is only dependent on its own price and independent of the price of any other good.

Indirect utility

Substitute the demand function into the utility

$$U(I, p; \theta) = \theta_0 \left(\frac{I}{p_1} \frac{\theta_1}{\theta_1 + \theta_2} \right)^{\theta_1} \left(\frac{I}{p_2} \frac{\theta_2}{\theta_1 + \theta_2} \right)^{\theta_2}$$

Indirect utility

Maximum utility that is achievable for a given set of prices and income

In discrete choice...

- only the indirect utility is used
- therefore, it is simply referred to as "utility"

Microeconomic theory of discrete goods

Expanding the microeconomic framework

- Continuous goods
- and discrete goods

The consumer

- selects the quantities of continuous goods: $Q = (q_1, \ldots, q_L)$
- chooses an alternative in a discrete choice set $i = 1, \ldots, j, \ldots, J$
- discrete decision vector: $(y_1, \ldots, y_J)$, $y_j \in \{0, 1\}$, $\sum_j y_j = 1$.

Note

- In theory, one alternative of the discrete choice combines all possible choices made by an individual.
- In practice, the choice set will be more restricted for tractability

Utility maximization

Utility

$$\widetilde{U}(Q, y, \tilde{z}^T y; \theta)$$

- Q: quantities of the continuous good
- y: discrete choice
- $\tilde{z}^T = (\tilde{z}_1, \ldots, \tilde{z}_i, \ldots, \tilde{z}_J) \in \mathbb{R}^{K \times J}$: K attributes of the J alternatives
- $\tilde{z}^T y \in \mathbb{R}^K$: attributes of the chosen alternative
- θ: vector of parameters

Utility maximization

Optimization problem

$$\max_{Q,y} \widetilde{U}(Q, y, \tilde{z}^T y; \theta)$$

subject to

$$\begin{aligned} & p^T Q + c^T y \leq I \\ & \textstyle\sum_j y_j = 1 \\ & y_j \in \{0, 1\}, \forall j. \end{aligned}$$

where $c^T = (c_1, \ldots, c_i, \ldots, c_J)$ contains the cost of each alternative.

Solving the problem

- Mixed integer optimization problem
- No optimality condition
- Impossible to derive demand functions directly

Solving the problem

Step 1: condition on the choice of the discrete good

- Fix the discrete good, that is select a feasible y.
- The problem becomes a continuous problem in Q.
- Conditional demand functions can be derived:

$$q_{\ell|y} = f(I - c^T y, p, \tilde{z}^T y; \theta),$$

or, equivalently, for each alternative i,

$$q_{\ell|i} = f(I - c_i, p, \tilde{z}_i; \theta).$$

- $I - c_i$ is the income left for the continuous goods, if alternative i is chosen.
- If $I - c_i < 0$, alternative i is declared unavailable and removed from the choice set.

Solving the problem

Conditional indirect utility functions

Substitute the demand functions into the utility:

$$U_i = U(I - c_i, p, \tilde{z}_i; \theta) \text{ for all } i \in \mathcal{C}.$$

Step 2: Choice of the discrete good

$$\max_y U(I - c^T y, p, \tilde{z}^T y; \theta)$$

- Enumerate all alternatives.
- Compute the conditional indirect utility function U_i.
- Select the alternative with the highest U_i.
- Note: no income constraint anymore.

Random utility model

Random utility

$$U_{in} = V_{in} + \varepsilon_{in} = \beta^T X_{in} + \varepsilon_{in}.$$

Similarity with linear regression

$$Y = \beta^T X + \varepsilon$$

Here, U is not observed. Only the choice is observed.

Choice model

$$P(i|\mathcal{C}_n) = \Pr(U_{in} \geq U_{jn}, \forall j \in \mathcal{C}_n),$$

Outline

Road map

Optimization

Microeconomics
in a nutshell

→

Discrete choice

Logit and
MEV models

Error term

Random utility

$$U_{in} = V_{in} + \varepsilon_{in}.$$

Assumptions about the distribution

- Probit: central limit theorem: the sum of many i.i.d. random variables approximately follows a normal distribution.
- Logit: Gumbel theorem: the maximum of many i.i.d. random variables approximately follows an Extreme Value distribution: EV(η, μ).

The Extreme Value distribution EV(η, μ)

Probability density function (pdf)

$$f(t) = \mu e^{-\mu(t-\eta)} e^{-e^{-\mu(t-\eta)}}$$

Cumulative distribution function (CDF)

$$\begin{aligned} P(c \geq \varepsilon) = F(c) &= \int_{-\infty}^{c} f(t)dt \\ &= e^{-e^{-\mu(c-\eta)}} \end{aligned}$$

Logit model

Assumptions

ε_{in} are i.i.d. EV(0, μ).

Choice model

$$P_n(i|\mathcal{C}_n) = \frac{y_{in} e^{\mu V_{in}}}{\sum_{j=1}^{J} y_{jn} e^{\mu V_{jn}}}.$$

Logit model

$$U_{in} = V_{in} + \varepsilon_{in}.$$

Why "logit"?

If U_{in} and U_{jn} are EV distributed, $U_{in} - U_{jn}$ follows a logistic distribution.

Availability of alternatives

$$y_{in} = \begin{cases} 1 & \text{if } i \in \mathcal{C}_n, \\ 0 & \text{otherwise.} \end{cases}$$

$y_{in=1}$ if alternative i is available to individual n.

Road map

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Profit maximization,
facility location

Outline

A simple example

Data

- $\mathcal{C}$: set of movies
- Population of N individuals
- Utility function:
  $U_{in} = \beta_{in} p_{in} + f(z_{in}) + \varepsilon_{in}$

Decision variables

- What movies to propose? y_{in}
- What price? p_{in}

Profit maximization

Data

- Two alternatives: my theater (m) and the competition (c)
- We assume an heterogenous population of N individuals

$$U_{cn} = 0 + \varepsilon_{cn}$$
$$U_{mn} = \beta_n p_m + c_{mn} + \varepsilon_{mn}$$

- $\beta_n < 0$
- Logit model: ε_{mn} i.i.d. EV

Heterogeneous population

Two groups in the population

$$U_{mn} = \beta_n p_m + c_{mn} + \varepsilon_{mn}$$

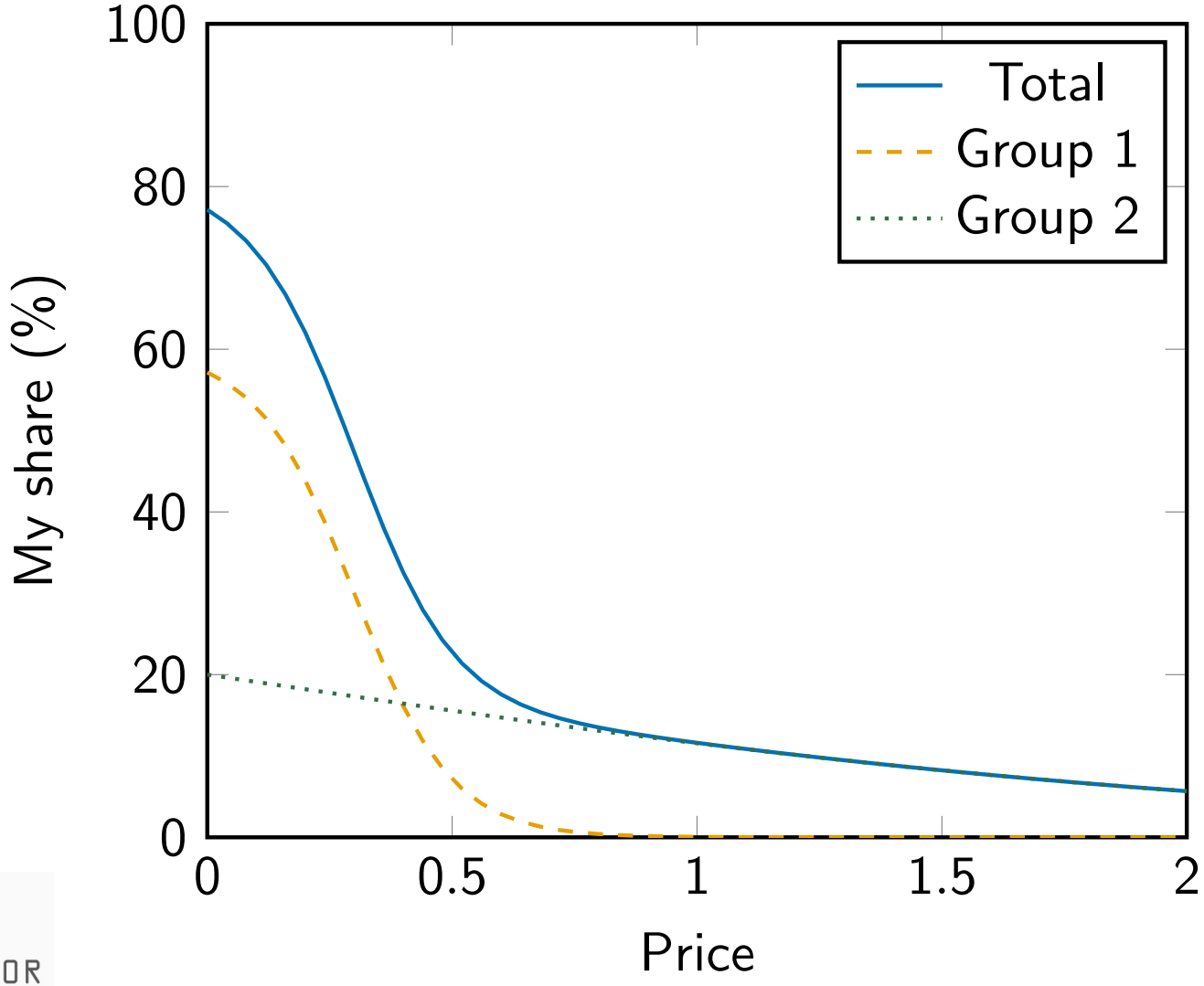
$n = 1$: Young fans: $2/3$

$\beta_1 = -10,\ c_{m1} = 3$

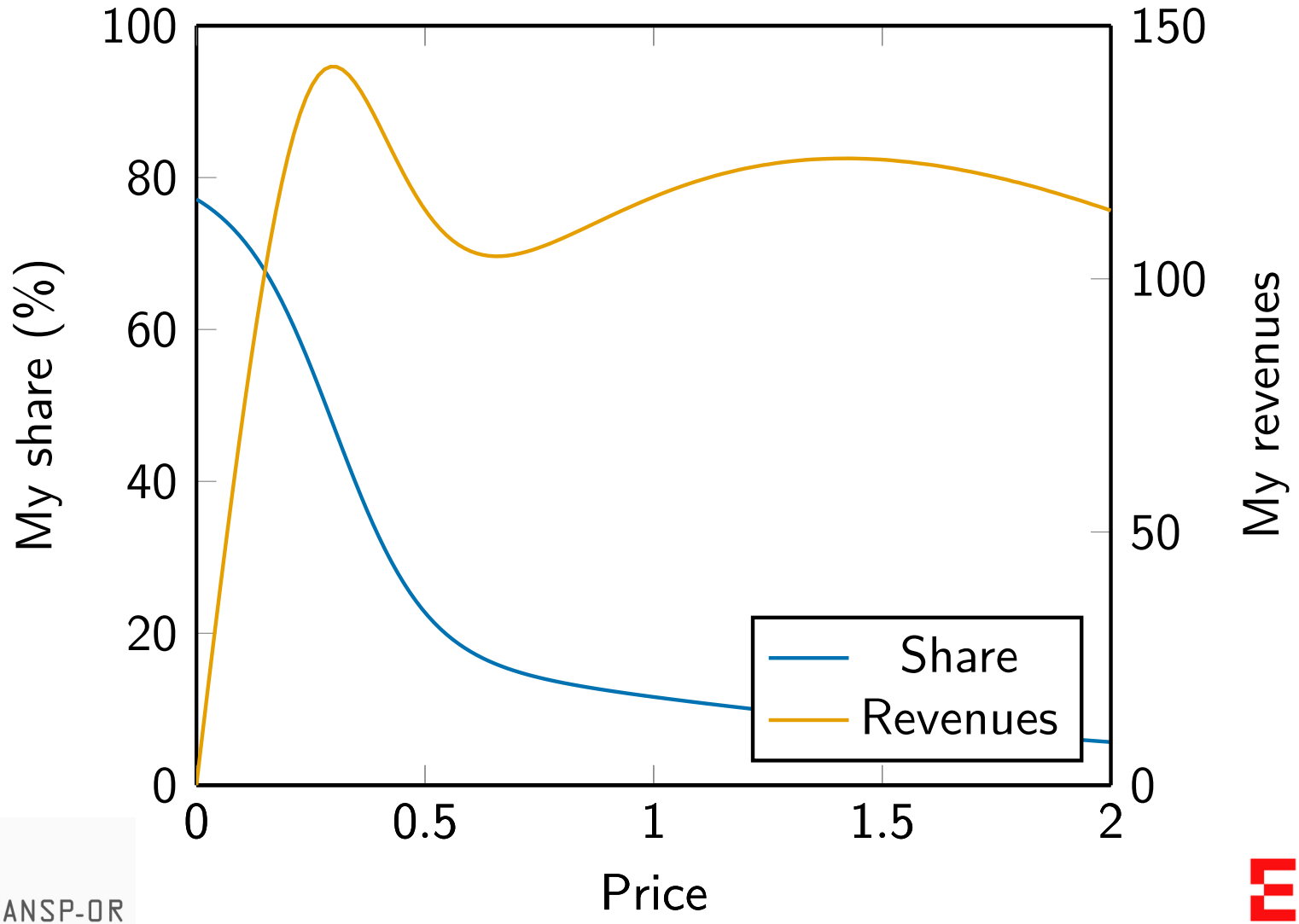
$n = 2$: Others: $1/3$

$\beta_1 = -0.9,\ c_{1m} = 0$

Illustrative example



Illustrative example



Optimization

Profit maximization

- Non linear
- Non convex

Facility location

- Discrete

The main idea

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Linearization

- Hopeless to linearize the logit formula (we tried...)
- Anyway, we want to go beyond logit.

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First principles

Each customer solves an optimization problem

The main idea

Linearization

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- Anyway, we want to go beyond logit.

First principles

Each customer solves an optimization problem

Solution

Use the utility and not the probability

A linear formulation

Utility function

$$U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + f(z_{in}) + \varepsilon_{in}.$$

Simulation

- Assume a distribution for ε_{in}
- E.g. logit: i.i.d. extreme value
- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- The choice problem becomes deterministic

Scenarios

Draws

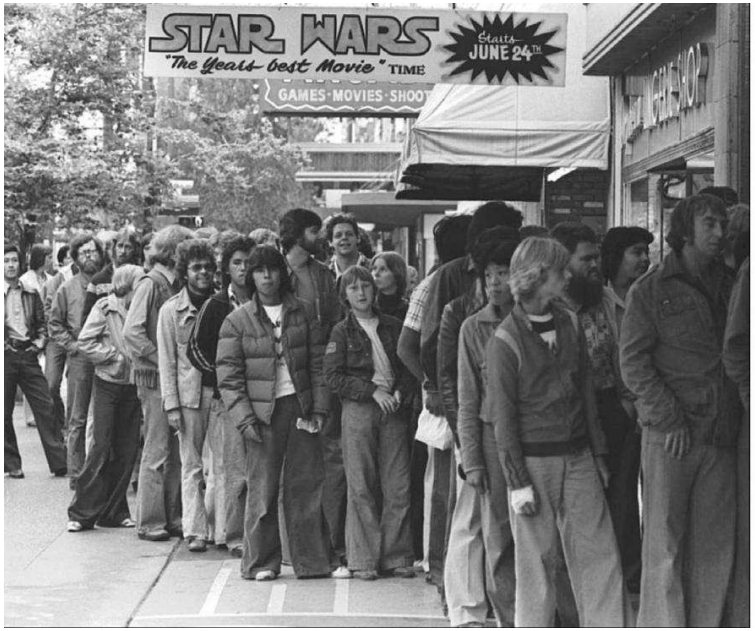
- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- We obtain R scenarios

$$U_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr}.$$

- For each scenario r, we can identify the largest utility.
- It corresponds to the chosen alternative.

Capacities

- Demand may exceed supply
- Each alternative i can be chosen by maximum c_i individuals.
- An exogenous priority list is available.
- Can be randomly generated, or according to some rules.
- The numbering of individuals is consistent with their priority.



Choice set

Variables

$y_i \in \{0,1\}$	operator decision
$y^d_{in} \in \{0,1\}$	customer decision (data)
$y_{in} \in \{0,1\}$	product of decisions
$y_{inr} \in \{0,1\}$	capacity restrictions

Constraints

$$y_{in} = y^d_{in} y_i \quad \forall i, n$$
$$y_{inr} \leq y_{in} \quad \forall i, n, r$$

Utility

Variables

U_{inr}	utility
$z_{inr} = \begin{cases} U_{inr} & \text{if } y_{inr} = 1 \\ \ell_{nr} & \text{if } y_{inr} = 0 \end{cases}$	discounted utility
(ℓ_{nr} smallest lower bound)	

Constraint: utility

$$U_{inr} = \overbrace{\beta_{in} p_{in} + q_d(x_d)}^{V_{in}} + \xi_{inr} \; \forall i, n, r$$

Utility (ctd)

Constraints: discounted utility

$$
\begin{aligned}
\ell_{nr} &\leq z_{inr} && \forall i, n, r \\
z_{inr} &\leq \ell_{nr} + M_{inr} y_{inr} && \forall i, n, r \\
U_{inr} - M_{inr}(1 - y_{inr}) &\leq z_{inr} && \forall i, n, r \\
z_{inr} &\leq U_{inr} && \forall i, n, r
\end{aligned}
$$

Choice

Variables

$$U_{nr} = \max_{i \in \mathcal{C}} z_{inr}$$

$$w_{inr} = \begin{cases} 1 & \text{if } z_{inr} = U_{nr} \\ 0 & \text{otherwise} \end{cases} \qquad \text{choice}$$

Constraints

$$z_{inr} \leq U_{nr} \qquad \forall i, n, r$$

$$U_{nr} \leq z_{inr} + M_{nr}(1 - w_{inr}) \qquad \forall i, n, r$$

$$\sum_i w_{inr} = 1 \qquad \forall n, r$$

$$w_{inr} \leq y_{inr} \qquad \forall i, n, r$$

Capacity

Capacity cannot be exceeded $\Rightarrow y_{inr} = 1$

$$\sum_{m=1}^{n-1} w_{imr} \leq (c_i - 1)y_{inr} + (n-1)(1 - y_{inr}) \; \forall i > 0, n > c_i, r$$

Capacity has been reached $\Rightarrow y_{inr} = 0$

$$c_i(y_{in} - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr}, \; \forall i > 0, n, r$$

Family of models

Constraints

- Set of linear constraints characterizing choice behavior
- Can be included in any relevant optimization problem.

Examples

- Profit maximization
- Facility location

Difficulties

- big M constraints
- large dimensions

Profit maximization

Profit

If p_{in} is the price paid by individual to purchase option i, the revenue generated by this option is

$$\frac{1}{R}\sum_{r=1}^{R}\sum_{n=1}^{N} p_{in} w_{inr}.$$

Linearization

If $a_{in} \leq p_{in} \leq b_{in}$, we define $\eta_{inr} = p_{in} w_{inr}$, and the following constraints:

$$\begin{aligned} a_{in} w_{inr} &\leq \eta_{inr} \\ \eta_{inr} &\leq b_{in} w_{inr} \\ p_{in} - (1 - w_{inr}) b_{in} &\leq \eta_{inr} \\ \eta_{inr} \leq p_{in} - (1 - w_{inr}) a_{in} & \end{aligned}$$

A case study

Challenge

- Take a choice model from the literature.
- It cannot be logit.
- It must involve heterogeneity.
- Show that it can be integrated in a relevant MILP.

A case study

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Parking choice

- [Ibeas et al., 2014]

Parking choices [Ibeas et al., 2014]

Alternatives

- Paid on-street parking
- Paid underground parking
- Free street parking

Model

- $N = 50$ customers
- $\mathcal{C} = \{\text{PSP}, \text{PUP}, \text{FSP}\}$
- $\mathcal{C}_n = \mathcal{C} \quad \forall n$
- $p_{in} = p_i \quad \forall n$
- Capacity of 20 spots
- Mixture of logit models

General experiments

Uncapacitated vs Capacitated case

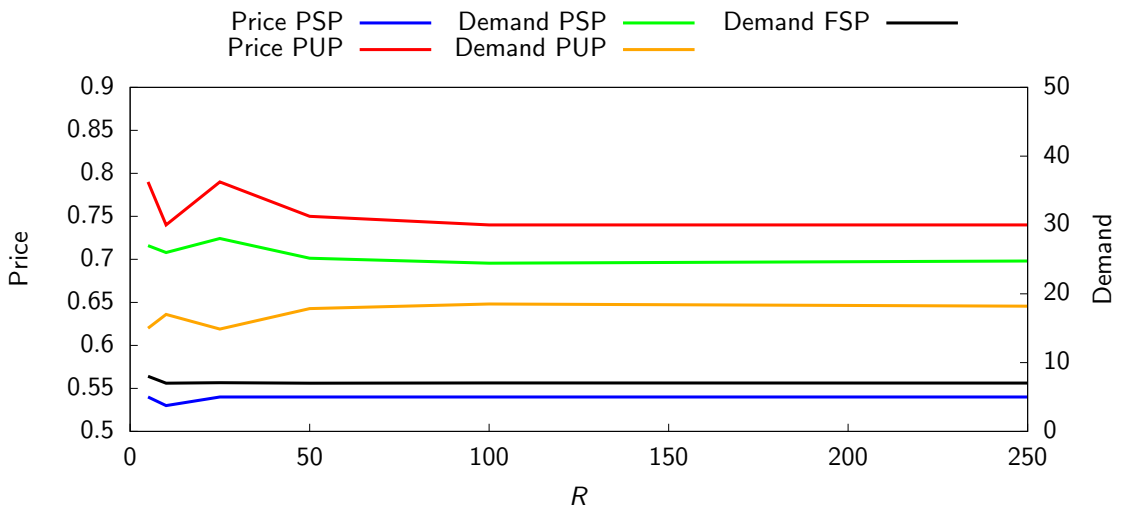
- Maximization of revenue
- Unlimited capacity
- Capacity of 20 spots for PSP and PUP

Price differentiation by population segmentation

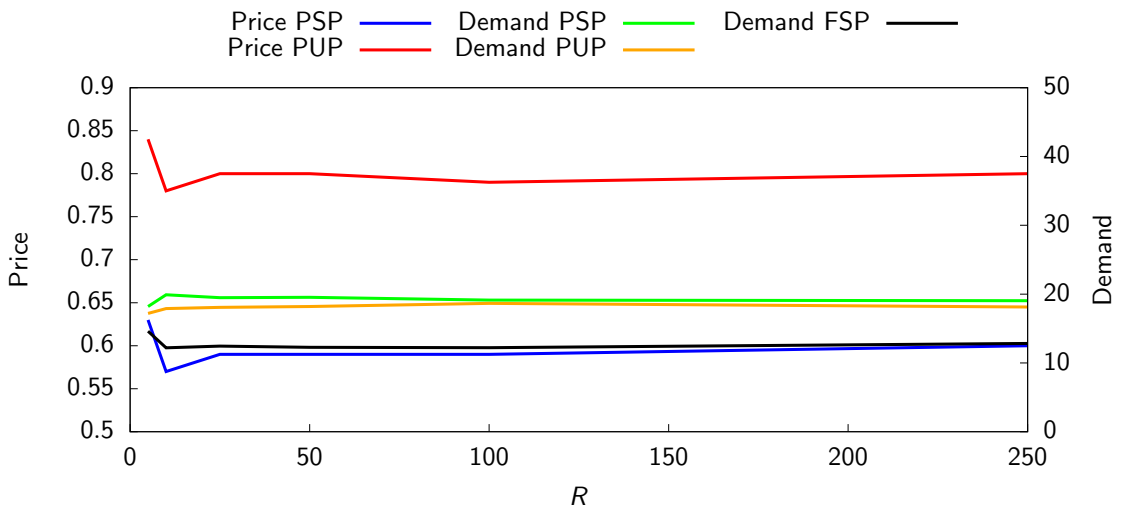
- Reduced price for residents
- Two scenarios
  1. Subsidy offered by the municipality
  2. Operator is forced to offer a reduced price

Uncapacitated vs Capacitated case

Uncapacitated



Capacitated



Computational time

	Uncapacitated case				Capacitated case			
R	Sol time	PSP	PUP	Rev	Sol time	PSP	PUP	Rev
5	2.58 s	0.54	0.79	26.43	12.0 s	0.63	0.84	25.91
10	3.98 s	0.53	0.74	26.36	54.5 s	0.57	0.78	25.31
25	29.2 s	0.54	0.79	26.90	13.8 min	0.59	0.80	25.96
50	4.08 min	0.54	0.75	26.97	50.2 min	0.59	0.80	26.10
100	20.7 min	0.54	0.74	26.90	6.60 h	0.59	0.79	26.03
250	2.51 h	0.54	0.74	26.85	1.74 days	0.60	0.80	25.93

Facility location

Data

- U_{in}: exogenous,
- C_i: fixed cost to open a facility,
- c_i: operational cost per customer to run the facility.

Objective function

$$\min \sum_{i \in \mathcal{C}_k} C_i y_i + \frac{1}{R} \sum_r \sum_i \sum_n c_i w_{inr}$$

Benders decomposition

$$\min \sum_{i \in \mathcal{C}_k} c_i y_i + \frac{1}{R} \sum_r \sum_i \sum_n c_i w_{inr}$$

subject to

$$\max_w U_{nr} = \sum_i U_{inr} w_{inr}$$

$$\sum_i w_{inr} \leq 1$$

$$w_{inr} \leq y_i$$

$$w_{inr} \geq 0$$

$$w_{inr}, y_i \in \{0, 1\}.$$

Benders decomposition

Customer subproblem: fix y_i^*

$$\max_w U_{nr} = \sum_i U_{inr} w_{inr}$$

subject to

$$\sum_i w_{inr} = 1$$

$$w_{inr} \leq y_i^*$$

$$w_{inr} \geq 0.$$

Property

Totally unimodular: no integrality constraint is required.

Benders decomposition

Primal

$$\min_{w} U = -\sum_{i} U_i w_i$$

subject to

$$\sum_{i} w_i = 1$$

$$w_i \leq y_i^* \qquad \forall i$$

$$w_i \geq 0.$$

Dual

$$\max_{\lambda,\mu} \lambda + \sum_{i} \mu_i y_i^*$$

subject to

$$\lambda + \mu_i \leq -U_i \qquad \forall i$$

$$\mu_i \leq 0 \qquad \forall i$$

Bender decomposition

Ongoing work

- Exploit the duality results to generate cuts for the master problem.
- Investigate the use of Benders for other problems.
  - profit maximization,
  - maximum likelihood estimation of the parameters.

Road map

Optimization

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Profit maximization, facility location

Activity-based models

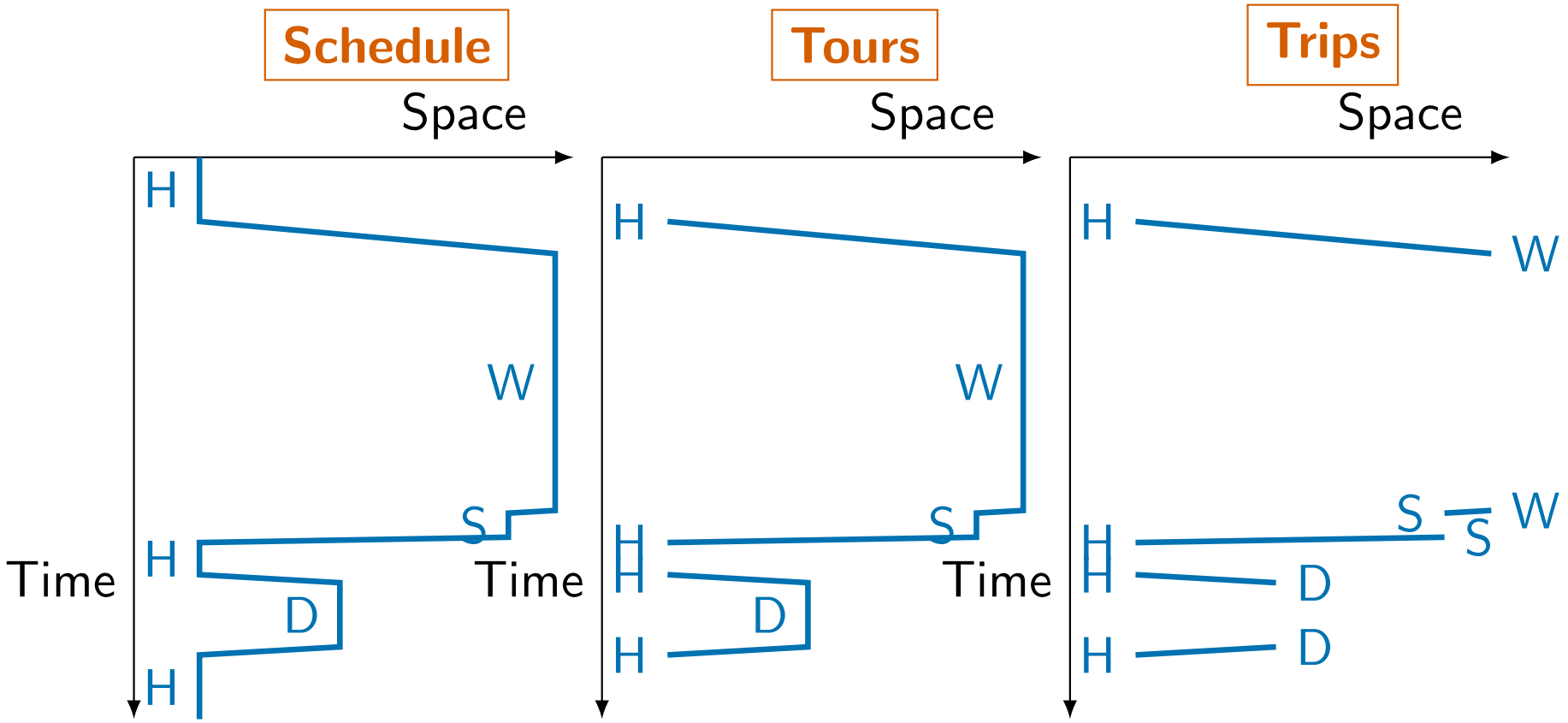
Outline

Introduction

- Travel demand is derived from activity demand.
- Activity demand is influenced by socio-economic characteristics, social interactions, cultural norms, basic needs, etc. [Chapin, 1974]
- Activity demand is constrained in space and time [Hägerstraand, 1970].

Activity-based models

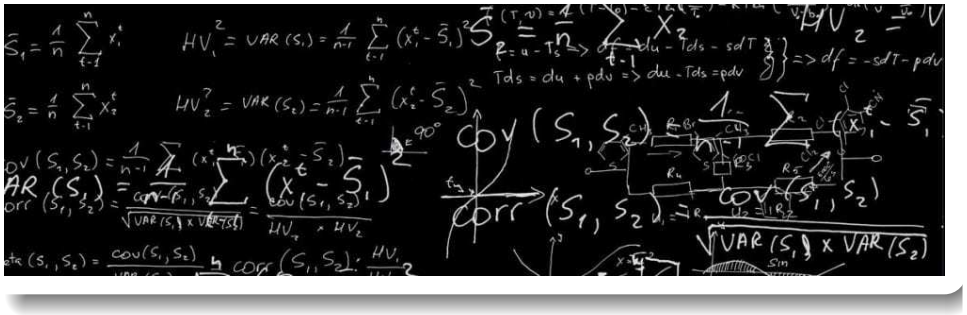
Travel demand models



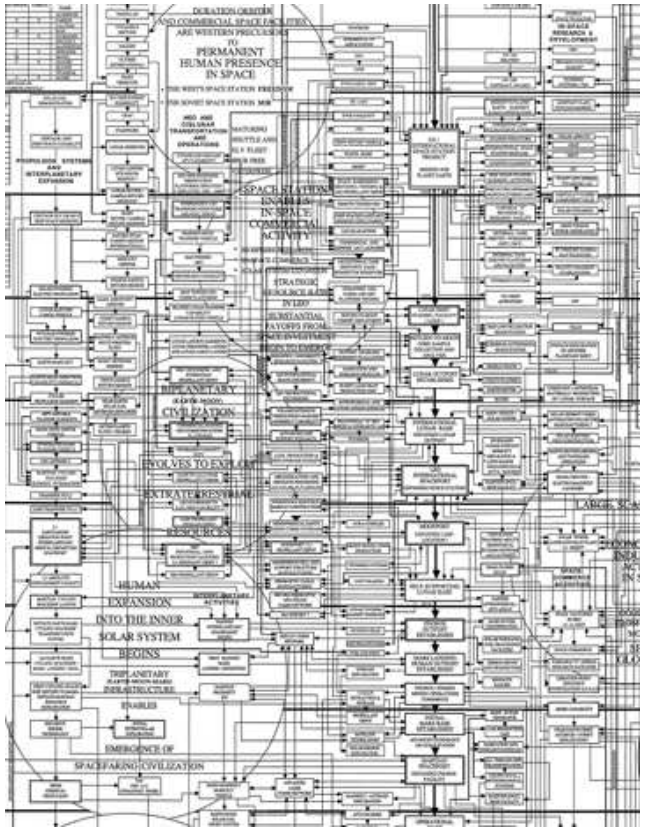
H: Home, W: Work, S: Shop, D: Dining out [Source: M. Ben-Akiva]

Literature

Econometric models



Rule-based models



Research question: can we combine the two?

	Econometric	Rule-based
Micro-economic theory	X	—
Parameter inference	X	—
Testing/validation	X	—
Joint decisions	—	X
Complex rules	—	X
Complex constraints	—	X

Integrated approach

Assumptions

- Individuals **are** utility maximizers.
- All decisions are made together.
- Decisions are subject to complex constraints and interactions.
  - Time constraint: to increase the activity duration, another activity is impacted.
  - Interaction constraints: if I leave home by bus, driving my car is not an option until I come back home.
  - Resource constraints: if my wife uses the only car in the household, driving the car is not an option for me.

Integrated approach

Integrate the econometric and the rule-based approaches

- Utility associated with activity participation, duration, etc.
- Disutility associated with traveling.
- Complex interactions and constraints are captured by rules.

Mathematical programming

- Individuals are solving an optimization problem.
- Decisions: activity participation and scheduling.
- Objective function: utilities.
- Constraints: complex rules.

First principles

- Each individual n has a time-budget (a day).
- Each activity a considered by n is associated with a utility U_{an}.
- Individuals schedule their activities as to **maximize** the total utility, subject to their time-budget constraint.

Further assumptions

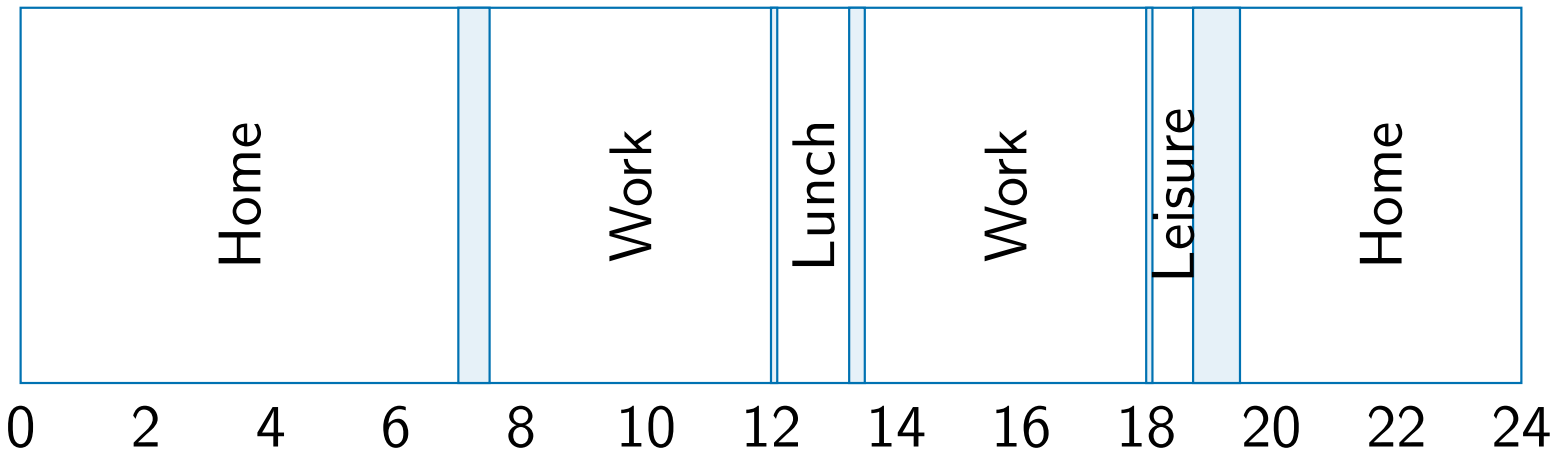
Individuals are **time sensitive**

- Have a desired start time, duration and/or end time for each activity
- Deviations from their desired times in the scheduling process decrease the utility function

Activities

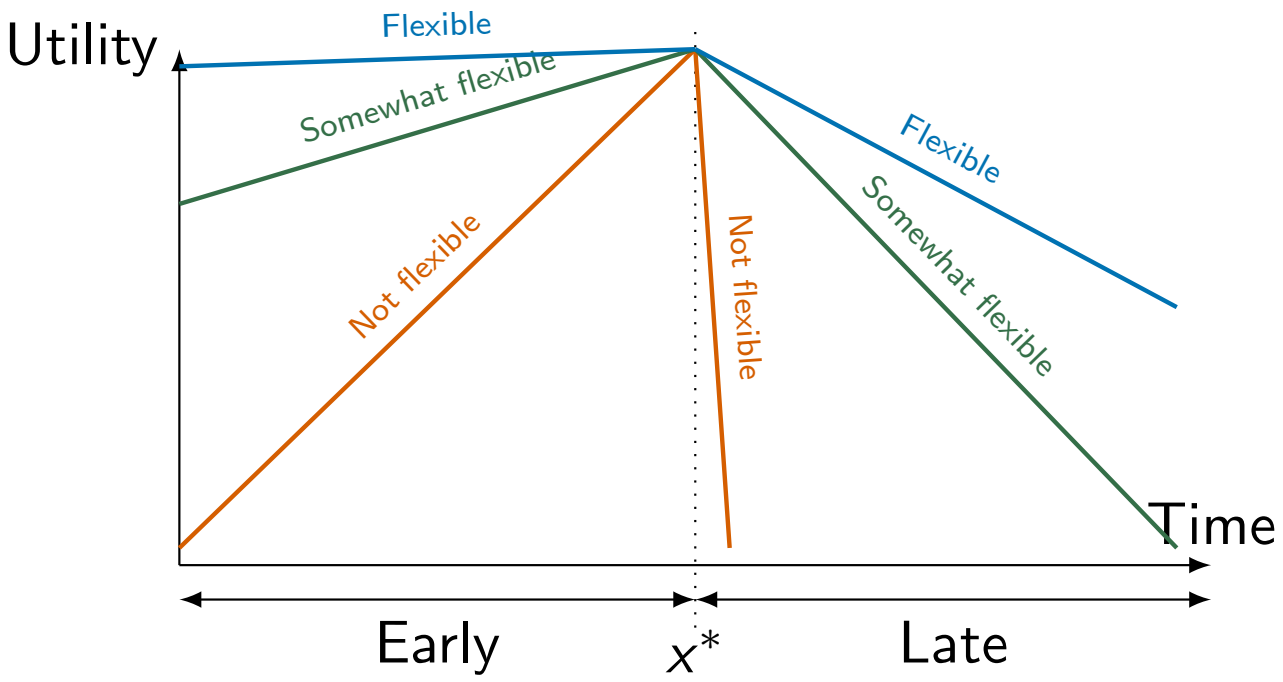
- Set A of activities.
- Location s_a.
- Transportation mode: m_a.
- Starting time x_a, $0 \leq x_a \leq T$.
- Duration: $\tau_a \geq 0$.
- Feasible time interval: $[\gamma_a^-, \gamma_a^+]$ (e.g. opening hours).

Scheduling



Penalty for early or late starting time

Parameters depend on the category type



Disutility of travel

Traveling is part of the activity

- Travel (time and cost) from a to a^+ negatively contributes to U_a: t_a, c_{t_a}.
- Exception: last activity of the day (home).

Utility function

An individual n derives the following utility from performing activity a, with a schedule flexibility k:

$$
\begin{aligned}
U_{an} = c_{an} &+ \theta_e \max(x_a^* - x_a, 0) \\
&+ \theta_\ell \max(x_a - x_a^*, 0) \\
&+ \theta_{ds} \max(\tau_a^* - \tau_a, 0) \\
&+ \theta_{d\ell} \max(\tau_a - \tau_a^*, 0) \\
&+ \theta_{tt} t_a + \theta_{tc} c_{t_a} \\
&+ \theta_c c_a + \xi_{an},
\end{aligned}
$$

where ξ_{an} is a random term with a known distribution.

Utility function

Error terms

- Rely on simulation.
- Draw ξ_{anr}, $r = 1, \ldots, R$.
- Optimization problem for each r.
- Utility: U_{anr}.

Households

Assumptions

- Members of the households are altruist.
- Everybody makes decisions for the sake of the household.
- Objective function: sum of the utilities of each individual

Model

- Similar model as for individuals.
- Resource constraints can easily be added.

Decision variables for individual n and draw r

For each (potential) activity a:

- Activity participation: $w_{anr} \in \{0, 1\}$.
- Starting time: $x_{anr} \in \{0, \ldots, T\}$.
- Duration: $\tau_{anr} \in \{0, \ldots, T\}$.
- Scheduling: $z_{abnr} \in \{0, 1\}$: 1 if activity b immediately follows a.
- Travel time: t_{anr}: travel time from a to the next activity.

Objective function

Additive utility

$$\max \sum_{n} \sum_{a \in A} w_{anr} U_{anr}$$

Constraints

Time budget

$$\sum_a \tau_{anr} + t_{anr} = T, \ \forall n, r.$$

Cost budget

$$\sum_a c_a w_{anr} + t_{c_{anr}} = B, \ \forall n, r.$$

Time windows

$$0 \leq \gamma_a^- \leq x_{anr} \leq x_{anr} + \tau_{anr} \leq \gamma_a^+ \leq T, \ \forall a, n, r.$$

Constraints

Precedence constraints

$$z_{abnr} + z_{banr} \leq 1, \ \forall a, b, n, r.$$

Single successor/predecessor

$$\sum_{b \in A \setminus \{a\}} z_{abnr} = w_{anr}, \ \forall a, n, r,$$

$$\sum_{b \in A \setminus \{a\}} z_{banr} = w_{anr}, \ \forall a, n, r.$$

Constraints

Travel time

$$t_{anr} = \sum_{b \in A} z_{abnr} \rho^{m_a}(s_a, s_b).$$

Consistent timing

$$(z_{abnr} - 1)T \leq x_{anr} + \tau_{anr} + t_{anr} - x_{bnr} \leq (1 - z_{abnr})T, \ \forall a, b, n, r.$$

Mutually exclusive duplicates

$$\sum_{a \in B_k} w_{anr} = 1, \ \forall k, n, r.$$

Constraints

Interaction constraint

- If I leave home by bus, driving my car is not an option until I come back home.
- $\delta^{\text{car}}_{anr} = 1$ if car is available for activity a.

$$\delta^{\text{car}}_{anr} \geq \delta^{\text{car}}_{bnr} + z_{abnr} - 1.$$

Constraints

Resource constraints

- Resource (e.g. private vehicle) considered as an agent with a schedule.
- Maximum one activity at each time-step.
- Constraint: resource must be accompanied by an adult agent throughout the tour.
- Except when idling (vehicle at the parking at home).

Constraints: other examples

Participation constraints

- Participation constraints: if I drop the children off, somebody needs to pick them up later.
- Drop-off: activity a.
- Pick-up: activity b.
- Activity participation: $w_{bnr} \geq w_{anr}$
- Timing: $x_{bnr} \geq x_{anr}$.

Sequence constraints

- If I go grocery shopping I need to go back home before doing another activity.
- Shopping: activity a.
- Home: activity b.

$$z_{abnr} \geq w_{anr}.$$

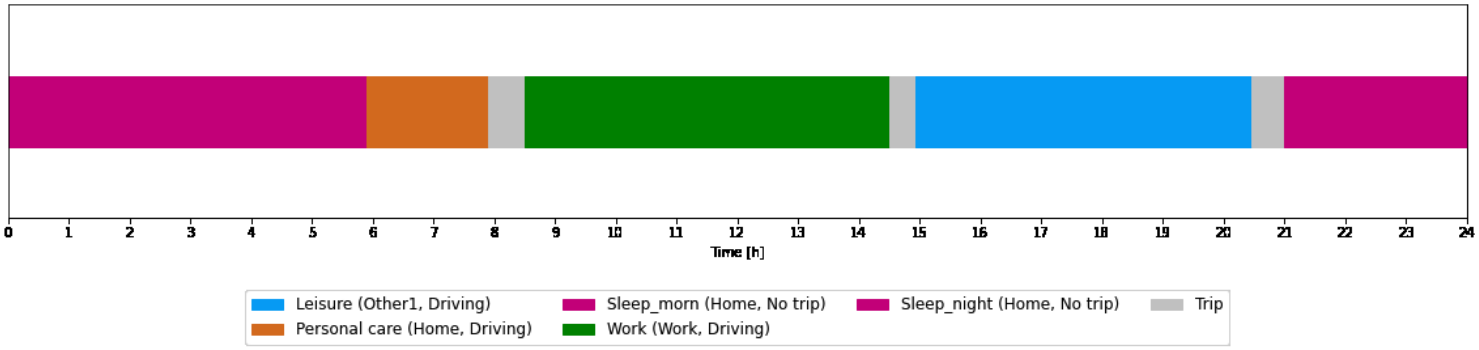
Integrated framework

Mathematical programming

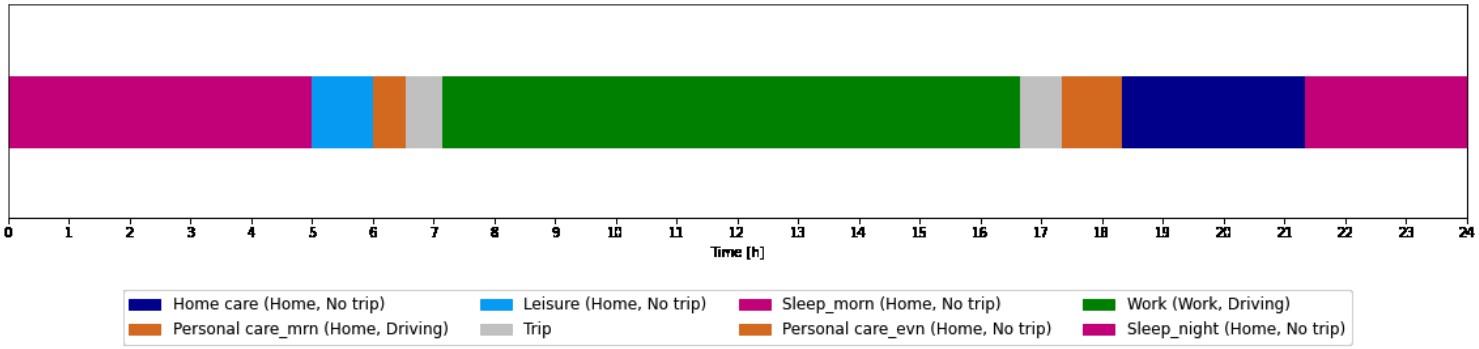
- Utility maximization.
- Scheduling problem.
- Rules are translated into additional constraints.
- Stochasticity is captured by simulation.

Simulation: From isolated individuals...

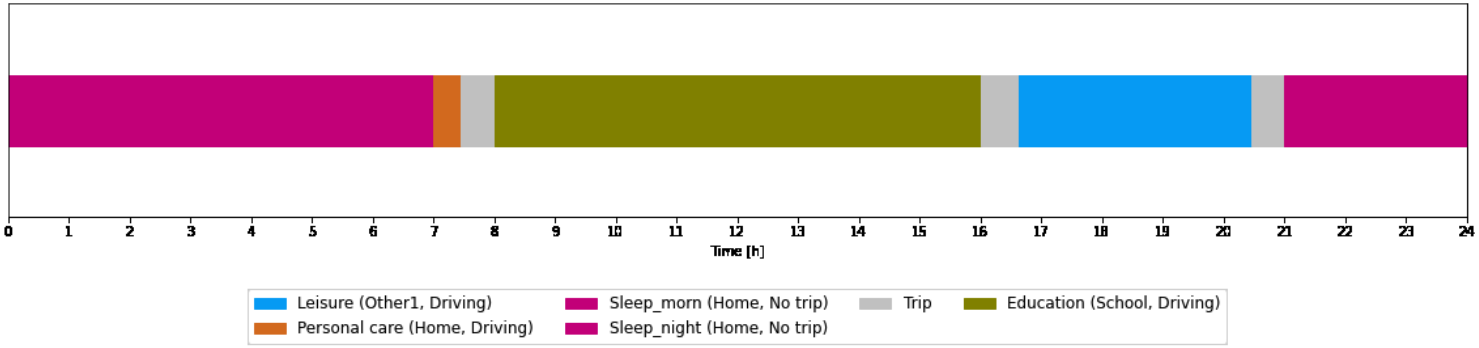
Sara



David

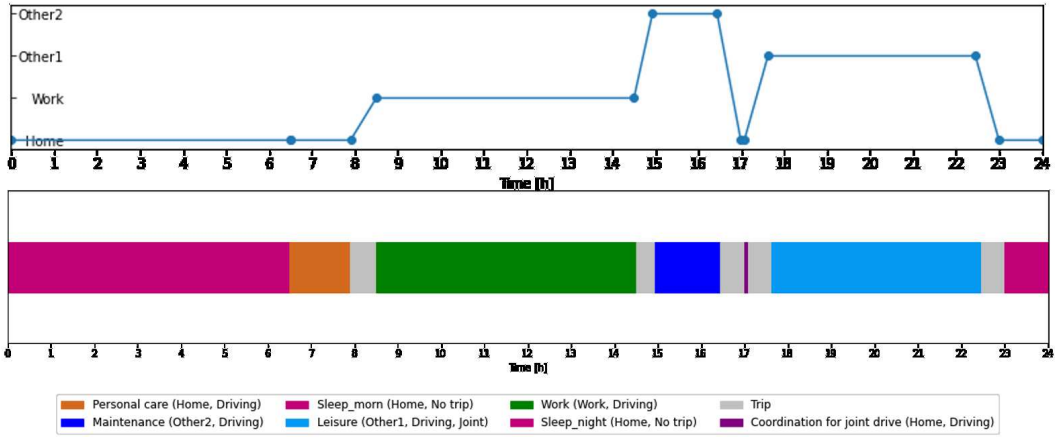


Alice



Simulation: To family of 2; 2 adults with no children...

Sara



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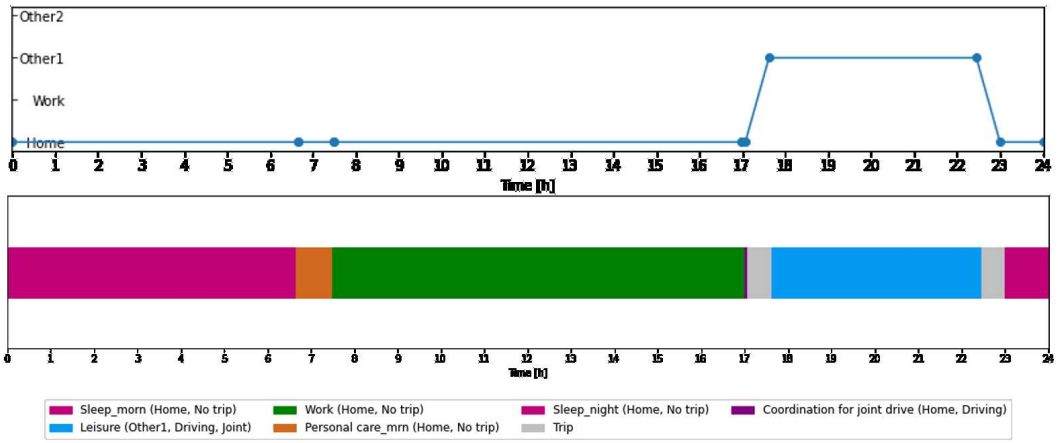
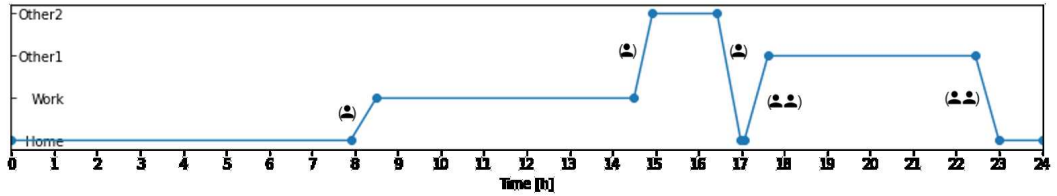


Car



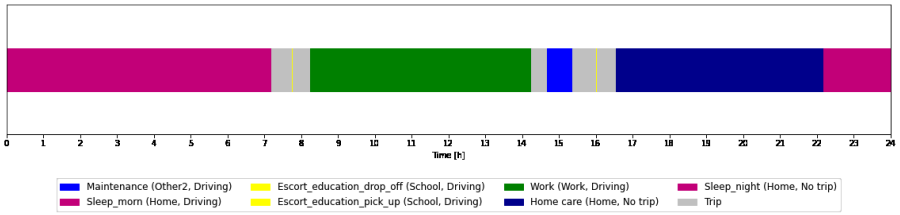
Simulation: Family of 2; 2 adults with no children...

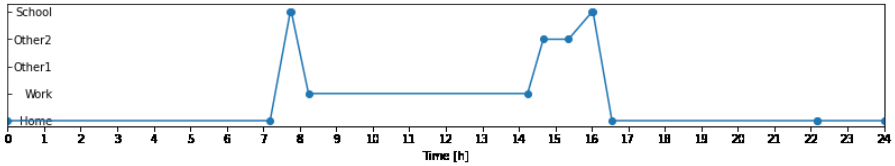
Table: Car location sequence and occupancy in the example of family of 2

Location	Start time (hh:mm)	End time (hh:mm)	Duration (hh:mm)	Person using	Parked_out indicator	Car occupancy
Home	00:00	7:54	7:54	-	0	0
On the road	7:54	8:30	0:36	1	0	1
Work	8:30	14:30	6:00	1	1	0
On the road	14:30	14:56	0:26	1	0	1
Other2	14:56	16:27	1:31	1	1	0
On the road	16:27	17:00	0:33	1	0	1
Home	17:00	17:05	0:05	-	0	0
On the road	17:05	17:38	0:33	1&2	0	2
Other1	17:38	22:27	4:49	1&2	1	0
On the road	22:27	23:00	0:33	1&2	0	2
Home	23:00	24:00	1:00	-	0	0

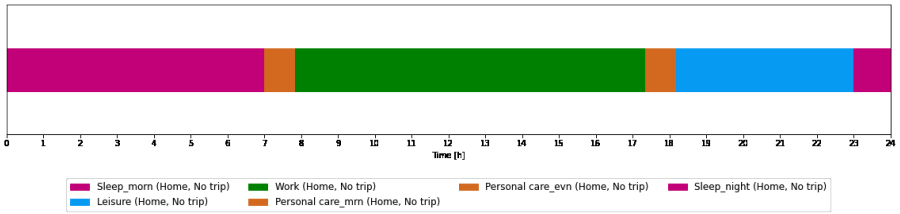
Simulation: To family of 3; 2 adults and 1 child...

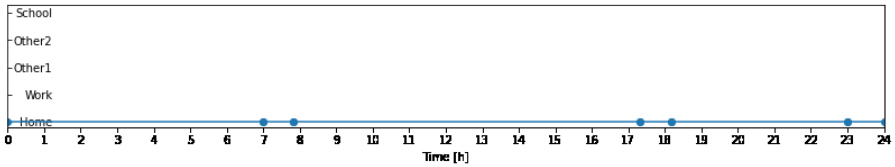
Sara



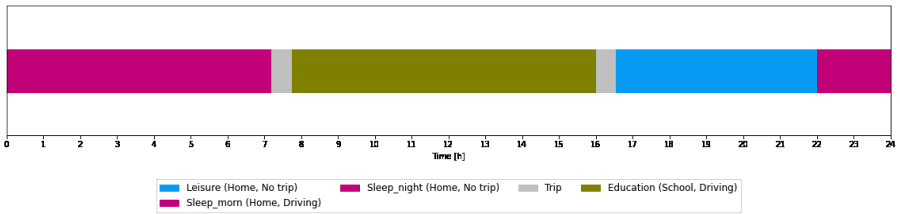


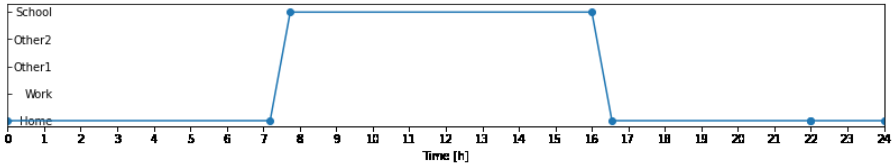
David



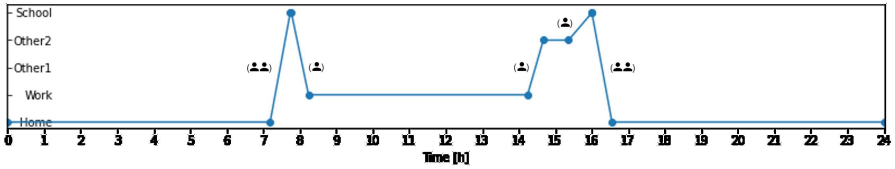


Alice





Car



Simulation: Family of 3; 2 adults and 1 child

Table: Car location sequence and occupancy in the example of family of 3

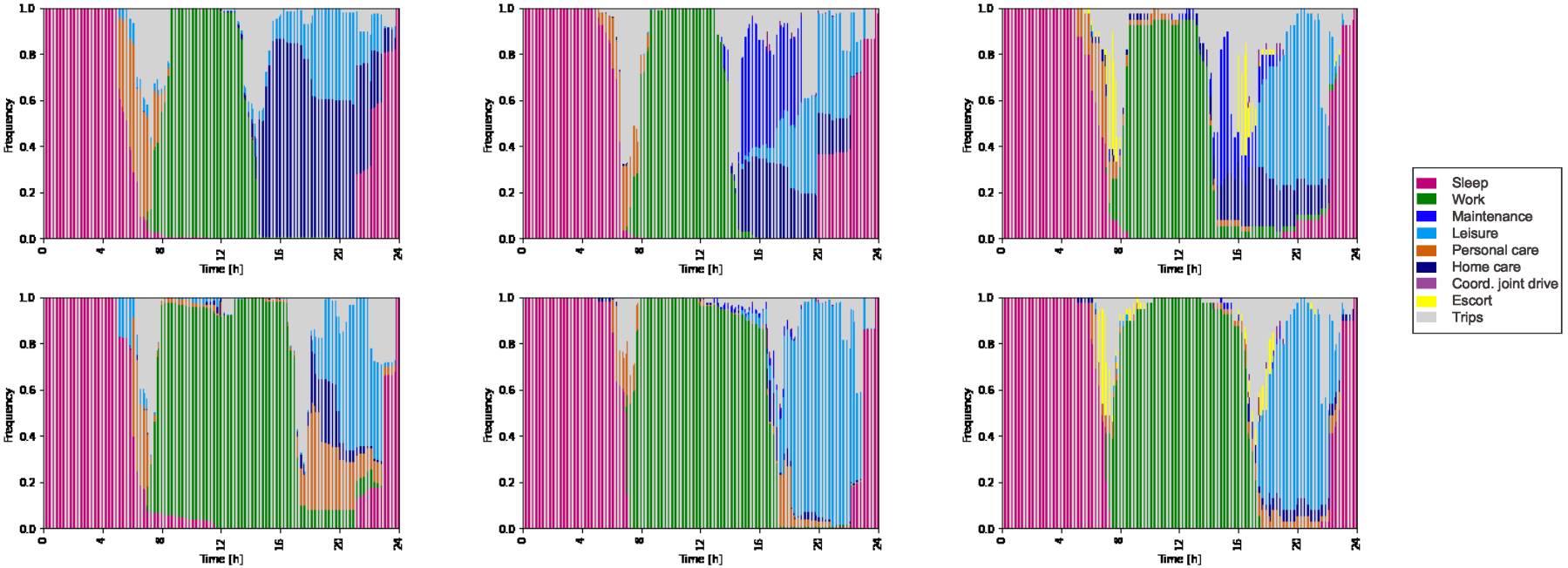
Location	Start time (hh:mm)	End time (hh:mm)	Duration (hh:mm)	Person using	Parked_out indicator	Car occupancy
Home	00:00	7:12	7:12	-	0	0
On the road	7:12	7:45	0:33	1&3	0	2
School	7:45	7:47	0:02	1	0	1
On the road	7:47	8:15	0:28	1	0	1
Work	8:15	14:15	6:00	1	1	0
On the road	14:15	14:40	0:25	1	0	1
Other2	14:40	15:22	0:42	1	1	0
On the road	15:22	16:00	0:38	1	0	1
School	16:00	16:02	0:02	1	0	1
On the road	16:02	16:33	0:31	1&3	0	2
Home	16:33	24:00	7:27	-	0	0

Distributions

Figure: Isolated indiv. Figure: Family of 2 Figure: Family of 3

Sara

David



Distributions

Alice

Figure: Isolated individual

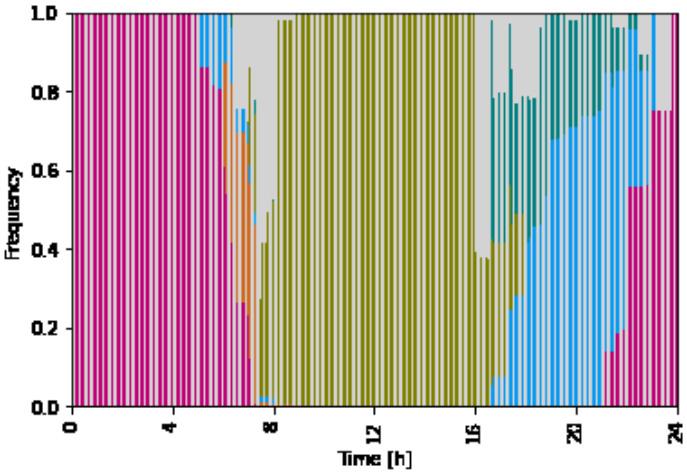
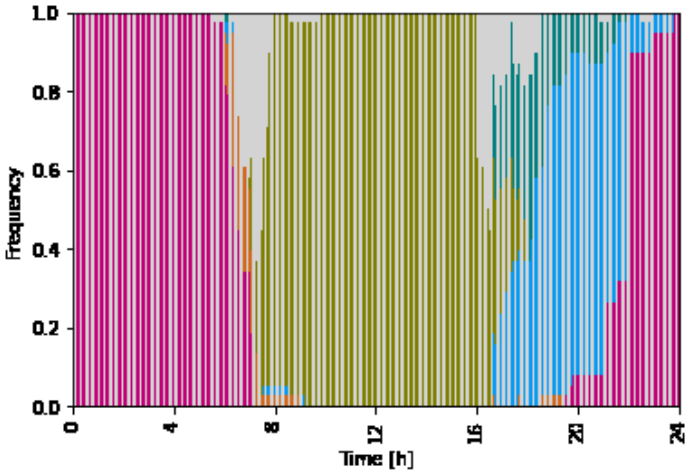
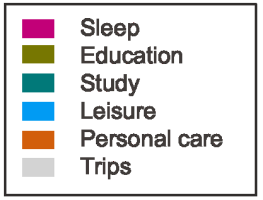


Figure: Family of 3





Schedule simulation

Data set

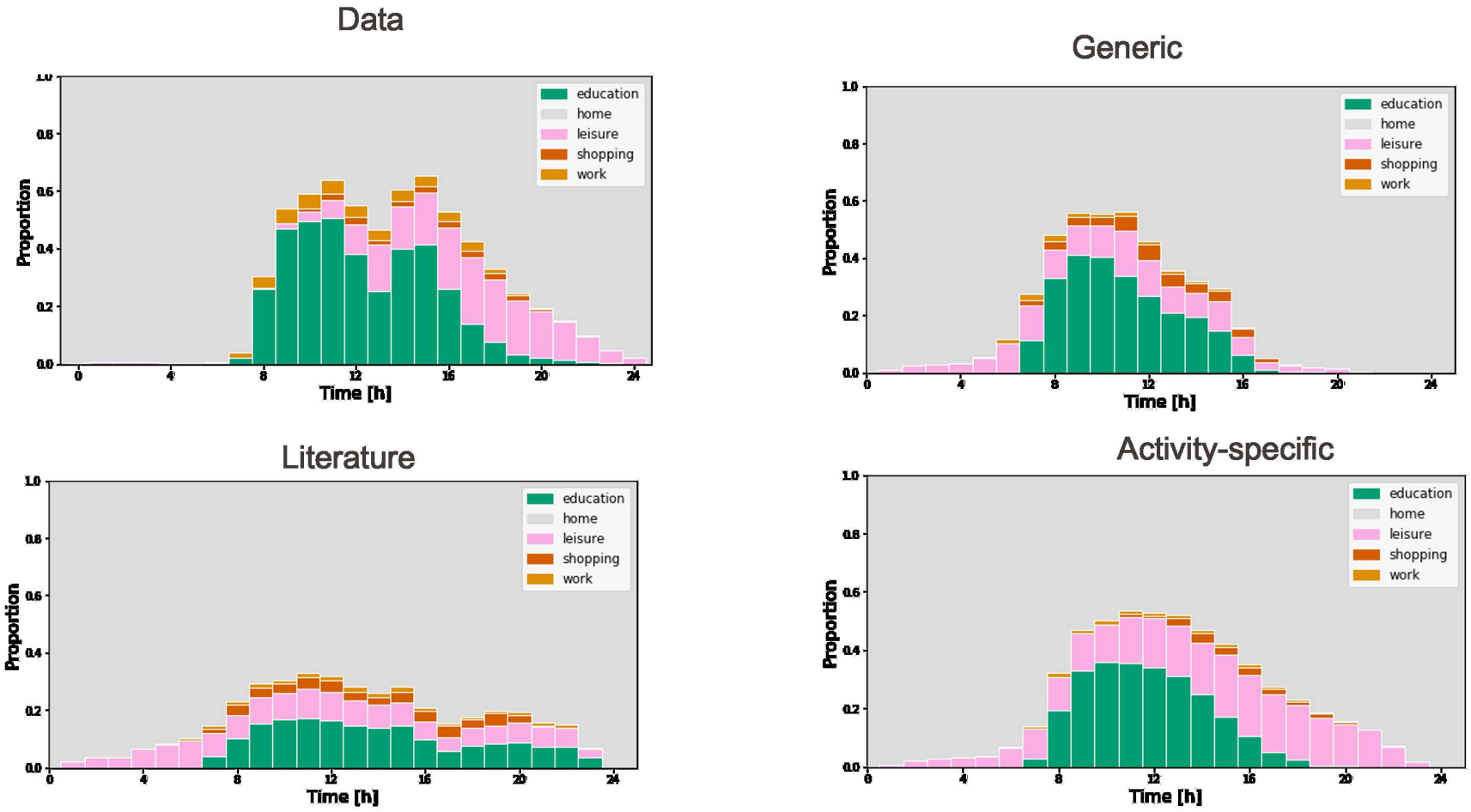
- 2015 Mobility and Transport Microcensus [ARE 2017]
- Nationwide travel survey conducted every 5 years
- Lausanne sample: 1118 individuals
  - Students: 236 individuals
  - Workers: 618 individuals

Visual validation

Distribution of activities over the day

- Data: Swiss microcensus (validation sample).
- Literature: model with 8 parameters, borrowed from the literature.
- Generic: model with generic coefficients, estimated from data (previous slide).
- Activity-specific: model with a set of coefficients for each activity type, estimated from data (20 parameters).

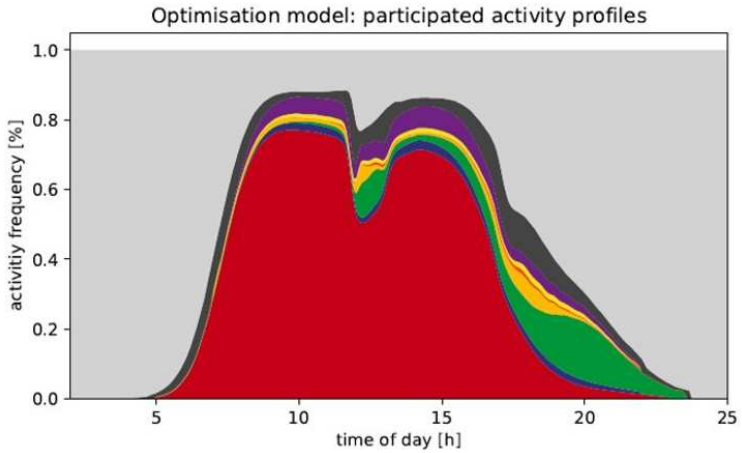
Visual validation

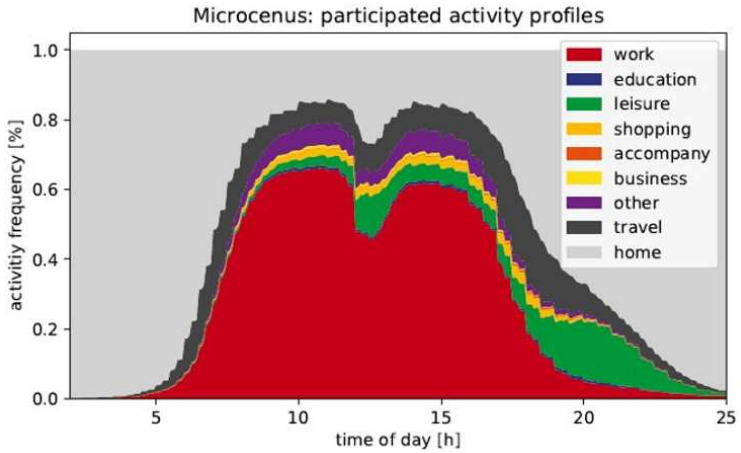


OPTIMs

OPTimization of **I**ndividual **M**obility **S**chedules, [Manser et al., 2022]

- Collaboration with Swiss Federal Railways.
- Integration of the optimization framework into their long-term travel demand forecasting tool (SIMBA MOBi).





Conclusions

Achievements so far

- Formulation of the model.
- Procedure for the estimation of the parameters.
- Simulation of complex and valid activity schedules.
- Simulation of complex resources constraints.
- Simulation of household coordination.
- Application to real case studies.

Challenges

- Latent preferences (desired start times, durations...)
- Validation.

TRANSP-OR

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Summary

- Motivation: design operational activity-based models.
- Combine the econometric and the rule-based approaches.
- Methodological contribution: use mathematical programming and simulation.
- Simulation of activity schedule: [Pougala et al., 2022a].
- Application with the Swiss Railways: [Manser et al., 2022].
- Estimation of the parameters: [Pougala et al., 2022b].
- Household interactions: under preparation.
- Main advantage of the framework: flexibility.

Summary

Long term research vision

- Synthetic population of households.
- Week-based activity scheduling.
- Real-time rescheduling.
- Applications to transportation and energy.

Outline

Conclusion

	Optimization	**Discrete choice**
Textbook	Microeconomics in a nutshell	Logit and MEV models
Research	Profit maximization, facility location	Activity-based models

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