Scheduling of daily activities: an optimization approach

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Travel demand is derived from activity demand.

Activity demand is influenced by socio-economic characteristics, social interactions, cultural norms, basic needs, etc. [Chapin, 1974]

Activity demand is constrained in space and time [Hägerstraand, 1970].
Econometric models

Rule-based models
[Pinjari et al., 2011]

- ... individuals make their activity-travel decisions to maximize the utility derived from the choices they make.
- These model systems usually consist of a series of ... discrete choice models ... that are used to predict ... individuals’ activity-travel decisions.
- these model systems employ econometric systems of equations ... to capture relationships between ... socio-demographics and ... attributes on the one hand and the observed activity-travel decision outcomes on the other.
State of the art: econometric approach

[Pinjari et al., 2011]: main criticisms

- individuals are not necessarily fully rational utility maximizers
- the approach does not explicitly model the underlying decision processes and behavioral mechanisms that lead to observed activity-travel decisions.
Research question

Relax the *series of discrete choice models* approach

- The interactions of all decisions is complex.
- Sequence of models is most of the time arbitrary.

Integrated approach

Develop a model involving all activity-based decisions:

- activity participation,
- activity pattern,
- location choice,
- time of day,
- duration.
Research objectives

• Integrated approach based on first principles.
• Theoretical framework: utility maximization.
• Individuals solve a scheduling problem.
• Important aspects: trade-offs on activity duration.
Outline

1. Introduction
2. Model
3. Mixed integer optimization problem
4. Examples
First principles

- Each individual $n$ has a time-budget (a day).
- Each activity $i$ considered by $n$ is associated with a utility $U_{in}$.
- Individuals schedule their activities as to maximize the total utility, subject to their time-budget constraint.
Further assumptions

Individuals are **time sensitive**
- Have a desired *start time*, *duration* and/or *end time* for each activity
- Deviations from their desired times in the scheduling process decrease the utility function
Time horizon: 24 hours.
Discretization: $T$ time intervals.
Trade-off between model accuracy and computational time.
Space

- Discrete and finite set $S$ of locations, indexed by $s$.
- Trips between location are modeled exogenously.
- For each $(s_o, s_d)$, $\rho(s_o, s_d)$ is the travel time.
- Extensions to include mode and route choices are possible.
Definition: Activity

An activity requires a trip to/from a given location.
Activities

- Set $A$ of activities.
- Location $s_a$.
- Starting time $x_a$, $0 \leq x_a \leq T$.
- Duration: $\tau_a \geq 0$.
- Feasible time interval: $[\gamma^-_a, \gamma^+_a]$ (e.g. opening hours).
- “Home”: same, except for boundary conditions.

Modeling location choice

An activity that can take place at $m$ locations is modeled as a set $B_k$ of $m$ activities with a unique location.
Scheduling of daily activities
Categories

- [Castiglione et al., 2014]: mandatory, maintenance, discretionary.
- Flexible, somewhat flexible, not flexible.

Category

Activities that share the same preference profile.
Preferences

- desired starting time $x_a^*$,
- desired duration $\tau_a^*$.

Penalties

- Starting early [Small, 1982]:
  $$\theta_e \max(x_a^* - x_a, 0).$$
- Starting late [Small, 1982]:
  $$\theta_\ell \max(x_a - x_a^*, 0).$$
- Shorter activity: $\theta_{ds} \max(\tau_a^* - \tau_a, 0).$
- Longer activity: $\theta_{d\ell} \max(\tau_a - \tau_a^*, 0).$
### Preferences

Parameters depend on the category type

![Diagram showing utility over time with different flexibility levels](image)

- **Flexible**
- **Somewhat flexible**
- **Not flexible**

- **Early**
- **Late**

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Scheduling of daily activities

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Disutility of travel

Traveling is part of the activity

- Travel from \( a \) to \( a^+ \) contributes to \( U_a: t_a \).
- Exception: last activity of the day (home).
- In this version, travel choices are exogenous.
Utility function

An individual $n$ derives the following utility from performing activity $a$, with a schedule flexibility $k$:

$$ U_{an} = \theta_e \max(x_a^* - x_a, 0) $$
$$ + \theta_\ell \max(x_a - x_a^*, 0) $$
$$ + \theta_{ds} \max(\tau_a^* - \tau_a, 0) $$
$$ + \theta_{d\ell} \max(\tau_a - \tau_a^*, 0) $$
$$ + \theta_{tt} t_a $$
$$ + c_{an} + \sigma_{an} \varepsilon_{an}, $$

where $\varepsilon_{an}$ is a random term with mean 0 and variance 1.
Utility function

Error terms
- Rely on simulation.
- Draw $\xi_{anr}, r = 1, \ldots, R$.
- Optimization problem for each $r$.
- Utility: $U_{anr}$. 
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Decision variables for individual $n$ and draw $r$

For each (potential) activity $a$:

- Activity participation: $w_{anr} \in \{0, 1\}$.
- Starting time: $x_{anr} \in \{0, \ldots, T\}$.
- Duration: $\tau_{anr} \in \{0, \ldots, T\}$.
- Scheduling: $z_{abnr} \in \{0, 1\}$: 1 if activity $b$ immediately follows $a$.
- Travel time: $t_{anr}$: travel time from $a$ to the next activity.
Mixed integer optimization problem

Objective function

Additive utility

\[ \max \sum_{a \in A} w_{anr} U_{anr} \]
### Constraints

**Time budget**

\[ \sum_a \tau_{anr} = T, \forall n, r. \]

**Time windows**

\[ 0 \leq \gamma^-_a \leq x_{anr} \leq x_{anr} + \tau_{anr} \leq \gamma^+_a \leq T, \forall a, n, r. \]
Constraints

Precedence constraints

\[ z_{abnr} + z_{banr} \leq 1, \, \forall a, b, n, r. \]

Single successor/predecessor

\[ \sum_{b \in A \setminus \{a\}} z_{abnr} = w_{anr}, \, \forall a, n, r, \]

\[ \sum_{b \in A \setminus \{a\}} z_{banr} = w_{anr}, \, \forall a, n, r. \]
Mixed integer optimization problem

Constraints

Travel time

$$t_{anr} = \sum_{b \in A} z_{abnr} \rho(s_a, s_b).$$

Consistent timing

$$(z_{abnr} - 1) T \leq x_{anr} + \tau_{anr} + t_{anr} - x_b \leq (1 - z_{abnr}) T, \ \forall a, b, n, r.$$  

Mutually exclusive duplicates

$$\sum_{a \in B_k} w_{anr} = 1, \ \forall k, n, r.$$
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Internal data set

Data collection
- One-week survey on planned daily schedules for 10 individuals.
- Planning across **week** and **day**.
- Resulting data (unavailable in traditional trip diaries):
  - Set of all considered activities and locations.
  - Preferences in terms of start times and duration.
  - Flexibilities for start times and duration.
Weekly plan: filled on Sunday, considerations for next Mon-Sun

Considered activities (*Which out-of-home activities do you plan to do this week?):
- Choice between 9 categories + 1 “other” option
- Frequency of the activity

Set of transportation modes (for travel time computations)

Routine preferences:
- Minimal daily duration at home (Mon-Fri and Sat-Sun)
- Typical daily duration at work
- Earliest departure from home (Mon-Fri and Sat-Sun)
- Latest arrival at home (Mon-Fri and Sat-Sun)
Examples

Data set

- **Daily plan**: filled each day (Sun-Sat), considerations for the next day
- Considered activities (*Which activities do you plan to do tomorrow?*)
- Preferred times:
  - Start time: absolute value or relative, e.g. “after work”
  - End time
  - Duration
- Considered location(s) and feasible time windows for each
- Flexibility (early, late, short, long):
  - -1: not flexible
  - 0: moderately flexible (threshold value to be specified in minutes)
  - 1: flexible
- Other constraints (e.g. drop-off at home after grocery shopping)
**Data set**

**Model input:**
- All possible activities for the week (Mon-Fri and Sat-Sun)
- All considered locations and travel times matrix (for the preferred mode(s), computed using Google Maps)
- Individual flexibilities and preferred times for each activity

**Output:**
- Optimal schedule for 1 day (Mon-Fri or Sat-Sun)
Individual 1 (weekday)

- Considered activities → location(s):
  - work → 1 (home), 2
  - education → 1
  - errands → 2, 3
  - fitness → 4
  - leisure/social → 5
  - lunch → 2

- Timing preferences and flexibility from daily schedules
- Preference for home time from week schedule
Individual 1 (weekday)

Optimal schedules generated for random draws of $\varepsilon_{an} \sim \mathcal{N}(0, 1)$
Examples

Individual 2 (weekday)

- Considered activities → location(s):
  - work → 1 (home), 2
  - errands → 3, 4, 5
  - fitness → 1
  - leisure/social → 2, 6
  - lunch → 2
Individual 2 (weekday)

Optimal schedules generated for random draws of $\varepsilon_{an} \sim \mathcal{N}(0, 1)$
Conclusions

Achievements so far

- Formulation of the model.
- Applied on a simple case.
- The results make sense.
- We are able to draw from a distribution of activity schedules.

Challenges

- Use of real data.
- Parameter estimation.
Conclusions

Real data

- 2015 Swiss Mobility and Transport Microcensus.
- Daily trip diaries for 20’000 individuals.
- Records of activities and visited location.
- Also: 2012–2015 London Travel Demand Survey.

Challenges: classical RP issues

- No information about unchosen alternatives.
- Latent preferences.
Parameter estimation

- Prior: \( f(\beta) \).
- Data: \( Y = (i_n, x_n)_{n=1}^N \).
- Likelihood: \( L(Y|\beta) \).
- Parameters:

\[
f(\beta|Y) \propto L(Y|\beta)f(\beta).
\]

Challenges

- Metropolis-Hastings algorithm.
- Calculation of the likelihood.
Bibliography I


