An Exact Method to Solve the Multi-Trip Vehicle Routing Problem with Time Windows

F. Hernandez

***

École Polytechnique de Montréal (MAGI) and CIRRELT, Canada

***

TRANSP-OR Seminar

August 2012

Summary

Problem statement

Instance

- an oriented graph $G = (V, A)$.
- $V = \{0, \cdots, n\}$ with 0 the depot and $1, \cdots, n$ the customers
- a cost c_{ij} and a travel time t_{ij} for each arc $(i, j) \in A$
- for each customer $i \in \{1, \cdots, n\}$
  - demand d_i
  - service time st_i
  - a time windows $[a_i, b_i]$
- U vehicles allowed
- a capacity Q
- planning time horizon $[0, T]$

- Each customer must be visited within its time window
- vehicles may arrive earlier and wait before start the service

Problem statement

Objective:

Find a set of trips with minimal cost visiting all customers and respecting capacity and time windows constraints such that :

- two trips are not traveled at the same time by the same vehicle
- at most U vehicles are used

- A trip is portion of a vehicle route issued from the depot and coming back to the depot

Litterature review

Meta-heuristics

- Fleishmann (1990) : first idea of multi-trip
- Tabu search : Taillard, Laporte and Gendreau (1996), Brandao and Mercer (1998)
- Genetic algorithm : Salhi and Petch (2004)
- Decomposition approach : Battarra, Monaci and Vigo (2009)

Exact methods for a variant where a limit duration is imposed on the trip

- Azi et al (2007 and 2010) and Macedo et al (2011)
  - Limit duration decrease the complexity that allows the use of a specific strategy
  - $\Rightarrow$ In our problem there is no limit duration

MTVRPTW vs VRPTW

MTVRPTW $\Rightarrow$ variant of the vehicle routing problem with time windows (VRPTW)

VRPTW	MTVRPTW

- Visit all customers (graph covering)
- 1 demand and 1 service time per customer
- 1 time windows per customer
- 1 cost and 1 travel time between each customer

VRPTW

- unlimited fleet
- 1 vehicle = 1 route

MTVRPTW

- limited number of vehicles
- 1 vehicle = many trips

A set covering problem

Like VRPTW

- Linear relaxation of explicit formulation is very weak
- $\Rightarrow$ Formulation where variables represent trips

MTVRPTW $\neq$ VRPTW

- Temporal constraints appear between two trips
- $\Rightarrow$ Trips must be located in time

MTVRPTW

- Trips definition is extended
- $\Rightarrow$ Structure definition

Definitions of *structure* and *trip*

Structure definition

A structure is defined by:

- sequence of visited customers
- length / cost
- duration
- time window $[\mathcal{A}, \mathcal{B}]$ where $\mathcal{A}$ is the depot earliest departure time and $\mathcal{B}$ is the depot latest arrival time for which this structure is valid and its duration is minimal (i.e., minimum waiting time)

Trip definition

A trip is defined by a structure and:

- start and end times

Many trips with different schedules can be derived from every structure

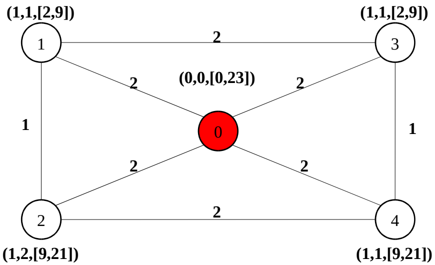
A set covering formulation for VRPTW

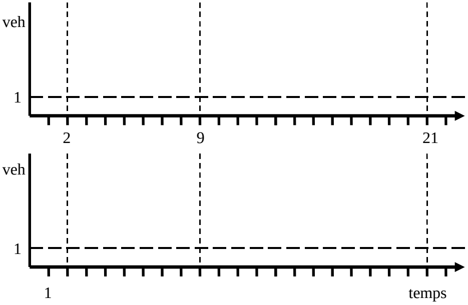
- Ω a set of feasible trips, fixed in time
- θ_k indicates the number of times where trip r_k is selected for covering, c_k cost of trip r_k
- $a_{ik} = 1$ if the customer i is visited by r_k, 0 else

$$
\begin{aligned}
& \text{minimize} \sum_{r_k \in \Omega} c_k \theta_k \\
& \sum_{r_k \in \Omega} a_{ik} \theta_k \geq 1 \qquad (i \in V \setminus \{0\}) \\
& \theta_k \in I\!N \qquad (r_k \in \Omega)
\end{aligned}
$$

How to model the temporal constraints ?

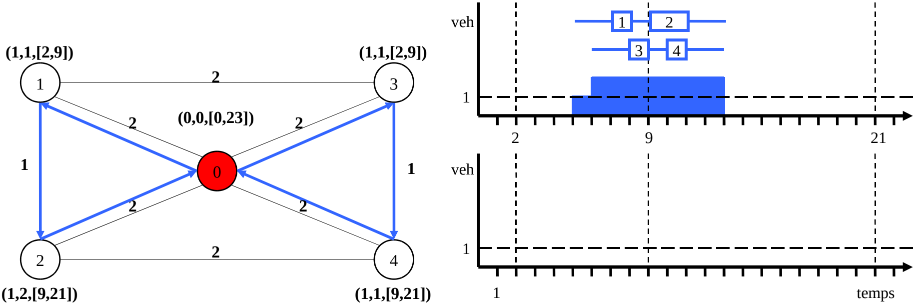
Trip succession





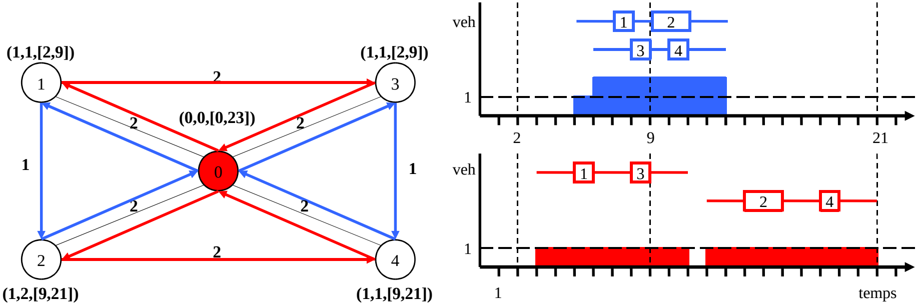
- 1 vehicle with a capacity of 2

Trip succession



- 1 vehicle with a capacity of 2
- VRPTW : Solution cost 10, not feasible

Trip succession



- 1 vehicle with a capacity of 2
- VRPTW : Solution cost 10, not feasible
- MTVRPTW : Solution cost 12, feasible

A set covering formulation for MTVRPTW

- Ω a set of feasible trips, fixed in time
- θ_k indicates the number of times where trip r_k is selected for covering, c_k cost of trip r_k
- $a_{ik} = 1$ if the customer i is visited by r_k, 0 else
- $b_{tk} \in \{0, 1\}$ indicates if the trip r_k includes the instant δ_t

$$\text{minimize} \sum_{r_k \in \Omega} c_k \theta_k$$

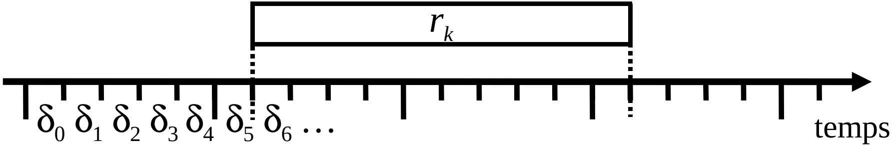
$$\sum_{r_k \in \Omega} a_{ik} \theta_k \geq 1 \qquad (i \in V \setminus \{0\})$$

$$\theta_k \in IN \qquad (r_k \in \Omega)$$

$$\sum_{r_k \in \Omega} b_{tk} \theta_k \leq U \qquad (\forall \delta_t)$$

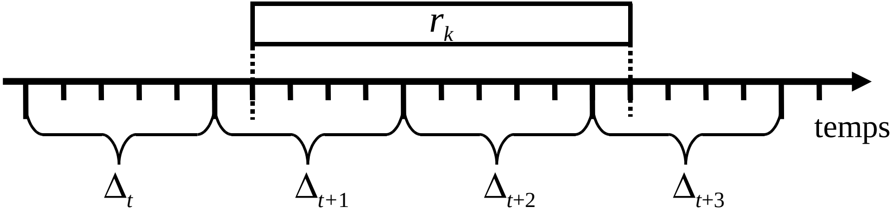
One time constraint by instant ?

- one time interval δ_t by instant $\Rightarrow$ b_{tk} are binary
- combinatorial explosion of constraint number related to temporal precision



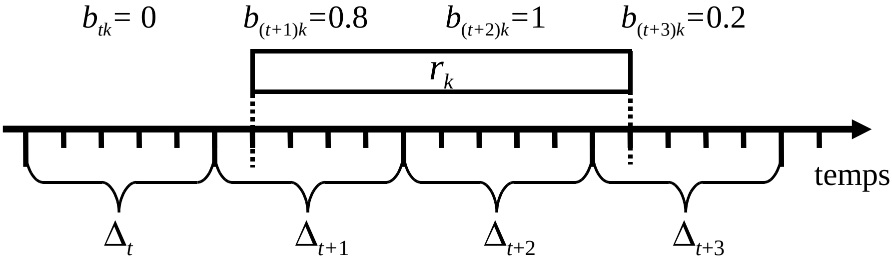
One time constraint by instant ?

- one time interval δ_t by instant $\Rightarrow$ b_{tk} are binary
- combinatorial explosion of constraint number related to temporal precision
- time interval length = minimal duration of possible trips



One time constraint by instant ?

- one time interval δ_t by instant $\Rightarrow b_{tk}$ are binary
- combinatorial explosion of constraint number related to temporal precision
- time interval length = minimal duration of possible trips
- $\Rightarrow b_{tk} \in [0, 1]$ become the fraction of time interval Δ_t occupied by trip r_k



A set covering formulation for MTVRPTW

- Ω a set of feasible trips, fixed in time
- θ_k indicates the number of times where trip r_k is selected for covering, c_k cost of trip r_k
- $a_{ik} = 1$ if the customer i is visited by r_k, 0 else
- $b_{tk} \in [0, 1]$ indicates if the trip r_k includes the instant Δ_t

$$\textit{minimize} \sum_{r_k \in \Omega} c_k \theta_k$$

$$\sum_{r_k \in \Omega} a_{ik} \theta_k \geq 1 \qquad (i \in V \setminus \{0\})$$

$$\theta_k \in IN \qquad (r_k \in \Omega)$$

$$\sum_{r_k \in \Omega} b_{tk} \theta_k \leq U \qquad (\forall \Delta_t)$$

Two issues

Time interval length = minimal duration of possible trips

- relaxation of the problem

Ω is a set of feasible trips, fixed in time

- too many variables

Two issues

Time interval length = minimal duration of possible trips

- relaxation of the problem

- $\Rightarrow$ if the solution found is not feasible then the problem have to be solved with one time interval δ_t by instant

Ω is a set of feasible trips, fixed in time

- too many variables

Two issues

Time interval length = minimal duration of possible trips

- relaxation of the problem

- $\Rightarrow$ if the solution found is not feasible then the problem have to be solved with one time interval δ_t by instant

Ω is a set of feasible trips, fixed in time

- too many variables

- $\Rightarrow$ adressed by column generation

What is the column generation ?

Inspiration

Simplexe algorithm

- Only basic varaibles are interesting

How does a nonbasic variable becomes a basic variable ?

- In minimisation case : only if its reduced cost is negative

Method

The column generation consists to solve iteratively two problems.

- the restricted master problem = problem restricted to a sub set of variables (basic and nonbasic)
- the pricing problem = Find new nonbasic variables that can become basic

Stop when the pricing problem don't find new nonbasic variables that can improve the solution.

Column generation

Problem decomposition

- Restricted master problem : Set covering problem where the integrity constraints are relaxed, reduced to a subset of variables Ω_w
- Subproblem : Find a negative reduced cost variable $\Rightarrow$ Elementary shortest path problem with resource constraints (ESPPRC)

Column generation

Problem decomposition

- Restricted master problem : Set covering problem where the integrity constraints are relaxed, reduced to a subset of variables Ω_w
- Subproblem : Find a negative reduced cost variable $\Rightarrow$ Elementary shortest path problem with resource constraints (ESPPRC)

Reduced cost

$$c_k^r = c_k - \sum_{i \in V \setminus \{0\}} a_{ik} \lambda_i + \sum_{\Delta_t} b_{tk} \mu_t$$

- λ_i dual value associated to customer i
- μ_t dual value associated to time interval Δ_t

Dynamic programming :

- labels $= L_{num} = (T^1_{num}, \cdots, T^n_{num})$
- each node has a label list
- during label extension
  - create a new label and insert it in correponding node label list
  - set resource consumption
  - check that the resource constraints are meet
- stop when no more label can be extended

Dynamic programming :

- labels = $L_{num} = (T^1_{num}, \cdots, T^n_{num})$
- each node has a label list
- during label extension
  - create a new label and insert it in correponding node label list
  - set resource consumption
  - check that the resource constraints are meet
- stop when no more label can be extended

Problem

- too many generated labels

Dynamic programming :

- labels = $L_{num} = (T^1_{num}, \cdots, T^n_{num})$
- each node has a label list
- during label extension
  - create a new label and insert it in correponding node label list
  - set resource consumption
  - check that the resource constraints are meet
- stop when no more label can be extended

Problem

- too many generated labels

Solution

- apply a dominance relation after each extension on corresponding label list

Subproblem

Objective 1

- take into account the loading times

The loading times are at the depot before departure

- add a customer to a trip $\Rightarrow$ delay time of service for the previous customers

Subproblem

Objective 1

- take into account the loading times

The loading times are at the depot before departure

- add a customer to a trip $\Rightarrow$ delay time of service for the previous customers
- $\Rightarrow$ extend the label in backward move

Subproblem

Objective 1

- take into account the loading times

The loading times are at the depot before departure

- add a customer to a trip $\Rightarrow$ delay time of service for the previous customers
- $\Rightarrow$ extend the label in backward move

Objective 2

- generate a trip with minimal reduced cost

Subproblem

Objective 1

- take into account the loading times

The loading times are at the depot before departure

- add a customer to a trip $\Rightarrow$ delay time of service for the previous customers
- $\Rightarrow$ extend the label in backward move

Objective 2

- generate a trip with minimal reduced cost

avoid the time interval associated with a dual value μ_t not null

- take into account all possible departure time

Subproblem

Objective 1

- take into account the loading times

The loading times are at the depot before departure

- add a customer to a trip $\Rightarrow$ delay time of service for the previous customers
- $\Rightarrow$ extend the label in backward move

Objective 2

- generate a trip with minimal reduced cost

avoid the time interval associated with a dual value μ_t not null

- take into account all possible departure time
  - too many labels

Subproblem

Objective 1

- take into account the loading times

The loading times are at the depot before departure

- add a customer to a trip $\Rightarrow$ delay time of service for the previous customers
- $\Rightarrow$ extend the label in backward move

Objective 2

- generate a trip with minimal reduced cost

avoid the time interval associated with a dual value μ_t not null

- take into account all possible departure time
  - too many labels
  - $\Rightarrow$ group labels that represent the same structure and select a representative

Subproblem: Label groups and representative label

Label group definition

A group of labels is a set of labels that represent the same partial path and whose arrival-to-destination dates belong to the same time interval

How to select a representative label

two main rules

- it can be dominated by an other label if and only if all labels of its group are dominated by this label
- it must accept all extensions accepted by at least one label of its group

Subproblem: Label groups and representative label

Label group definition

A group of labels is a set of labels that represent the same partial path and whose arrival-to-destination dates belong to the same time interval

How to select a representative label

two main rules

- it can be dominated by an other label if and only if all labels of its group are dominated by this label
- it must accept all extensions accepted by at least one label of its group

In our previous studies

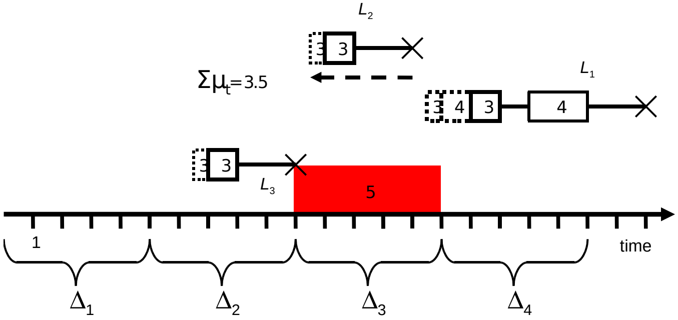
- To compare the time dependent reduced cost of two different labels L_k and $L_{k'}$ during the dominance relation process we use this formula
- $c_k^r + \sum_{t>h_{k'}}^{t=h_k} \mu_t \leq c_{k'}^r$

Subproblem: Select a representative label

Select relative to the reduced cost

Let L_2 and L_3 in the same group (their represent the same partial path)

- reduced cost formula : $c_k^r + \sum_{t>h_{k'}}^{t=h_k} \mu_t \leq c_{k'}^r$
- $c_1^r = 3$; $c_3^r = 5$; $c_2^r = c_3^r + 3.5 = 8.5$
- comparaisons :

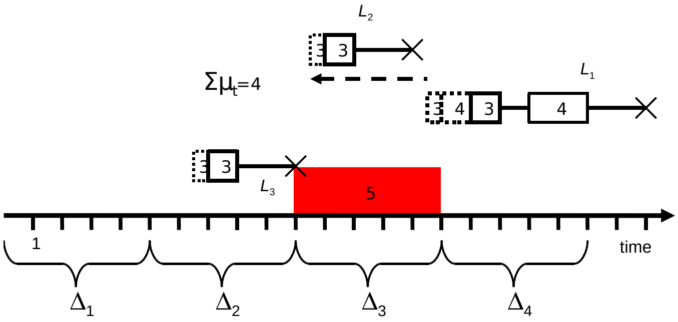


Subproblem: Select a representative label

Select relative to the reduced cost

Let L_2 and L_3 in the same group (their represent the same partial path)

- reduced cost formula : $c_k^r + \sum_{t>h_{k'}}^{t=h_k} \mu_t \leq c_{k'}^r$
- $c_1^r = 3$; $c_3^r = 5$; $c_2^r = c_3^r + 3.5 = 8.5$
- comparaisons : $c_1^r + 4 < c_2^r$

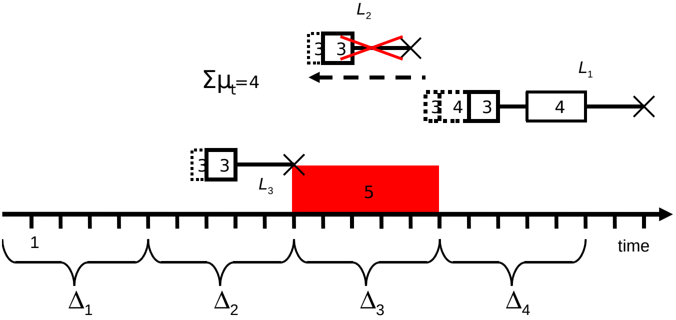


Subproblem: Select a representative label

Select relative to the reduced cost

Let L_2 and L_3 in the same group (their represent the same partial path)

- reduced cost formula : $c^r_k + \sum_{t>h_{k'}}^{t=h_k} \mu_t \leq c^r_{k'}$
- $c^r_1 = 3$; $c^r_3 = 5$; $c^r_2 = c^r_3 + 3.5 = 8.5$
- comparaisons : $3 + 4 < 8.5$

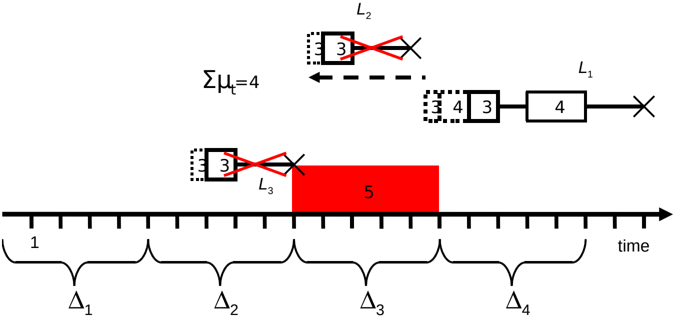


Subproblem: Select a representative label

Select relative to the reduced cost

Let L_2 and L_3 in the same group (their represent the same partial path)

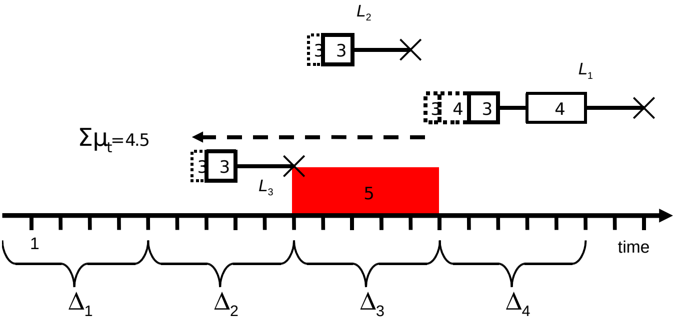
- reduced cost formula : $c_k^r + \sum_{t>h_{k'}}^{t=h_k} \mu_t \leq c_{k'}^r$
- $c_1^r = 3$; $c_3^r = 5$; $c_2^r = c_3^r + 3.5 = 8.5$
- comparaisons :



Select relative to the reduced cost

Let L_2 and L_3 in the same group (their represent the same partial path)

- reduced cost formula : $c^r_k + \sum_{t>h_{k'}}^{t=h_k} \mu_t \leq c^r_{k'}$
- $c^r_1 = 3$; $c^r_3 = 5$; $c^r_2 = c^r_3 + 3.5 = 8.5$
- comparaisons : $c^r_1 + 4.5 > c^r_3$

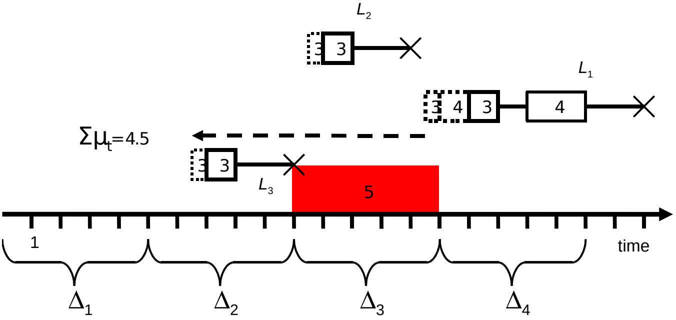


Subproblem: Select a representative label

Select relative to the reduced cost

Let L_2 and L_3 in the same group (their represent the same partial path)

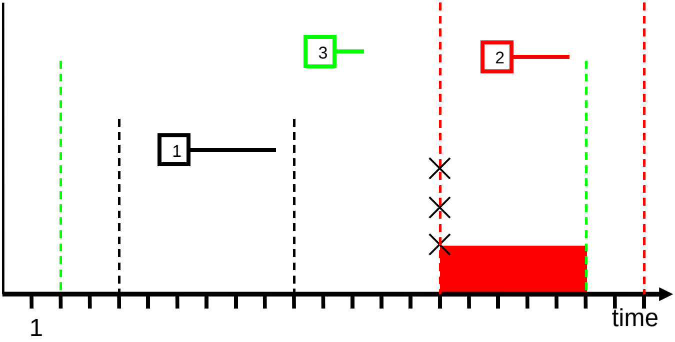
- reduced cost formula : $c_k^r + \sum_{t>h_{k'}}^{t=h_k} \mu_t \leq c_{k'}^r$
- $c_1^r = 3$; $c_3^r = 5$; $c_2^r = c_3^r + 3.5 = 8.5$
- comparaisons : $3 + 4.5 > 5$



Subproblem: Select a representative label

Select relative to the possible extensions

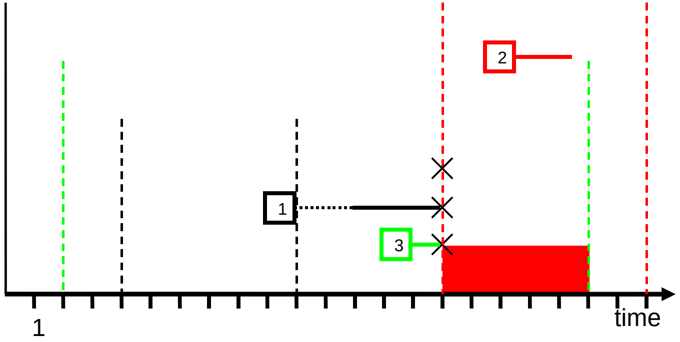
- it must accept all extensions accepted by at least one label of its group



Subproblem: Select a representative label

Select relative to the possible extensions

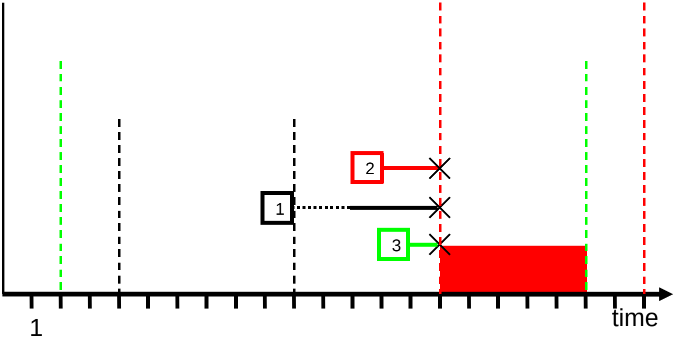
- it must accept all extensions accepted by at least one label of its group



Subproblem: Select a representative label

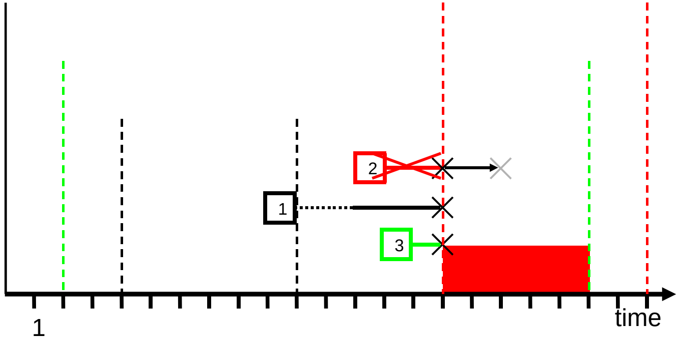
Select relative to the possible extensions

- it must accept all extensions accepted by at least one label of its group



Select relative to the possible extensions

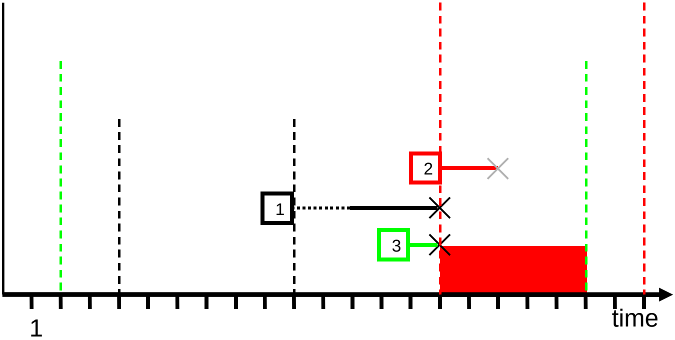
- it must accept all extensions accepted by at least one label of its group
- all labels of the group have to take into account ⇒ retardation



Subproblem: Select a representative label

Select relative to the possible extensions

- it must accept all extensions accepted by at least one label of its group
- all labels of the group have to take into account $\Rightarrow$ retardation



New parameter : retardation of the arrival time to the depot

New definition of representative labels

Définition

A path p from j to 0 is represented by the label:
$L_p = (c_p^r, h_p, q_p, d_p, rd_p, V_p^1, \cdots, V_p^n)$, where:

- c_p^r is the reduced cost of p
- h_p is the starting time of the service of j
- q_p is the carried quantity
- d_p is the arrival time to the depot
- rd_p is the possible retardation of the arrival time of the depot
- V_p^i =1 if the customer i is unreachable by p, 0 else

Initialisation

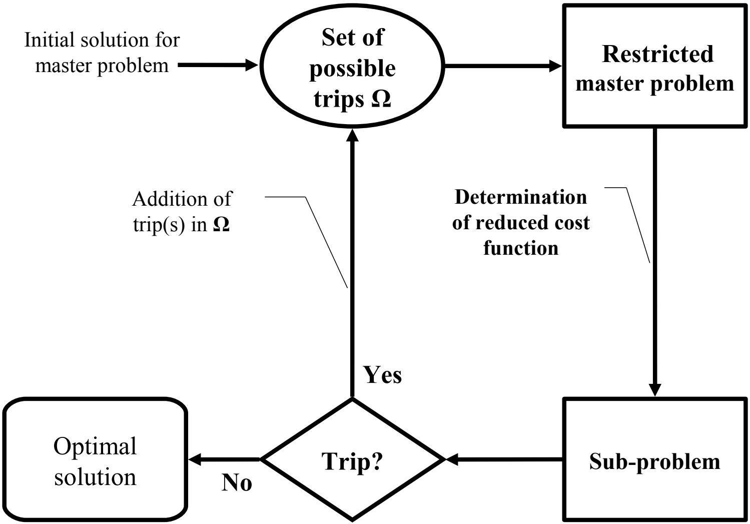
- One label is created at the end of the planning time horizon and one label at the beginning of each time interval with a not null dual value

Dominance relation

Label L_1 dominates label L_2 if and only if:

- $q_1 \leq q_2$ (carried quantity)
- All customers unreachable by L_1 are not by L_2 too
- $h_2 + rd_2 \leq h_1$ (starting time of the service)
- $c_1^r + \sum_{t>h_2}^{t=h_1} \mu_t \leq c_2^r$

Column generation scheme



Branch and Price

Recall

Branch and bound	Branch and Price
Simplexe	Column generation

Branch and Price

Recall

Branch and bound	Branch and Price
Simplexe	Column generation

Branching on arcs

- select an arc (i,j) with a fractional flow
- $x_{ij} = 1 \Rightarrow \theta_k = 0$ if trip k visits customer i or j without using arc (i, j)
- $x_{ij} = 0 \Rightarrow \theta_k = 0$ if trip k uses arc (i, j)

Branch and Price

Recall

Branch and bound	Branch and Price
Simplexe	Column generation

Branching on arcs

- select an arc (i,j) with a fractional flow
- $x_{ij} = 1 \Rightarrow \theta_k = 0$ if trip k visits customer i or j without using arc (i,j)
- $x_{ij} = 0 \Rightarrow \theta_k = 0$ if trip k uses arc (i,j)

Problem

- having all arcs with an integer flow ($x_{ij} \in IN$) does not imply that the solution is integer ($\theta_k \in \{0,1\}$)

Recall

Branch and bound	Branch and Price
Simplexe	Column generation

Branching on arcs

- select an arc (i,j) with a fractional flow
- $x_{ij} = 1 \Rightarrow \theta_k = 0$ if trip k visits customer i or j without using arc (i, j)
- $x_{ij} = 0 \Rightarrow \theta_k = 0$ if trip k uses arc (i, j)

Problem

- having all arcs with an integer flow ($x_{ij} \in IN$) does not imply that the solution is integer ($\theta_k \in \{0, 1\}$)

- $\Rightarrow$ call a repair strategy

Case when the flow matrix is integer and some variables fractional

- The flow matrix is such that every customer has an unique successor
- The flow matrix represents a set of structures
- In the actual fractional solution some structures are represented by several trips with different time positions

We consider the following issue:

- Is it possible to assign a single time position to every structure?
- Equivalent to determining the existence of an integer solution supported by the integer flow matrix

Solved using a VRPTW modeling

- Nodes: structures
- Arcs: feasible successions

Branch and Price: case of an integer flow matrix

Instance of VRPTW

- customers
  - service time
  - demande
  - time window
- arcs (i,j)
  - travel time
  - cost

Our branching problem

- structures
  - duration
  - time window
- arcs (i,j)
  - cost of the structure j

Branch and Price: case of an integer flow matrix

Instance of VRPTW

- customers
  - service time
  - demande
  - time window
- arcs (i, j)
  - travel time
  - cost

Our branching problem

- structures
  - duration
  - 0
  - time window
- arcs (i, j)
  - 0
  - cost of the structure j

Branch and Price: case of an integer flow matrix

Instance of VRPTW

- customers
  - service time
  - demande
  - time window
- arcs (i, j)
  - travel time
  - cost

Our branching problem

- structures
  - duration
  - 0
  - time window
- arcs (i, j)
  - 0
  - cost of the structure j

We solve it with:

- Standard branch and price scheme

Branch and Price: case of an integer flow matrix

if a solution of the VRPTW is found

- Update the upper bound
- prune the node

If no VRPTW solution exists

- Select an arc not involved in previous branching constraints
- Branch on this arc

Implementation

Hardware and software

- Language: C++
- Solver: GLPK (open source)
- Computer: Intel Core2Duo E7300 2.66GHz, 3Gb RAM

Benchmarks

- based on Solomon's instances: R2, RC2, C2
- 25 customers: 2 vehicles allowed
- 50 customers: 4 vehicles allowed
- computation time limited to 30h per instance

Results

Instances with 25 customers

Instances	% GAP			Time (sec)		
	Min	Max	Avg	Min	Max	Avg
C	0.55	4.85	2.27	12	371	170
R	0.44	4.70	2.41	22	3769	1006
RC	3.52	8.98	5.41	9	20038	12537

- 25/27 instances closed
- great variation of computation times and GAPs

Results

Instances with 50 customers

Instances	% GAP	Time (sec)
C	-	-
R201-50	1.58	237
R202-50	2.42	78880
R205-50	2.39	24062
RC201-50	2.24	662
RC202-50	2.40	99346

- Only 5/27 instances closed
- great variation of computation times

Conclusion

Conclusion

- Master problem: time constraint between trips
  - time aggregation
  - all solution found with time aggregation are feasible that imply a good relaxation of time constraint
- Sub-problem: time dependent reduced cost
  - appropriate dynamic programming $\Rightarrow$ representative labels

Perspectives

- Problem with a great temporal dependence
  - mainly solved with a structural branching scheme
  - $\Rightarrow$ improve the branching scheme with temporal branching politic

Thank you for your attention

florent.hernandez@cirrelt.ca