The berth allocation problem in bulk ports

Nitish Umang
Michel Bierlaire
Ilaria Vacca

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Abstract

Maritime transportation is a major channel of international sea trade which has increased significantly over the past few decades. The proper planning and management of port operations in view of the ever growing demand represents a big challenge. From the past research, it is well established that operations research methods and techniques can be successfully used to optimize port operations and enhance terminal efficiency. While significant contributions have been made in the field of container terminal management, relatively little attention has been directed to bulk port operations. In general, the bulk terminal managers are faced with the challenge of maximizing efficiency both along the quay side and the yard; the objective is usually to minimize the service times of vessels, which includes both the waiting times and the handling times of vessels at the berth. Moreover, the large number of complexities and uncertainties involved in bulk port operations which can potentially disrupt the normal functioning of the port and require quick real time action, also need to be considered at the planning level.

In this work, we start with a general description of bulk port operations, along with a brief review of the past literature related to bulk ports. Through our collaboration with the biggest bulk port in the Middle East, SAQR port in Ras Al Khaimah, UAE, we have identified some key issues and possible sources of disruption. We focus on the problem of waiting times at the berth and review the literature on the berth allocation problem in port terminals. Then, we present a mixed integer linear optimization model for the berth allocation problem in bulk ports, which considers interactions between the decision problems arising at the berth and yard management. We also present preliminary computational results for instances inspired by port real data. We conclude the paper with suggestions for future work and open issues.

Keywords
bulk ports, integrated planning, robustness, berth allocation, hybrid layout, dynamic arrivals
1 Introduction to bulk ports

Maritime transportation is a major channel of international trade. The international sea borne trade has increased by more than 120% by weight, from 1980 to 2008 (UNCTAD, 2009). Some of the major contributing factors to the continuing growth in maritime transportation are population growth, increasing standard of living, rapid industrialization, exhaustion of local resources, road congestion, and elimination of trade barriers. Since the beginning of the decade all forms of cargo (general, dry bulk and liquid bulk) have registered an increase in shipping tonnage. The figures for dry bulk, liquid bulk and containerized cargo are particularly impressive at 52%, 48% and 154% respectively. It is also interesting to note that the total volume of dry bulk cargoes loaded in 2008 stood at 5.4 billion tons, accounting for 66.3 per cent of total world goods loaded (UNCTAD, 2009).

The plot in Figure 1 represents the development in international sea borne trade over the last four decades. As we can see, from 2000 to 2008 alone, oil trade including crude and oil products has risen by more than 27%, while the trade in major bulks including iron ore, grain, coal, bauxite/alumina and phosphate has risen by almost 63%. The total growth in international sea borne trade in this period is over 36%. The proper planning and management of port operations in view of this ever growing demand represents a big challenge.

Maritime cargo can be broadly classified into general cargo and bulk cargo. The former consists of break bulk (sacks, cartons, crates, drums, bags), neo bulk (lumber, paper, steel, autos) and containerized cargo (lift on/ lift off and roll on/ roll off). Bulk cargo consists of dry bulk cargo

![Graph: Development in international sea borne trade](image-url)
such as grains, sand, metal, coal, fertilizer etc. and liquid bulk cargo such as LNG, petroleum, chemicals, vegetable oil etc.

A bulk port terminal is typically a zone of the port where sea-freight docks on a berth and is stored in a buffer area called yard for loading, unloading or transshipment of cargo. A material handling system for iron ore in a bulk port is illustrated in Figure 2.

Bulk port terminals typically have the following five operations that may be evaluated for port productivity:

1. berth and vessel activities;
2. ship loading or discharge;
3. apron to storage transfer;
4. storage;
5. intermodal transfer and inland distribution.

### 1.1 Berth and vessel activities

These activities comprise the berth availability for vessels and berth limitations on vessel capacity. These operations estimate the cargo capacity of ships calling at the facility, the percentage of cargo transferred at each call, the berth occupancy ratio and the number of vessel calls which are possible in a year. Research work done in this area has primarily focused on the problem of allocating vessels to berths, with constraints such as the vessel length, vessel draft, berth draft, time windows for arrivals, priorities assigned to vessels, favorite berthing locations etc. Queuing approaches for modeling and simulation of the vessel arrival process in bulk ports have been studied by Altiok (2000) and Jagerman and Altiok (2003). These authors consider the vessel arrival process as a SHIP/G/1 queuing system, and study the impact on port performance of ratio between fixed inter-arrival times and lay period for arrivals, and correlation between inter-arrival times.

### 1.2 Ship loading or discharge

The equipment used for loading or unloading cargo onto or from the vessel depends on the characteristics of both the vessel and the cargo. An example of loading and unloading operations is provided in Figures 3 and 4 respectively. Dry bulk cargo is typically transferred from the quay side to the vessel (or vice versa) using equipment such as mobile harbor cranes, ship loaders, bucket wheel unloaders (MHS’s), clamshell grabs, loading spouts, etc.; they are illustrated in Figures 5, 6 and 7. A wide variety of specialized equipment is also used. Conveyor systems are used to directly transfer the cargo from a nearby factory or storage terminal to the
vessel (cf. Figure 8). Liquid bulk cargo is typically loaded by hoses and pumps located on the pier, while discharge may be accomplished through the use of ships pumps or pipelines that directly transfer the cargo from the vessel to a tank farm on or near the terminal. The number and productivity of available cranes (or unloaders), and thus the loading or unloading rate for each vessel is estimated by this function. An interesting problem to explore in this context is the assignment of cranes (Daganzo, 1989) to specific tasks as well as the scheduling of loading or unloading operations, taking into account the operational constraints. Work on evolutionary optimization in belt conveyor design and conveyor loading chute design has been done by Wensrich (2003) and Wheeler et al. (2007).

1.3 Apron to Storage Transfer

The equipment used for transfer of cargo from the apron to the terminal storage facility depends on the characteristics of the cargo. Dry bulk cargo is typically transferred from the quay side to the storage location on the yard (or vice versa) using a wide variety of auxiliary equipment such as loading shovels, mini loaders, wheel loaders etc. This terminal operation allows to evaluate the productivity of the transfer equipment such as loading shovels etc. within the terminal.

1.4 Storage

The buffer area for loading, discharging or transshipment of cargo is called the yard. The management of yard operations involves a wide range of decision problems in accordance with the cargo characteristics, such as routing and scheduling of cranes for transfer of cargo within the yard, and storage allocation of multiple brands of cargo on the yard. Dry bulk cargo can be stored in a variety of enclosures or open yard configurations. The storage component for both dry and liquid bulk cargo can also include other value-added activities such as blending or processing. Storage facilities determine the storage yard’s peak static capacity. The yard throughput is largely determined by the efficient management of yard operations depending on the cargo turnover rate and yard utilization factor. Relevant work done in this research area related to handling of materials includes Kim et al. (2009), who solve a MIP model for yard allocation using CPLEX and compare their results with real world data showing cost savings of up to 21.3%. Ago et al. (2007) solve a MILP using Lagrangian decomposition for simultaneous optimization of storage allocation and routing problems for belt conveyor transportation.
Figure 2: *Material handling system for iron ore.*

Figure 3: *Loading operations.*

Figure 4: *Unloading operations.*

Figure 5: *Load shovel.*

Figure 6: *Wheel loader.*

Figure 7: *Mobile harbor crane.*

Figure 8: *Conveyor.*
1.5 Intermodal transfer and inland distribution

Usually, dry bulk cargo is distributed inland by rail mode. Trucks are also used when the cargo is to be distributed in areas local to the port. Liquid bulk cargo is usually distributed by rail, truck or pipeline. The choice of the transfer mode is governed by cargo characteristics as well as volume of the bulk material to be transferred.

1.6 Performance indicators

As evident from the above description of bulk ports, there could be several performance indicators for bulk terminals such as berth occupancy, yard tank occupancy, turnover factor, throughput per berth per quay, number of vessels, average waiting time, vessel turnaround time, revenue per vessel, revenue per m³ tank volume, realized loading efficiency, berth capacity etc. To get an idea of how the existing port infrastructure is performing, and determine if an investment in new infrastructure and expansion of terminal capacity is worthwhile, the answers may be provided by one or more of these performance indicators.

We remark that the complexity in evaluation of port productivity is exacerbated due to the following reasons: 1) The wide variation in port operations ranging from loading or discharging cargo to transfer operations and inland distribution 2) Evaluation and identification of the specific units of productivity to be measured. 3) Complexity in resource allocation at a single port, for example, multiple marine terminals at a single port may share common resources such as berths, cranes or gates etc. To quantify the productivity of port terminal operations, two different approaches are generally used. To evaluate and improve specific components of the terminal operations, a micro-analysis of each step in the cargo handling process over a day-to-day or even hour-to-hour time frame is appropriate. On the other hand, if the objective to evaluate the overall effectiveness of an entire terminal or port, a more global approach may be used to carry out macro-analysis of port operations on a much longer time period basis.

The paper is structured as follows. In Section 2 we describe the port of SAQR, the biggest bulk port in the Middle East, focusing on critical operations and major issues occurring at the port. A general discussion on research trends and challenges in optimizing bulk port operations is provided in Section 3. In the second part of the paper, we present a model for the berth allocation problem, taking into account specific features of bulk ports. The problem is described in Section 4 and modeled in Section 5. The presented mixed integer linear program is validated and tested on instances based on real data. Preliminary computational results are discussed in Section 6, while Section 7 concludes the paper.
2 The SAQR Port

In this section we discuss here the example of SAQR port in Ras Al Khaimah, UAE, which is a major bulk handling port and also our main collaborator in this research.

2.1 Background

SAQR port is strategically located in Ras Al Khaimah (RAK), the first emirate at the entrance of the Arabian Gulf. It is the biggest bulk commodity port in the entire Middle East, handling 30 million tons of bulk and assorted cargo annually. The port plays a key role in the economic growth of the RAK emirate, which has registered a significant growth in GDP from AED 6.6 billion in 2002 to AED 13.6 billion in 2008. The port is excellently positioned to distribute goods within UAE and beyond, owing to its unique geographical position, quality of service and excellent connectivity to the main road networks. It has regular sailings from the gulf countries, and services to and from the MENA region, Indian Sub Continent and other worldwide countries. As of March 2010 the operational management of all Ras Al Khaimah Ports has been placed under the responsibility of SAQR Port Authority.

Cargo is handled at SAQR port on port operated terminals. The port’s cargo handling depart-

Figure 9: Port layout of SAQR, Ras Al Khaimah, UAE.
ment specializes in dealing with a wide variety of imported and exported commodities: consignments of aggregates, cement, coal, clinker, iron ore, feldspar, clay, soda ash, silica sand, grain, animal feedstock, steel, project cargoes and petroleum products. In 2008, Saqr handled a total of 30.5 million tons of bulk and other assorted cargo, including 22.27 million tons of exports (risen drastically from 8.71 million tons in 2004) and 8.32 million tons of imported cargo (risen more slowly from 6.54 million tons in 2004).

The port layout is illustrated in Figure 9. The port has 12 berths, all having an alongside depth of 12.2 metres at mean low water spring tide. These consist of 8 x 200 metres bulk handling berths, 3 x 200 metres container handling berths and 1 general purpose roll-on/roll-off berth. The port also has two ramps with specialized berths for handling bulk cement and aggregates.

2.2 Port resources and operations

We visited SAQR port in Ras Al Khaimah, UAE from 6th-10th November, 2010, to study the port practices and identify the key issues and sources of disruption at the port. The container terminal at SAQR port, opened in 2007 is managed and operated by Kuwait Gulf Links Port International (KGLPI). The container terminal has 3 x 200 berths with capacity to handle 350,000 TEUs container traffic, and is supported by 3 x 50 tonnes Ship-to-Shore (STS) gantry cranes and 6 Rubber Tired Gantry Cranes (RTG’s).

The bulk handling terminal at SAQR has a wide variety of equipment including a fleet of 13 units of mobile harbor cranes, fleet of fork lift trucks up to 40 tons SWL, 24 units of load-
ing shovels, mobile conveyors and mobile hoppers, 2 units of ship loaders, 27 units of wheel loaders, 5 units of mini loaders, 6 units of tug masters and 35 units of trailers.

The terminal has 8x200 meters bulk handling berths. Some berths are more in demand than others. In particular, specialized equipment such as conveyors and pipelines installed on certain berths enhance the demand for those berths. The conveyor system used for loading rock aggregates and limestone from a nearby factory to incoming vessels is installed at berth 5 (cf. Figure 10), and another conveyor for loading bulk cement from the cement factory is installed at berth 7 (cf. Figure 11). The pipelines used for discharging liquid bulk from vessels to liquid tank farms at the port are installed at berths 6, 7 and 11. The conveyors systems do not belong to the port.

Due to environmental reasons, coal and other dirty products are handled on the far side of the port on berths 11 and 12 to minimize pollution and dust generation at the port. Export of clinker is dedicated to berths 6 and 12.

2.3 Key issues and sources of disruption

During our visit to SAQR, we identified some key issues and sources of disruption at the port. In particular, it was seen that the delays at the berth were significant resulting in high waiting times for vessels at the berths and anchorage. These delays can be attributed to:

- unavailability of berths due to congestion of incoming vessels;
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• unavailability of required number and type of equipment at the desired time, either because the equipment is engaged in other tasks, or owing to unexpected breakdown in equipment disrupting the schedule of operations;
• uncertainty in arrival of cargo trucks for pickup or delivery of cargo.

In case of loading operations, in many cases, the full quantity of cargo doesn’t reach the yard at the time of loading, or there is an insufficient number of trucks to transfer cargo form the yard to the vessels. For discharging operations, it is often the case that the cargo discharged from a vessel and dumped adjacent to the quay, is not picked up by the cargo trucks for many days even after the vessel has sailed away. This makes that section of the quay unavailable for berthing other vessels resulting in important delays which propagate through the system.

Delays on the yard were also found to be significant. Trucks are used to collect the cargo from the yard for inland distribution of cargo to the local areas. The trucks are loaded with cargo using loading shovels. However, there is a lot of uncertainty in arrival times of trucks which is a major source of disruption, as it results in either the loading shovels being idle till enough trucks are sent by the agent, and conversely the port may be unable to provide the sufficient number of loading shovels when the trucks actually arrive. This is illustrated in the following pictures.

3 Research challenges in bulk ports

Research work done on optimization of port terminal operations suggests that integrated planning of related port operations significantly enhances the terminal efficiency by more effective utilization of the limited resources of the port and allows the terminal to have much improved control on its performance.

From the past OR literature on terminal operations, it can be seen that significant contribution has been made in the field of large scale optimization and integrated planning of operations in container terminals. Park and Kim (2003), Meisel and Bierwirth (2006), Giallombardo et al. (2010), Vacca (2011) study the integration of berth allocation and quay crane scheduling, while Bish et al. (2001) and Kozan and Preston (2006) analyze the integration of yard allocation and container transfers and many others.

Comprehensive literature surveys on the use of OR methods and techniques in context of container terminal operations can be found in Steenken et al. (2004), Stahlbock and Voss (2008) and Bierwirth and Meisel (2010). Bulk port terminals on the other hand have received far less attention. However, work done for container terminals can be used as a starting point for research in the context of bulk ports.
The large number of uncertainties involved in bulk port operations such as uncertainty in weather conditions, mechanical problems etc. can potentially disrupt the normal functioning of a port and require quick real time action to prevent damage to the minimum possible level. Some of the common problems and sources of disruption are uncertainty of information, changing estimated time of arrival of vessels, barges and trucks, last-minute changes (cargo suppliers and traders), change of modalities, variety of product conditions, variety of ship’s conditions, damages, weather, reliability equipment, change of vessel (un)loading rotation by shipmaster etc. To account for these various complexities and uncertainties in bulk port operations, it is crucial to include robustness in planning operations to minimize the probability of disruption in operations, and enable fast recovery in real time with minimum possible damage in the event of a disruption. The major objective of planning robust port operations is to minimize operational costs while maximizing system reliability. In particular, the aim is to minimize port vacancy while assuring that the service rendered to the vessels is in line with the widely accepted standards. In the context of container terminals, robust planning methods have been used by Gao et al. (2010) by considering stochasticity in vessel arrivals and by Han et al. (2010) by considering stochasticity in both vessel arrival times and handling times.

According to this analysis, the next step for improving bulk port operations is to see to what extent the work done on robust optimization in container terminals and other applications can be extended to bulk ports. It is crucial to identify similarities in applications, as well as identify specific issues and bottlenecks for bulk terminal operations. In particular, the design large scale optimization models for bulk port operations with emphasis on integrated planning and maintenance of operations represents, in our opinion, an interesting research challenge for the future.

### 3.1 The case of SAQR port

The issues and sources of disruption identified at the port call for proper planning and management of port operations; in specific, better coordination between berthing activities and yard operations. Furthermore, a primary issue that also needs to be taken into account during the planning phase is the enormous amount of uncertainty involved in the arrival times of vessels as well as the trucks belonging to the cargo agent. We believe that integrated planning of port operations and robust solutions would allow the terminal to reduce congestion, lower delay costs and enhance efficiency.

In particular, we focus on two crucial optimization problems.

**Berth Allocation** It refers to the problem of allocating vessels to berths while minimizing the total service times of vessels. Constraints and issues to be taken into account in the
optimization process include the vessel length, berth draft, time windows, availability of equipment such as tug boats, priorities assigned to ships, favorite berthing areas etc.

**Yard Allocation** It refers to decisions that concern the storage location and the routing of materials. This affects the travel distance of the material (between the berth and storage location on the yard) and the storage efficiency of the yard. When multiple brands of cargo are stored in the same area, as in the case of bulk ports like SAQR, the clearance distances between different brands also need to be considered in the modeling of yard operations.

To account for the various uncertainties in operations that result in unforeseen disruptions and delays, it is important to include the concept of robustness in the planning process, in order to minimize the probability of disruption in operations, as well as enable fast recovery in real time with minimum possible damage in the event of a disruption.

## 4 The Berth Allocation Problem

The Berth Allocation Problem (BAP) refers to the problem of serving a set of vessels for a given berth layout within the given planning horizon. The objective is usually to minimize the service times to vessels, though there could be several other objectives such as minimization of port stay time, minimization of number of rejected vessels, minimization of deviation between actual and planned berthing schedules etc.

There are several spatial and temporal constraints involved in the BAP, which lead to a multitude of BAP formulations. The existing models for BAP in literature can be classified on the basis of both these temporal attributes such as vessel arrival process, start of service, handling times of vessels as well as the spatial attributes relating to the berth layout, draft restrictions and others. We now attempt to provide a brief classification of the various BAP formulations based on some of these attributes.

According to Bierwirth and Meisel (2010), the vessel arrival process can be considered as *static* or *dynamic*. In the static case, there are no arrival times given for the vessels or the arrival times impose merely a soft constraint on the berthing times. In the dynamic variant, expected arrival times for vessels are given and vessels cannot berth before their arrival. The vessel arrivals can be further considered as *deterministic* in which fixed expected values of arrival times are given, or *stochastic* in which a distribution of arrival times may be given to account for uncertainty in arrivals.

The handling times for vessels can be assumed as fixed and unchangeable, or dependent on the berthing positions of vessels and/or work schedule and number of cranes assigned to ves-
sels. The handling times may also be considered as stochastic to account for uncertainty in handling times due to unforeseen disruptions such as equipment breakdown or unavailability of equipment or cargo due to any other reason.

Spatial constraints limit the feasible berthing positions of vessels according to a preset partitioning of the quay into berths. On the basis of berth layout, the BAP can be classified as discrete, continuous or hybrid (Bierwirth and Meisel, 2010). In the discrete case, the quay is divided into a set of sections or berths, and only one vessel can be served by each single berth at any given time. In the continuous case, there is no partitioning of quay, and a vessel can occupy any arbitrary position along the quay. This leads to better utilization of the quay space, but is computationally more complicated. In the hybrid case, the quay is partitioned into a set of sections, but a vessel can occupy more than one section at a time, and multiple vessels are also allowed to share the same berth at the same time. In addition, the draft restrictions on vessels which limit the feasible berthing positions of vessels to only those berths which have a draft higher than the draft of the vessel may or may not be considered in the BAP.

### 4.1 Literature review

In this section we present a brief review of past literature on the berth allocation problem in the context of container terminals.

**Discrete BAP** The static variant of discrete BAP has been studied by Imai *et al.* (1997) which minimizes the total service times of vessels and the deviation between arrival order and service order of vessels, Imai *et al.* (2001) and Imai *et al.* (2008). The dynamic discrete BAP problem is considered by Imai *et al.* (2001), Monaco and Sammarra (2007) and Imai *et al.* (2003). More recent approaches, such as Zhou and Kang (2008) and Han *et al.* (2010), solve the problem considering stochasticity in both arrival times and handling times of vessels. Cordeau *et al.* (2005) uses a Tabu Search method to solve the discrete dynamic BAP with due dates, which is further improved upon by Mauri *et al.* (2008) using a column generation approach that delivers better solutions in shorter runtime.

**Continuous BAP** The static continuous BAP has been considered by Li *et al.* (1998), Guan *et al.* (2002) and Park and Kim (2003). Guan and Cheung (2004) consider continuous dynamic BAP with fixed handling times using a tree search procedure to minimize the total weighted port stay time of vessels. Gao *et al.* (2010) use a robust planning approach to solve a dynamic continuous BAP with stochastic vessel arrivals via feedback procedure in the planning stage. Minimization of tardiness as an objective in continuous dynamic BAP is considered by Park and Kim (2002) using a sub-gradient method and by Kim and Moon (2003) using
simulated annealing approach. Minimization of quay length with given berthing times as an objective is studied by Lim (1998) and Tong et al. (1999). The continuous BAP with handling times depending on berthing positions is studied by Imai et al. (2005) and Chang et al. (2008) who further considers draft restrictions in the BAP model.

Hybrid BAP The dynamic hybrid BAP with fixed handling times is considered by Moorthy and Teo (2006), which considers a robust planning approach with respect to stochastic vessel arrivals, and further studied by Dai et al. (2008). The dynamic hybrid BAP with position-dependent handling times is studied by Imai et al. (2007) for indented berths, and Cordeau et al. (2005). Draft restrictions in dynamic hybrid BAP are considered by Nishimura et al. (2001) and Cheong et al. (2010).

Comprehensive literature surveys on the BAP in context of container terminal operations can be found in Bierwirth and Meisel (2010), Steenken et al. (2004) and Stahlbock and Voss (2008). To our knowledge, the problem has not been investigated thus far in the context of bulk port terminals, which is the primary focus of our research.

4.2 Problem description

We consider a set of vessels $N$, to be berthed on a continuous quay of length $L$ for a time horizon $H$. We consider dynamic vessel arrival process and the berth layout used in our model is an extension of the hybrid case. We discretize the quay boundary into a set of sections $M$ of variable length. While a given vessel can occupy multiple sections, it is assumed that each section can be occupied by at most one vessel or part of a vessel at any given time. Partitioning the quay space into sections of variable length brings more flexibility to the model, and the manner in which sections are defined along the quay is critical. A comparison of continuous, discrete and hybrid layouts is illustrated in Figure 12.

Integration with yard assignment of cargo One major difference that distinguishes the Berth Allocation Problem (BAP) in bulk ports from that in container terminals is the fixed specialized equipment facilities such as conveyors and pipelines at bulk ports. In a container terminal, all cargo is packed into containers, and thus there is no need for any specialized equipment to handle any particular type of cargo. In contrast in bulk ports, depending on the vessel requirements and cargo properties, a wide variety of equipment is used for discharging or loading operations. For example, liquid bulk is generally discharged using pipelines which are installed at only certain sections of the quay. Similarly, a vessel may require the conveyor facility to load cargo from a nearby factory outlet to the vessel. For a given vessel, we consider handling time values which are dependent on both the berthing position of the vessel along the
quay and the cargo type to be loaded or discharged from the vessel.

5 The model

In this section we present a mixed integer programming formulation for the extended hybrid berth allocation problem with dynamic arrivals in the context of bulk ports.

5.1 Notation

We assume the following input data to be available:

- \( N \) set of vessels berthing at the port, indexed from \( i = 1 \) to \( i = |N| \);
- \( M \) set of sections along the quay, indexed from \( k = 1 \) to \( k = |M| \);
- \( A_i \) expected arrival time of vessel \( i \in N \);
- \( U_i \) upper bound to the arrival time of vessel \( i \in N \);
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\(D_i\) draft of vessel \(i \in N\);
\(L_i\) length of vessel \(i \in N\);
\(Q_i\) quantity of cargo to be loaded on or discharged from vessel \(i\)
\(W_i^w\) set of cargo types to be loaded/discharged from vessel \(i \in N\), indexed from \(w = 1\) to \(w = |W_i^w|\);
\(h_{ikw}\) handling time for unit quantity of cargo \(w \in W_i^w\) when vessel \(i \in N\) is berthed in section \(k \in M\);
\(d_k\) draft of section \(k \in M\);
\(l_k\) length of section \(k \in M\);
\(b_k\) starting coordinate of section \(k \in M\) along the quay;
\(L\) total length of the quay;
\(B\) large positive constant.

The clearance distances between adjacent vessels as well as end-clearances are considered implicitly in vessel lengths. Similarly, the clearance times between two successive vessels overlapping in space are considered implicitly in the handling times.

Furthermore, the following coefficients can be determined by data preprocessing:

\[
x_{ikp} = \begin{cases} 
1 & \text{if a vessel } i \in N, \text{ berthed at starting section } k \in M, \text{ will occupy also section } p \in M; \\
0 & \text{otherwise.}
\end{cases}
\]

5.2 Mathematical formulation

In order to model the problem, we define the following decision variables:

\(a_i\) \(\geq 0\), represents the arrival time of vessel \(i \in N\);
\(m_i\) \(\geq 0\), represents the starting time of handling of vessel \(i \in N\);
\(h_i\) \(\geq 0\), represents the total handling time of vessel \(i \in N\);
\(s_k\) binary, equals 1 if section \(k \in M\) is the starting section of vessel \(i \in N\), 0 otherwise;
\(x_{ik}\) binary, equals 1 if vessel \(i \in N\) occupies section \(k \in M\), 0 otherwise;
\(y_{ij}\) binary, equals 1 if vessel \(i \in N\) is berthed to the left of vessel \(j \in M\) without any overlapping in space, 0 otherwise;
\(z_{ij}\) binary, equals 1 if handling of vessel \(i \in N\) finishes before the start of handling of vessel \(j \in N\), 0 otherwise;
\(r_i\) binary, equals 1 if vessel \(i \in N\) is risk averse with respect to arrival time, 0 otherwise.
The berth allocation problem is formulated as follows:

\[
\min \sum_{i \in N} (m_i - a_i + h_i)
\]
\[
\text{s.t. } m_i - a_i \geq 0 \quad \forall i \in N,
\]
\[
a_i = A_i r_i + U_i (1 - r_i) \quad \forall i \in N,
\]
\[
\sum_{k \in M} (b_k s_k^i) + B(1 - y_{ij}) \geq \sum_{k \in M} (b_k s_k^j) + L_i \quad \forall i, j \in N, i \neq j,
\]
\[
m_j + B(1 - z_{ij}) \geq m_i + h_i \quad \forall i, j \in N, i \neq j,
\]
\[
y_{ij} + y_{ji} + z_{ij} + z_{ji} \geq 1 \quad \forall i, j \in N, i \neq j,
\]
\[
\sum_{k \in M} s_k^i = 1 \quad \forall i \in N,
\]
\[
\sum_{k \in M} (b_k s_k^i) + L_i \leq L \quad \forall i \in N,
\]
\[
\sum_{p \in M} (x_{ipk}s_{ipk}) = x_{ik} \quad \forall i \in N, \forall k \in M,
\]
\[
(d_k - D_i)x_{ik} \geq 0 \quad \forall i \in N, \forall k \in M,
\]
\[
h_i \geq h_{iw}^i (l_i/L_i) Q_i x_{ik} \quad \forall i \in N, \forall k \in M, \forall w \in W^i
\]
\[
s_k^i \in \{0, 1\} \quad \forall i \in N, \forall k \in M,
\]
\[
x_{ik} \in \{0, 1\} \quad \forall i \in N, \forall k \in M,
\]
\[
y_{ij} \in \{0, 1\} \quad \forall i, j \in N,
\]
\[
z_{ij} \in \{0, 1\} \quad \forall i, j \in N.
\]

The objective function (1) minimizes the total service time of berthed vessels. Constraints (2) ensures that vessels are serviced only after their arrival. Constraints (3) determines which vessels are averse to risk with respect to arrival times.

Constraints (4)–(6) are the non-overlapping restrictions for any two vessels berthing at the port, to ensure that no two vessels overlap both in time and space. Constraints (7) ensure that each vessel can have only one starting section. Constraints (8) ensure that the vessel is berthed such that it does not exceed beyond the total length of the quay. Constraints (9) ensure that each vessel occupies only as many number of sections as determined by its length and starting section. Constraints (10) ensure that the draft of the vessel does not exceed the draft of any occupied section. Constraints (11) determine the total handling time for any given vessel. Finally, constraints (12)–(15) define decision variables’ domain.
6 Computational results

In this section we present some preliminary computational results. The mixed integer linear model (1)–(15) is tested using CPLEX 10.2 for different instance based on a small sample of data received from SAQR port.

6.1 Description of instances

In all instances, the quay length $L$ is 1600 meters and vessel lengths $L_i$ lie in the range 80-260 meters as in SAQR. We consider instances containing $|N| = 5, 10$ and $15$ vessels and we discretize the quay in $|M| = 10, 20$ and $30$ sections.

The length $l_k$ of sections can be constant or variable. For instances with 10 sections and variable section lengths, we randomly generate sections with lengths between 80 to 240 meters. Similarly, for 20 sections, we consider section lengths varying between 40 and 120 meters, and for 30 sections case, between 20 and 88 meters.

The expected arrival times $A_i$, the upper bounds $U_i$ and the handling times $h_{iw}$ are expressed in hours and randomly generated between a suitable range of values which closely represent the values in the data sample from SAQR. For each size of $N$ and $M$, we test 4 different instances: instances A, B, C consider variable section length, while constant section length is taken into account by instance D.

We remark that the drafts of all vessels $D_i$ are less than the minimum draft for all sections, as in the data provided by the port. Therefore, constraints (10) become redundant for the tested instances.

6.2 Preliminary results

Preliminary results for the above instances obtained from CPLEX 12.1 are shown in Tables 1 and 2. The time limit is set to 2 hours for all instances. For each tested instance we report the number of vessels ($N$), the number of sections ($M$), the instance id ($A, B, C, D$), the value of the best solution found ($\text{obj}$), the optimality gap ($\text{gap}$) and the computational time ($\text{t(s)}$) expressed in seconds.

As seen in Table 1, all instances with 5 vessels can be easily solved in less than one second for different number of sections. Instances containing 10 vessels can be solved within a few seconds. However, for 15 vessels only 4 out of 9 instances can be solved within the time limit, while for the remaining instances the gap varies between 0.5% and 7.8%. Clearly, the
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complexity of the problem is highly affected by the problem size and increases exponentially.

In most cases, for same number of vessels and quay length, the computation time increases with the number of sections along the quay. This is clear especially for instances with 10 vessels. The increased computational effort is paid off by a reduction of the objective function, explained by a more accurate discretization of the quay. However, the definition of handling times for different discretizations is one aspect of the model that needs to be further looked into.

Table 2 reports results for instance D (constant section length) for 5, 10 and 15 vessels. The number of sections has been set to 20 in each case. For 5 and 10 vessels, the optimal solution is found within a few seconds, as for variable section length. For 15 vessels, the solver fails to find the optimal solution within the time limit, and the best solution found has gap 1.55%.

7 Conclusions

In this work, we provide a description of the operations in bulk ports and review the OR literature of decision problems related to the management of bulk port terminals. We present a case study on SAQR port in Ras Al Khaimah, UAE and highlight the key issues facing the port. In particular, we believe that the waiting times and delays at the berth and yard are significant, which calls for better coordination between berthing activities and yard management at the port. Furthermore, there is a large degree of uncertainty involved in the port operations which can potentially disrupt the normal functioning of the port and result in huge delays in operations. We believe that integrated planning of operations and robust solutions would allow the terminal to lower delay costs and enhance efficiency. We present some potential research directions and focus on the berth allocation problem. We present a mixed linear integer model on the allocation of vessels along the quay which attempts to considers the interaction between the berth and yard activities. Preliminary results show that the problem is complex and general purpose solvers fails to produce good solutions as soon as the problem size increases. As a next step, we plan to investigate heuristic approaches in order to produce near-optimal solutions in a reasonable time. Furthermore, we plan to extend our model for berth allocation and take into account the integrated planning of berth and yard space allocation.

References

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Table 1: Results for instances with variable section length.

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Table 2: Comparison between instances of constant and variable section length.


