

# Optimizing Fueling Decisions for Locomotives in Railroad Networks

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# Agenda

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- Problem Overview
- Literature Review
- Mathematical Formulation
- Key Results
- Conclusion

# Basic Problem

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- Given a train time-table / schedule, locomotive assignment to the trains and all costs related to fueling, **determine the fueling plan** that minimizes the overall fueling costs
- Additional fueling constraints:
  - Number of fueling stops
  - Capacity of locomotive fuel tank
  - Daily capacity of the fueling truck at yards
- Assumptions:
  - Fueling stops do not delay the trains (instantaneous refueling)
  - Fuel consumption rate between two station yards known

# Problem Features

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- Fueling costs have three parameters – unit cost of fuel, cost of making a fueling stop at a yard and cost of hiring a fueling truck at a yard
- Cost of fuel at yards vary (due to logistics, marketing cost & taxes)
- Restriction on the amount of fuel available at a yard
- Restriction on the number of fueling stops at intermediate yards
- The refueling plan **MUST** ensure that locomotives have enough fuel to run all trains during the planning horizon

# Motivation

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- A practical problem impacting the cost of operations of railroad industry
- Increasing cost of fuel and increased competition in recent times
- Availability of a slice of real-life data for a major US railroad

# Simple Example

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- Four yards: Y1, Y2, Y3 and Y4
- Tracks directly connect Y1-Y2, Y2-Y3 and Y3-Y4
- Two trains T1 and T2 run daily
- Yard sequence for T1: (Y1, Y2, Y3, Y4)
- Yard sequence for T2: (Y4, Y2, Y1)
- Though in this example train runs daily and exactly takes one day for journey, real problems could be more complex
- Yard-Yard distances given as Y1-Y2: 106, Y2-Y3: 146, Y3-Y4: 16 and Y2-Y4: 162 (Y2-Y3 + Y3-Y4)

# Fueling Costs

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- Fixed cost for halting and waiting for fueling: \$250
- Locomotive Fuel Consumption Rate: 3.5 gallons / mile
- Fueling truck weekly contracting cost: \$4,000
- Locomotive fueling tank capacity: 4,500 gallons
- Fueling truck capacity: 25,000 gallons / day

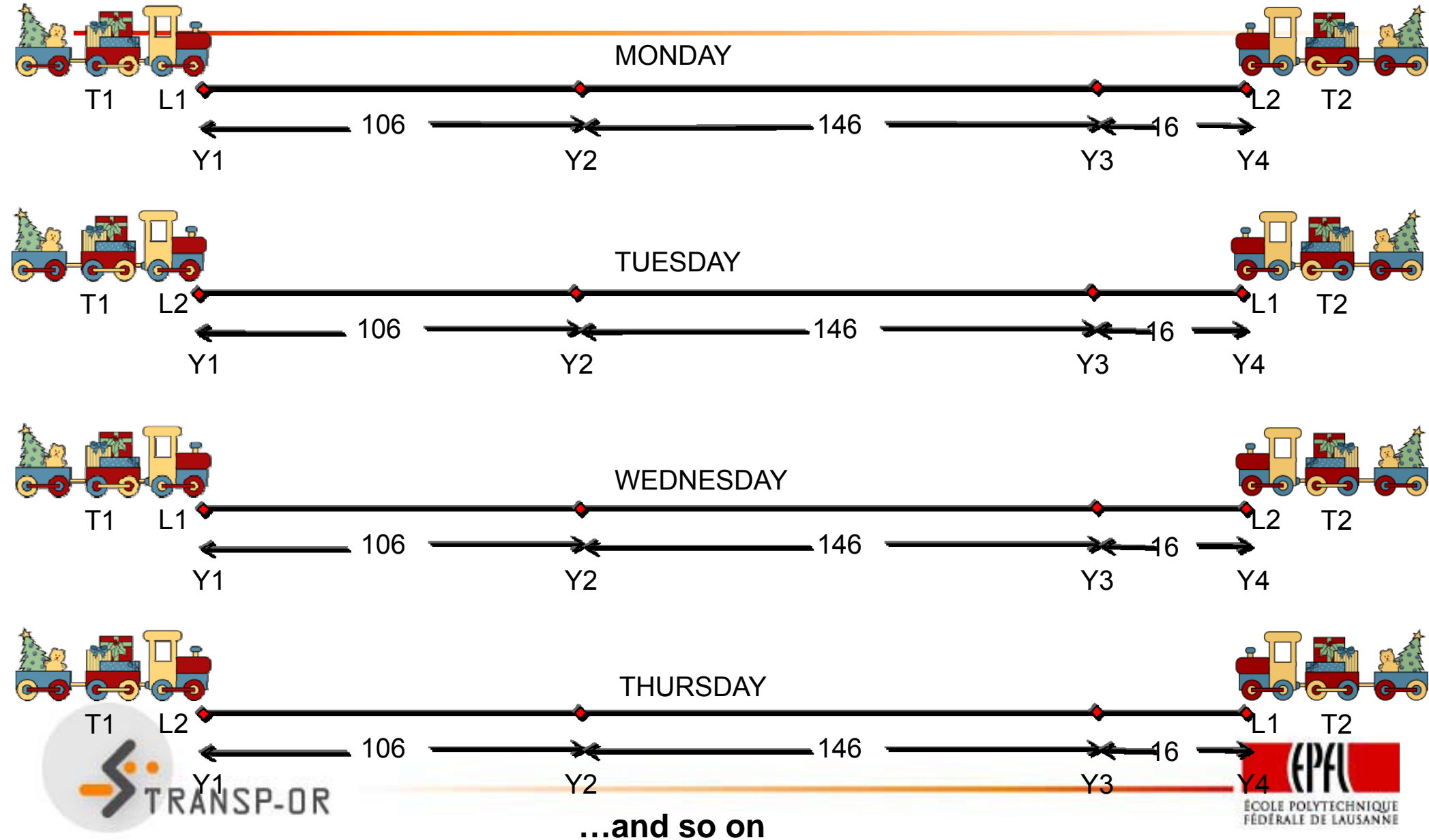
Yard	Fuel Price (\$/gallon)
Y1	\$3.25
Y2	\$3.05
Y3	\$3.15
Y4	\$3.15

# Loco-Train Assignment Schedule

LocoID	Train	Train Start Day	Week	Cycle Sequence	Horizon Day
L1	T1	MON	1	1	1
	T2	TUE	1	2	2
	T1	WED	1	3	3
	T2	THU	1	4	4
	T1	FRI	1	5	5
	T2	SAT	1	6	6
	T1	SUN	1	7	7
L2	T2	MON	1	1	1
	T1	TUE	1	2	2
	T2	WED	1	3	3
	T1	THU	1	4	4
	T2	FRI	1	5	5
	T1	SAT	1	6	6
	T2	SUN	1	7	7



# Pictorial Representation



# Problem Analysis

- In reality, trains have NO ROLE in locomotive refueling, excepting the information they carry about the sequence of yards that the locomotive will power the train to
- Yard information is critical because it plays a role on the decision regarding placement of refueling trucks
- Loco-yard assignment can be written as:

LocoID	Yard	Stop Number	StationType	Horizon Day	Fuel or Not
L1	Y1	1	Origin	1	?
L1	Y2	2	Intermediate	1	?
L1	Y3	3	Intermediate	1	?
L1	Y4	4	Origin	2	?
L1	Y2	5	Intermediate	2	?
L1	Y1	6	Origin	3	?

# Literature Review

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- Nourbakhsh and Ouyang (2010)
  - No restriction of fuel at yards
  - Solve using Lagrangean relaxation – difficult sub-problem by shortest path
- Nag and Murthy (2010)
  - Solve using greedy algorithm

# Mathematical Model

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- Sets
  - $J$ : set of all locomotives, denoted by  $j$
  - $I$ : set of all station yards, denoted by  $i$
  - $R$ : set of all train routes, denoted by  $r$
  - $S$ : set of stop sequence, denoted by  $s$
- Known Parameters:
  - $\text{Param\_refuel}_{jis}$ : 1 if locomotive  $j$  visits yard  $i$  on sequence  $s$  and 0 otherwise
  - $\text{Day}_{jst}$ : 1 if locomotive  $j$  visits yard sequence  $s$  on day  $t$  and 0 otherwise
  - $\text{Train}_{rjs}$ : Flag for intermediate yards; 1 if yard sequence  $s$  for locomotive  $j$  on train route  $r$  is Intermediate, 0 otherwise (pre-processed)
  - $d_{js}$ : Distance between yards appearing in sequence  $s$  and next (in case it is the last non-destination yard, then the first yard of the sequence) for locomotive  $j$
  - $\text{rate}$ : Amount of fuel consumed to run one mile
  - $\text{Min\_fuel}_{js}$ : Minimum fuel required to reach next yard ( $=d_{js} * \text{rate}$ )

# Mathematical Model

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- Known Parameters:
  - $c_i$ : Cost of fuel at yard  $i$
  - $c_{\text{FIXED}}$ : Fixed cost for refueling
  - $c_{\text{CONTRACT}}$ : Weekly cost of operating a refueling truck
  - CAP: Refueling truck capacity
  - TANK: Locomotive tank capacity
- Decision variables:
  - $x_{js}$ : Flag to represent refueling of locomotive  $j$  at the yard appearing in sequence  $s$  on its route
  - $y_{js}$ : Amount of fuel in locomotive  $j$  at the time of entering the yard appearing in sequence  $s$
  - $w_{js}$ : Amount of fuel filled in locomotive  $j$  at the yard appearing in sequence  $s$
  - $z_i$ : Number of refueling trucks at yard  $i$

# Mathematical Model

- Minimize Cost 
$$\sum_j \sum_s c_{FIXED} x_{js} + \sum_i \sum_j \sum_s Param\_refuel_{jis} c_i w_{js} + \sum_i c_{CONTRACT} z_i$$

- Constraints:

- A locomotive  $j$  on yard sequence  $s$  is refueled if and only if there is a halt at that yard.

$$w_{js} \leq TANK \cdot x_{js} \quad \forall j, s \quad \dots (1)$$

- Fuel in the locomotive at any time cannot exceed the tank capacity

$$y_{js} + w_{js} \leq TANK \quad \forall j, s \quad \dots (2)$$

- Fuel conservation in the locomotive before and after crossing a yard sequence  $s$

$$y_{js} + w_{js} - Min\_fuel_{js} = y_{js+1} \quad \forall j, s: s \cap \{S\} \quad \dots (3a)$$

$$y_{js:s=\{S\}} + w_{js:s=\{S\}} - Min\_fuel_{js:s=\{S\}} = y_{js:s=\{1\}} \quad \forall j \quad \dots (3b)$$

# Mathematical Model

- Constraints:
  - A locomotive can be refueled in at most NFP intermediate yards along a route (excluding the origin and the destination)

$$\sum_s Train_{rjs} x_{js} \leq NFP \quad \forall j, r \quad \dots (4)$$

- There is a limit on the amount of fuel that can be filled at each yard every day

$$\sum_j \sum_s Day_{jst} Param\_refuel_{jis} w_{js} \leq CAP \cdot z_i \quad \forall i, t \quad \dots (5)$$

# Further Improvements: Valid Inequalities

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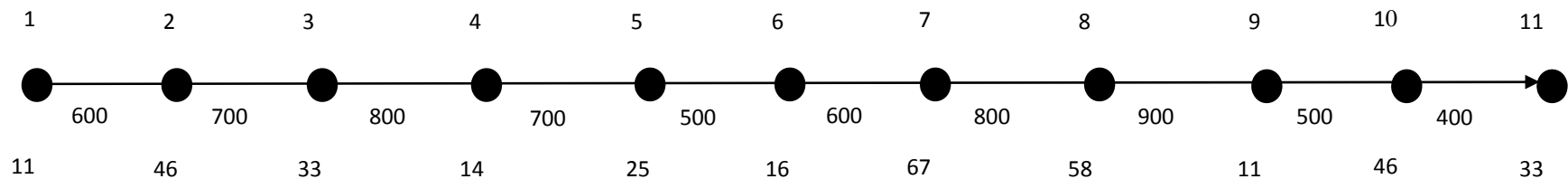
- MIP Cuts:
  - If a station yard does not have a contracted refueling truck, it is not possible to fuel the locomotive at that yard. It would effectively mean that the  $x$  variable for a particular locomotive and stop sequence has to be less than or equal to the  $z$  variable corresponding to the yard in the optimal solution

$$\sum_j \sum_s \text{Param\_refuel}_{jis} x_{js} \leq z_i \quad \forall i$$



# Further Improvements: Valid Inequalities

- MIP Cuts:
  - For every yard sequence  $s_k \in s$ , there exists a finite set of station yards  $S_k \in \{s_k + 1, \dots, s_k + n\}$  such that the locomotive must be refueled in at least one of them to be able to continue the journey. The following cut represents the introduction of this constraint



$$x_{j,s_k+1} + \dots + x_{j,s_k+n} \geq 1$$

$$\forall j, s_k \in s: \{s \cup (S+1 \equiv 1 \in s)\}$$

$$z_{i_p} + \dots + z_{i_q} \geq 1$$

$$\forall i_p, \dots, i_q \in i$$

# Further Improvements: Valid Inequalities

- MIP Cuts:
  - If there was no cost for contracting a fueling truck or that unlimited fuel was always available at every station yard, the problem of minimizing fuel purchase cost and halting cost can be decomposed at locomotive level. Thus the optimal cost of minimizing fuel purchase decision and the cost of halting for each locomotive is same as the optimal cost obtained by considering all these locomotives together

$$\sum_i \sum_s \text{Param\_refuel}_{jis} c_i w_{js} \geq \sum_i \sum_s \text{Param\_refuel}_{jis} c_i w_{js}^*$$

$$\sum_s c_{FIXED} x_{js} \geq \sum_s c_{FIXED} x_{js}^*$$

# Real Instance

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- 73 yards
- Up to 14 Intermediate yards for each train
- 214 trains – some run daily, others with lesser frequency
- 214 locomotives
- Planning Horizon: 2 weeks
- Expanded the same network to two, four and eight times the size (i.e., with up to 1768 locomotives and 584 station yards) to evaluate the model performance on larger networks
- Advantage is that we can compare solution quality without any extrapolation

# Results

Model Name	Constraints Included	Time Limit (s)	Solution (mil \$)	Optimality Gap (%)
Base	(1) – (6)	600	11.41150	1.53%
MIPCut#1	(1) – (7)	600	11.40419	0.85%
MIPCut#2	(1) – (9)	600	11.40068	0.21%
MIPCut#3	(1) – (11)	600	11.39967	0.08%

- Implementation using C and solver used was Gurobi 4.3
- The model could not prove the optimality but we were left with a small absolute optimality gap of less than \$10 after 24 hours run

# Results: Larger Instances

Network Size	Model Name	Constraints Included	Time Limit (s)	Solution (mil \$)	Optimality Gap (%)	Best Known Solution Gap (%)
Double	Base	(1) – (6)	600	22.84456	1.89%	0.20%
Double	MIPCut#1	(1) – (7)	600	22.82189	1.06%	0.10%
Double	MIPCut#2	(1) – (9)	600	22.81712	0.48%	0.08%
Double	MIPCut#3	(1) – (11)	600	22.80818	0.36%	0.04%
Quadruple	Base	(1) – (6)	600	45.74321	2.12%	0.32%
Quadruple	MIPCut#1	(1) – (7)	600	45.69412	1.34%	0.21%
Quadruple	MIPCut#2	(1) – (9)	600	45.68753	0.66%	0.19%
Quadruple	MIPCut#3	(1) – (11)	600	45.67923	0.47%	0.18%
Eight Times	Base	(1) – (6)	600	91.72433	2.56%	0.58%
Eight Times	MIPCut#1	(1) – (7)	600	91.65912	1.73%	0.51%
Eight Times	MIPCut#2	(1) – (9)	600	91.63665	1.01%	0.48%
Eight Times	MIPCut#3	(1) – (11)	600	91.61742	0.79%	0.46%

# Dealing with Uncertainty

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- Assumption of deterministic fuel consumption throughout the locomotive network is strong and impractical
- Fuel requirement for a locomotive varies due to changes in train speeds, braking needs, atmospheric pressure, wind conditions and temperature
- If the locomotive consumed 10% more fuel at every section (between two yards), there might have been **as many as 700 occasions** (about 66% of refueling halts) when the train would have run out of fuel before arriving at the next yard with an available fueling truck
- While this measure is indicative of the extent to which the theoretical model has been optimized, it also indicates that this solution could be of little interest to the practitioners

# Dealing with Uncertainty

- Handle uncertainty with a reserve fuel
- Change the objective function to:

$$\sum_j \sum_s c_{FIXED} x_{js} + \sum_i \sum_j \sum_s Param\_refuel_{jis} c_i w_{js} + \sum_i c_{CONTRACT} z_i - \alpha \cdot y_{min}$$

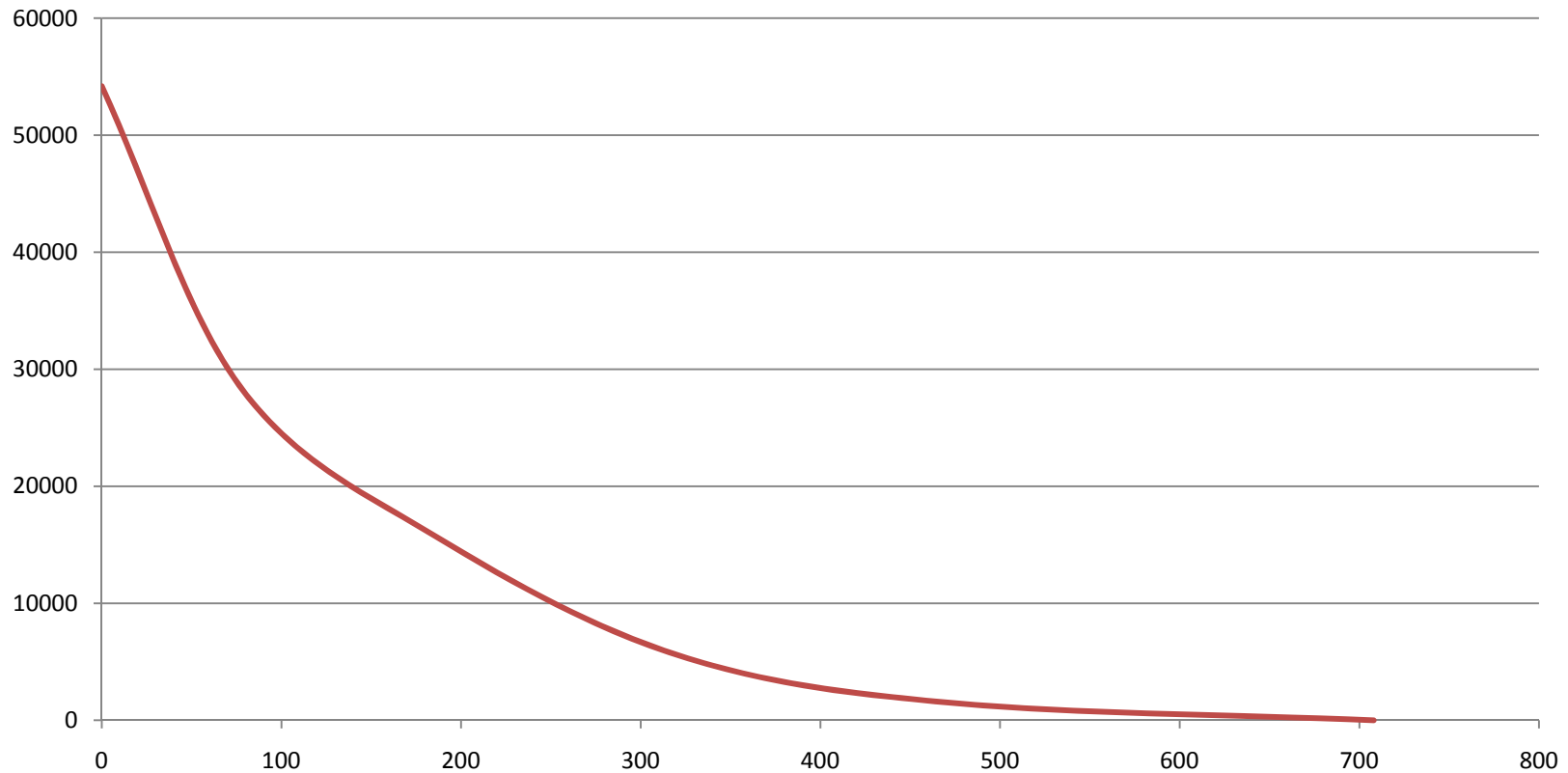
- Add another constraint

$$y_{min} \leq y_{js} \quad \forall j,s$$

- The more we increase the value of  $\alpha$ , the more we tend to increase the reserve fuel in locomotives
- We find that increasing  $y_{min}$  gradually to 103.5 gallons results in ensuring that the locomotive never goes without fuel if the consumption increases by 10% on any section

# Dealing with Uncertainty

Pareto Curve (Cost \$ Versus No. of Stock Out Points)





# Conclusion

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- While existing approaches rely on heuristics, we have shown that this paper that realistic instances of the problem can be solved to optimality with exact methods, thanks to adequate valid inequalities
- We could solve very large instances with around 1800 locomotives and 600 station yards – which shows that our approach is practical
- We have shown that the concept of uncertainty features are appropriate to generate robust solutions, without impacting the complexity of the model, or the performance of the algorithm
- Concept can surely be extended to airline fueling and other facilities location problems such as car sharing
- Future research must dwell into robust optimization and recoverability in the event of disruptions

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Thank you!