

# Optimization of uncertainty features for transportation problems

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## Abstract

In this work we present the concept of Uncertainty Feature Optimization (UFO), an optimization framework to handle problems due to noisy data. We show that UFO is an extension of standard methods as robust optimization and stochastic optimization and we show that the method can be used when no information of the data uncertainty sets is available. We present a proof of concept for the multiple knapsack problem and we show applications to some routing problems: vehicle routing with stochastic demands and airline scheduling.

**Introduction** Many real-life problems, especially in transportation, are due to uncertainty. Standard methods such as robust optimization (Bertsimas and Sim, 2004) or stochastic optimization (Kall and Wallace, 1994), (Herroelen and Leus, 2005), (Sahinidis, 2004), provide solutions that either stay feasible for any possible realization or provide a solution performing best in average case. The main concept of UFO is that the fundamental hypothesis needed for both robust and stochastic optimization, namely that we are given an explicit characterization of the possible scenarios, is relaxed: we are seeking for solution properties improving robustness under noisy data in general, independently of any uncertainty characterization.

The motivations for this research come from the conclusions of some works related to optimization under uncertainty. Authors report that solutions tend to the optimization of simple properties. For example, in airline scheduling, such measures are to increase the number of plane crossings (Klabjan et al., 2002), (Bian et al., 2004), reducing the length of plane rotations (Rosenberger et al., 2004) or increasing idle time (Al-Fawzana and Haouari, 2005).

**The UFO Framework** is based on a standard optimization problem  $z^* = \min\{f(\mathbf{x}) \mid A(\mathbf{x}) \leq \mathbf{b}, \mathbf{x} \in X\}$ .

The aim is to append an uncertainty feature given as a function  $\mu(\mathbf{x})$  to the objective. We are now faced with a multi-objective optimization problem defined over the same feasible set  $\{A(\mathbf{x}) \leq \mathbf{b}, \mathbf{x} \in X\}$  and new objective

$$z^{\text{UFO}} = [\min f(\mathbf{x}), \max \mu(\mathbf{x})].$$

Note that the feasibility of solution  $\mathbf{x}$  is not affected by the uncertainty measure. Moreover, if  $\mu(\mathbf{x})$  is of the same type than  $f(\mathbf{x})$  and  $A(\mathbf{x})$ , then the obtained problem is of same difficulty than the initial problem ( $P$ ).

UFO is based on an implicit uncertainty handling. However, with an appropriate choice of uncertainty features we can retrieve stochastic optimization formulations as well as the robust formulation of Bertsimas and Sim (2004). Details are reported in the full paper.

**Applications** Results on the multiple knapsack problem show that simple uncertainty features such as maximizing diversification or minimizing the maximum taken object tend to improve the robustness of a solution for different classes of problems. Results are obtained with an extensive simulation procedure.

Finally, we show how to apply UFO to transportation problems under uncertainty such as vehicle routing with stochastic demands and airline scheduling.

## References

- Al-Fawzana, M. and Haouari, M. (2005), ‘A bi-objective model for robust resource-constrained project scheduling’, *International Journal of Production Economics* **96**, 175–187.
- Bertsimas, D. and Sim, M. (2004), ‘The price of robustness’, *Operations Research* **52**, 35–53.
- Bian, F., Burke, E., Jain, S., Kendall, G., Koole, G., Mulder, J. L. S. J., Paelinck, M., Reeves, C. and Suleman, I. R. M. (2004), ‘Measuring the robustness of airline fleet schedules’.  
**URL:** <http://www.cs.nott.ac.uk/gxk/papers/jdsmista2003sel.pdf>
- Herroelen, W. and Leus, R. (2005), ‘Project scheduling under uncertainty: Survey and research potential’, *EJOR* **165**, 289–306.
- Kall, P. and Wallace, S., eds (1994), *Stochastic Programming*, John Wiley & Sons, New York, N.Y.
- Klabjan, D., Johnson, E., Nemhauser, G., Gelman, E. and Ramaswamy, S. (2002), ‘Airline crew scheduling with time windows and plane-count constraints’, *Transportation Science* **36**(3), 337–348.
- Rosenberger, J., Johnson, E. and Nemhauser, G. (2004), ‘A robust fleet assignment model with hub isolation and short cycles’, *Transportation Science* **38**(3), 357–368.
- Sahinidis, N. V. (2004), ‘Optimization under uncertainty: state-of-the-art and opportunities’, *Computers and Chemical Engineering* **28**, 971–983.