A general methodology and a free software for the calibration of DTA models

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1 Introduction

This note presents an integrated calibration approach for dynamic traffic assignment (DTA) models that is applicable both in a macroscopic (analytical) and microscopic (simulation-based) setting. It constitutes a generalization of previous work, where arbitrary demand dimensions, including path and origin-destination (OD) flows are estimated from traffic counts (Flötteröd et al., 2010). The added value of this text is that it generalizes the previously developed methodology to the joint estimation of the demand together with structural parameters of both the demand model and the supply (network loading) model.

Despite of their very preliminary nature, the presented experiments clearly indicate that it is possible to jointly estimate travel behavior, demand model parameters, and supply model parameters in a consistent Bayesian setting that makes, apart from differentiability, no assumptions about the nature of the underlying demand and supply model. A particularly noteworthy feature of the new approach is that it solves the parameter estimation problem subject to a given assignment logic in terms of *an optimization problem that goes without equilibrium constraints*.

The remainder of this note is organized as follows. Section 2 derives the proposed estimator. Section 3 provides a simple example that demonstrates the feasibility of the approach. Section 4 describes how the estimator can be applied to the

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calibration of general DTA microsimulations and presents a freely available software tool in which the proposed methodology is implemented. Finally, Section 5 summarizes the article and gives an outlook on current and future research topics.

2 Methodology

For notational simplicity, the time index is omitted in the following presentation. Turning all involved quantities into time-dependent ones and assuming the involved mappings to be time-dependent as well is formally straightforward.

Denote by d_n the total demand of OD pair n, $n = 1 \ldots N$, and by d_{ni} the demand for path $i \in C_n$, where C_n is the path set of OD pair n. Collecting all path flows in $\mathbf{d} = (d_{ni})$, an SUE (stochastic user equilibrium) is attained if

$$d_{ni} = P_n(i|\mathbf{x}(\mathbf{d};\gamma);\beta)d_n \quad n = 1 \ldots N, i \in C_n \tag{1}$$

holds where $P_n(i|\mathbf{x};\beta)$ represents the route choice model that defines which share of OD demand d_n takes route $i \in C_n$ given a vector of network conditions $\mathbf{x}$ and the demand model parameters β. $\mathbf{x}(\mathbf{d};\gamma)$ represents the network loading, which maps the path flows $\mathbf{d}$ on the network conditions $\mathbf{x}$; its parameters are denoted by γ.

In earlier work, a path flow estimator for given demand and supply parameters β and γ was developed (Flötteröd et al., 2010). (This reference also demonstrates the dynamic nature of the entire methodology.) In the following, this result is extended for the joint estimation of path flows, demand parameters and supply parameters. In Flötteröd et al. (2010), it is shown that path flows that satisfy the SUE condition (1) can be equivalently written as maximizers of the prior entropy function

$$\begin{aligned} W(\mathbf{d}|\beta,\gamma) &= \sum_{n=1}^{N} \left[d_n \ln d_n + \sum_{i \in C_n} d_{ni} \ln P_n(i|\mathbf{x}(\mathbf{d};\gamma);\beta) - \sum_{i \in C_n} d_{ni} \ln d_{ni} \right] \\ \text{s.t.} & \sum_{i \in C_n} d_{ni} = d_n \; \forall n = 1 \ldots N, \end{aligned} \tag{2}$$

which represents the logarithm of the probability of the occurrence of the path flows $\mathbf{d}$ given the route choice model $P_n(i|\mathbf{x};\beta)$ and the network loading model

$\mathbf{x}(\mathbf{d};\gamma)$. For path flow estimation, the maximizer of the posterior entropy function

$$\begin{aligned} & W(\mathbf{d}|\mathbf{y};\beta,\gamma) = \ln p(\mathbf{y}|\mathbf{x}(\mathbf{d};\gamma)) + W(\mathbf{d}|\beta,\gamma) \\ \text{s.t.} & \sum_{i\in C_n} d_{ni} = d_n \; \forall n = 1\ldots N \\ & d_{ni} \geq 0 \; \forall n = 1\ldots N,\, i \in C_n \end{aligned} \tag{3}$$

is proposed where $\mathbf{y}$ is the vector of available traffic counts and $\ln p(\mathbf{y}|\mathbf{x}(\mathbf{d};\gamma);\beta)$ is its log-likelihood function. The posterior entropy models the logarithm of the probability that a certain path flow pattern $\mathbf{d}$ occurs given both the route choice model $P_n(i|\mathbf{x};\beta)$, the network loading model $\mathbf{x}(\mathbf{d};\gamma)$, and the measurements $\mathbf{y}$.

For the derivation of the combined path flow and parameter estimator, Bayes' law is applied to obtain the posterior entropy function

$$W(\mathbf{d},\beta,\gamma|\mathbf{y}) = \ln p(\mathbf{y}|\mathbf{x}(\mathbf{d};\gamma)) + W(\mathbf{d}|\beta,\gamma) + W(\beta,\gamma), \tag{4}$$

which represents (apart from a constant offset) the logarithm of the probability of the occurrence of the path flows $\mathbf{d}$ and the parameters β and γ given the route choice model $P_n(i|\mathbf{x};\beta)$, the network loading model $\mathbf{x}(\mathbf{d};\gamma)$, the measurements $\mathbf{y}$ and the logarithm of the parameters' prior distributions, $W(\beta,\gamma)$. A natural generalization of the path flow estimator (3) now becomes to maximize the posterior entropy function

$$\begin{aligned} & W(\mathbf{d},\beta,\gamma|\mathbf{y}) = \ln p(\mathbf{y}|\mathbf{x}(\mathbf{d};\gamma)) + W(\mathbf{d}|\beta,\gamma) + W(\beta,\gamma) \\ \text{s.t.} & \sum_{i\in C_n} d_{ni} = d_n \; \forall n = 1\ldots N. \\ & d_{ni} \geq 0 \; \forall n = 1\ldots N,\, i \in C_n \end{aligned} \tag{5}$$

also with respect to the parameters β and γ. The result constitutes a Bayesian joint estimator of the path flows $\mathbf{d}$, the demand parameters β, and the supply parameters γ.

A noteworthy feature of this estimator is that it solves the parameter estimation problem subject to a given SUE assignment in terms of *an optimization problem that goes without equilibrium constraints*. This operationally relevant feature results from the relaxation of the coupling between the log-likelihood function and the assignment logic through the joint estimation of the path flows that link these two functions.

If no measurements are given, the estimator is underspecified because a maximization of $W(\mathbf{d}|\beta,\gamma)$ with respect to $\mathbf{d}$ alone yields an optimal objective function value of zero for arbitrary parameter values. That is, for a plain assignment, the parameters should either be fixed or stabilized at their initial values through the use of an appropriate prior $W(\beta, \gamma)$. This carries over to any application of the estimator with available measurements: whenever it is possible that the measurements are insufficient to identify the parameters, a prior needs to be used. If the prior parameter values are obtained from a different calibration, the estimated parameter covariances can be used to specify, e.g., a multivariate normal instance of $W(\beta, \gamma)$.

3 Example

A simple network that consists of four parallel routes is considered. During the analysis period, 1000 vehicles enter the network and travel along one of the routes, which are numbered from 1 through 4. For simplicity (and due to the preliminary nature of these results), the time dimension is again omitted. The feasibility of the calibration approach in fully dynamic conditions is demonstrated (for fixed parameters) in Flötteröd et al. (2009).

The route travel times t_i result from the following congestion-dependent performance functions

$$\begin{aligned} t_1(q_1;\gamma) &= 1 + (q_1/100)^\gamma \\ t_2(q_2;\gamma) &= 2 + (q_2/200)^\gamma \\ t_3(q_3;\gamma) &= 4 + (q_3/400)^\gamma \\ t_4(q_4;\gamma) &= 8 + (q_4/800)^\gamma \end{aligned} \tag{6}$$

where q_i is the number of vehicles on route i and γ is a positive parameter. That is, route 1 has the lowest free-flow travel time of 1 but also the lowest capacity of 100, whereas route 4 has the highest free-flow travel time of 8 but also the highest capacity of 800.

Travelers select their route according to a multinomial logit model with the utility function

$$V_i = \beta t_i \tag{7}$$

where β is a dispersion parameter. Assuming the parameter values $\beta = -1$ and $\gamma = 2$, an SUE is computed (e.g., by maximizing (2)), which yields the following flows (rounded to four digits) on route 1, 2, 3, and 4:

$$\begin{aligned} y_1 &= 216.0583 \\ y_2 &= 356.0135 \\ y_3 &= 406.9378 \\ y_4 &= 20.9904. \end{aligned} \tag{8}$$

These measurements are used to reconstruct the path flows, the demand parameter β, and the supply parameter γ. The following log-likelihood function is used:

$$\ln p(\mathbf{y}|\mathbf{x}(\mathbf{d};\gamma);\beta) = -\frac{1}{2}\sum_{i \in Y}(q_i - y_i)^2 \tag{9}$$

where $Y \subseteq \{1, 2, 3, 4\}$ in different experiments. This implies a very small measurement variance of 1 on each route, which puts a high weight on the measurements and should enforce their precise reproduction. For notational consistency with Section 2, one may also want to define the path flows $\mathbf{d} = (q_1 \ldots q_4)$ and the network conditions $\mathbf{x}(\mathbf{d};\gamma) = (\mathbf{d}\ t_1(\mathbf{d};\gamma) \ldots t_4(\mathbf{d};\gamma))$.

The calibration problem is solved by maximization of (5) with respect to $\mathbf{d}$, β, and γ. To investigate the identifiability of this setting, no prior on the parameters is used. The calibration is started with $d_1 = d_2 = d_3 = d_4 = 250$, $\gamma = 1$ and $\beta = -2$. Using any three out of four measurements, both the parameters and the path flows are recovered, with the parameter precision being at least three digits in all experiments and the objective function being strictly concave at the solution point.

Using two out of four measurements, the optimization routine (the Maple function NLPSolve is used) is in some cases unable to improve the initial point because the objective function is too flat. This is not a deficiency of the calibration methodology but only indicates that the problem at hand is not (or hardly) identifiable with only two measurements. However, since the path flows for given parameters are always well-defined, the estimation of two parameters is in principle possible from only two measurements.

4 Application to microsimulations; free software tool

It is demonstrated in Flötteröd et al. (2010) how the path flow estimator with fixed parameters can be applied to general DTA microsimulations: Essentially, the OD pairs are replaced by individually simulated travelers, and the routes are replaced by individual travel plans per traveler, which comprise a sequence of trips that connect intermediate stops during which activities are conducted, including all associated timing information. The derivatives required by the estimation are obtained from regression models that are recursively fitted to the microsimulation (Flötteröd and Bierlaire, 2009). The additional estimation of demand and supply parameters within a microsimulation framework appears possible along the same lines.

A free software tool (Cadyts – "Calibration of dynamic traffic simulations" Flötteröd (2009)) that implements the proposed estimator is available on the Internet (Cadyts, accessed 2010). The current implementation estimates the choice distributions for arbitrary demand dimensions from traffic counts, but it does not yet implement the joint estimation of demand and supply parameters. Preliminary results, however, have been obtained, where parameters of a simple mode choice model are estimated from traffic counts on a large real-world network.

5 Summary and outlook

This note demonstrates how an existing method for the estimation of arbitrary choice dimensions from traffic counts can be generalized towards the joint estimation of travel behavior and structural model parameters of both the demand model and the supply (network loading) model. The estimator is phrased in terms of a Bayesian posterior distribution for the travel behavior and the model parameters. A simple example demonstrates the feasibility of the approach. It is indicated how the mathematical estimator can be carried over DTA microsimulations, and the freely available software tool Cadyts that serves this purpose is introduced.

Ongoing and future work comprises the following items:

- Use of different data sources but traffic counts. Apart from speed and travel time measurements, point-to-point observations (vehicle re-identifications) constitute emerging and rich data sources.
- Complete implementation of the approach in the Cadyts software tool. The current implementation estimates travel behavior, but the estimation of model parameters is only experimentally implemented.
- Applications. Cadyts is currently applied in three different DTA microsimulations (MATSim, SUMO, and DRACULA). Further applications will help to better understand the practical properties of the method and to identify new (both methodological and technical) challenges.

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