Conceptual model of the shippers’ choice between sea, rail and road transport

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Background: Transfer of goods from road to sea or rail

- According to the National Transport Plan of Norway, growth in long distance freight transport is to be taken by rail or sea to the largest possible extent.
  - Several attempts have been made to achieve this transition, but the mechanisms behind each individual shipper’s choice between road, sea and rail are not well understood.

- Two elements considered to be of importance:
  - Uncertainty
  - Economies of scale
Background: The Norwegian commodity flow survey

- Collected by Statistics Norway. Meant to give a complete picture of all freight flows in Norway during one year (2014).
- All shipments from the 8 largest transport operators in Norway registered
- As well as a survey among 4224 firms
  - Varying sizes and industries
  - All shipments in 2014 were registered …
  - … Including mode of transport
  - 91% of the firms had answered by 1/12-2015
- By now, data regarding 70,000,000 shipments is collected
- Data will (should) be ready some time during May

- We have very good data on freight flows and chosen transport modes – but not on actual transport costs
  - How to utilize this data in the best possible way?
Project idea

- Focus on interesting commodity groups and origin-destination pairs; choose some case studies
- For each shipper, calculate logistics costs associated with each (potential) mode based on generic cost parameters
- Use these calculations as input in a discrete mode choice model
- Aggregating over shippers to predict the size of each mode specific commodity flow
- This affects the freight rate for each shipper due to economies of scale (think large container ship)
- To assess the potential for transferring goods from road to sea or rail: Simulate a policy change and iterate until convergence
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The supply chain

- We are considering supply chains where goods are regularly **supplied from a single source** and **sold at a single outlet**. (At least) two possible transport chains:

\[
\begin{align*}
\text{Expected one-way transport time: } & \overline{t} \\
\text{Distance: } & a \\
\text{Price: } & P
\end{align*}
\]

\[
\begin{align*}
\text{Expected one-way transport time: } & \overline{t}_1 \\
\text{Distance: } & a_1 \\
\text{Turn-around time/ unproductive time: } & u \\
\text{Loading/unloading time per unit commodity: } & t_l \\
\text{Expected one-way transport time: } & \overline{t}_3 \\
\text{Distance: } & a_3
\end{align*}
\]
Transport costs

- The shipper can always choose the most appropriate truck size \( C \) from a continuous interval \([C_{min}, C_{max}]\).

- We assume economies of scale in vehicle size:
  - A linear relationship between vehicle size and kilometer cost:
    \[ k = k_0 + k_1 C \]
  - And between vehicle size and hourly vehicle capital cost:
    \[ i = i_0 + i_1 C \]

- Shipment size \( Q \) can never be larger than \( C \) (but smaller).

- Let \( x \) be annual demand, \( Y \) be annual transport capacity and assume no unnecessary trips are made:
  - If \( Q \in [C_{min}, C_{max}] \), then \( Y = x \) (and consequently \( C = Q \))
  - If \( Q \leq C_{min} \) then \( Y = x \cdot \frac{C_{min}}{Q} \) (> \( x \))
Transport costs

- This allows us to write the annual transport costs of the shipper \((K_T)\) as a function of two choice variables: shipment size \((Q)\) and annual transport capacity \((Y)\)

\[
K_T = \{2k_0(a_1 + a_3) + (w + i_0)[2(\bar{t}_1 + \bar{t}_3) + (u_1 + u_3)]\} \frac{x}{Q} \\
+ \{i_l t_l\}QY + \{(w + i_0 + w_l)t_l + P\}x \\
+ \{2k_1(a_1 + a_3) + i_1[2(\bar{t}_1 + \bar{t}_3) + (u_1 + u_3)]\}Y
\]
Ordering costs and inventory holding costs, no uncertainty

- The ordering cost: $b$ per shipment
- The expected annual cost of the stationary inventory:
  \[
  \frac{1}{2} H (1 + \varepsilon) Q
  \]
  Inventory holding cost per year and unit of stationary inventory

- The inventory holding cost for units tied up in transport:
  \[
  \frac{J}{\eta} (\bar{t} + t_l Q) x
  \]
  Inventory holding cost per year and unit of mobile inventory
Uncertainty – demand during lead time

- Assume commodities are demanded one at the time; demand is generated by a stationary stochastic process; and demand per business hour is $\sim N(\mu_D, \sigma_D^2)$, where $\mu_D = \frac{x}{\eta}$
- Lead time: from a shipment is ordered until the commodity is on the shelf (including transport time). Assume lead time $\sim N(\mu_T, \sigma_T^2)$
- Then, demand during lead time is $\sim N(\mu_L, \sigma_L^2)$, where (Hadley and Whitin, 1963):
  \[ \mu_L = \mu_D \mu_T \]
  \[ \sigma_L^2 = \mu_T \sigma_D^2 + \mu_D^2 \sigma_T^2 \]
Uncertainty – demand during lead time

- Stock-outs are allowed to occur, but at a cost.
  - Assume **stock-outs are backordered** with a cost per instance plus a cost depending on the time until delivery takes place.
- The firms trade off **stock-out costs** and **inventory holding costs** by fixing a reorder point $R$.
  - Whenever the inventory position reaches $R$, a new shipment is ordered.
- Adding $R$ as an instrument, we say that the firm follows a $(Q, Y, R)$ policy.
  - $Q$ and $Y$ must be strictly positive, but $R$ might be negative.
Safety stock and stock-out costs

- Let
  - $E$: The average number of backorders per year
  - $B$: The average number of backorders at any point in time
- Hadley and Whitin (1963) show that:
  \[
  E = E(Q, R) = \frac{x}{Q} \alpha(R)
  \]
  \[
  B = B(Q, R) = \frac{1}{Q} \beta(R)
  \]
- Where:
  \[
  \alpha(R) = \sigma_L \left[ \left( 1 + \left( \frac{R - \mu_L}{\sigma_L} \right)^2 \right) \left( 1 - F \left( \frac{R - \mu_L}{\sigma_L} \right) \right) - \frac{R - \mu_L}{\sigma_L} f \left( \frac{R - \mu_L}{\sigma_L} \right) \right]
  \]
  \[
  \beta(R) = \frac{1}{2} \sigma_L^2 \left[ \left( 1 + \left( \frac{R - \mu_L}{\sigma_L} \right)^2 \right) \left( 1 - F \left( \frac{R - \mu_L}{\sigma_L} \right) \right) - \frac{R - \mu_L}{\sigma_L} f \left( \frac{R - \mu_L}{\sigma_L} \right) \right]
  \]
Safety stock and stock-out costs

- Then, the stock-out costs will be:

\[ \pi E(Q, R) + \hat{\pi} B(Q, R) \]

- The average excess inventory compared to the deterministic case will be:

\[ R - \mu_L + B(Q, R) \]
The logistics cost function

Adding all the cost elements gives:

\[
K = [\gamma_1 + \psi(R)] \frac{x}{Q} + \gamma_2 Qx + \gamma_3 QY + \gamma_4 Q + \gamma_5 x + \gamma_6 Y + pH(R - \mu_L)
\]

Where:

\[
\begin{align*}
\gamma_1 &= 2k_0(a_1 + a_3) + (w + i_0)[2(\bar{t}_1 + \bar{t}_3) + (u_1 + u_3)] + b \\
\gamma_2 &= 2J\eta^{-1}t_l \\
\gamma_3 &= i_1t_l \\
\gamma_4 &= \frac{1}{2}H(1 + \varepsilon) \\
\gamma_5 &= (w + i_0 + w_l)t_l + P + J\eta^{-1}(\bar{t}_1 + \bar{t}_2 + \bar{t}_3) \\
\gamma_6 &= 2k_1(a_1 + a_3) + i_1[2(\bar{t}_1 + \bar{t}_3) + (u_1 + u_3)] \\
\psi(R) &= \pi\alpha(R) + (H + \hat{\pi})x^{-1}\beta(R)
\end{align*}
\]
The logistics cost function

- The decision maker’s problem is to minimize logistics costs subject to \( C_{min} \leq C \leq C_{max} \) and \( Y \geq x \). This can be written as:

\[
\begin{align*}
\text{Max}_{Q,Y,R} & \quad -K \\
\text{s.t.} & \quad -QY \leq -C_{min}x \quad (\lambda_1) \\
& \quad QY \leq C_{max}x \quad (\lambda_2) \\
& \quad -Y \leq -x \quad (\lambda_3)
\end{align*}
\]

Given the Lagrangian:

\[
L = -K - \lambda_1 (C_{min}x - QY) - \lambda_2 (QY - C_{max}x) - \lambda_3 (x - Y)
\]

The Kuhn-Tucker conditions for an optimum are:

\[
\begin{align*}
\frac{\partial L}{\partial Q} = \frac{\partial L}{\partial Y} = \frac{\partial L}{\partial R} = 0 \\
\lambda_1 \geq 0 \quad (= 0 \text{ if } QY > C_{min}x) \\
\lambda_2 \geq 0 \quad (= 0 \text{ if } QY < C_{max}x) \\
\lambda_3 \geq 0 \quad (= 0 \text{ if } Y > x)
\end{align*}
\]
8 potential cases

\[ L = -K - \lambda_1 (C_{min}x - QY) - \lambda_2 (QY - C_{max}x) - \lambda_3 (x - Y) \]

<table>
<thead>
<tr>
<th>Cases</th>
<th>1st constraint $C \geq C_{min}$</th>
<th>2nd constraint $C \leq C_{max}$</th>
<th>3rd constraint $Y \geq x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$C = C_{min}$</td>
<td>$C = C_{max}$</td>
<td>$Y = x$</td>
</tr>
<tr>
<td>2</td>
<td>$C = C_{min}$</td>
<td>$C = C_{max}$</td>
<td>$Y &gt; x$</td>
</tr>
<tr>
<td>3</td>
<td>$C = C_{min}$</td>
<td>$C &lt; C_{max}$</td>
<td>$Y = x$</td>
</tr>
<tr>
<td>4</td>
<td>$C = C_{min}$</td>
<td>$C &lt; C_{max}$</td>
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</tr>
<tr>
<td>5</td>
<td>$C &gt; C_{min}$</td>
<td>$C = C_{max}$</td>
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</tr>
<tr>
<td>6</td>
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<tr>
<td>7</td>
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<td>$C &lt; C_{max}$</td>
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</tr>
<tr>
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4 possible cases

\[ L = -K - \lambda_1 (C_{\text{min}}x - QY) - \lambda_2 (QY - C_{\text{max}}x) - \lambda_3 (x - Y) \]

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<th>Cases</th>
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<tr>
<td>1</td>
<td>( C = C_{\text{min}} )</td>
<td>( C = C_{\text{max}} )</td>
<td>( Y = x )</td>
<td></td>
<td>Contradiction</td>
</tr>
<tr>
<td>2</td>
<td>( C = C_{\text{min}} )</td>
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<td>( Y &gt; x )</td>
<td></td>
<td>Contradiction</td>
</tr>
<tr>
<td>3</td>
<td>( C = C_{\text{min}} )</td>
<td>( C &lt; C_{\text{max}} )</td>
<td>( Y = x )</td>
<td></td>
<td>Possible case</td>
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### 4 possible cases - solutions

<table>
<thead>
<tr>
<th>Case</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
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<tbody>
<tr>
<td>( Q^* = )</td>
<td>( \sqrt{\frac{x(\gamma_1 + \psi(R) + \gamma_6 C_{min})}{\gamma_2 x + \gamma_4}} ) ( &lt; C_{min} )</td>
<td>( C_{min} )</td>
<td>( \sqrt{\frac{\gamma_1 + \psi(R)x}{(\gamma_2 + \gamma_3)x + \gamma_4}} ) ( \in [C_{min}, C_{max}] )</td>
<td>( C_{max} )</td>
</tr>
<tr>
<td>( Y^* = )</td>
<td>( C_{min}x \sqrt{\frac{\gamma_2 x + \gamma_4}{x(\gamma_1 + \psi(R) + \gamma_6 C_{min})}} ) ( &gt; x )</td>
<td>( x )</td>
<td>( x )</td>
<td>( x )</td>
</tr>
<tr>
<td>( R^* = )</td>
<td>from ( \frac{\partial L}{\partial R} = 0 )</td>
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</tbody>
</table>

- As the cost per shipment \( \gamma_1 + \psi(R^*) \) increase, the optimal solution moves from the first of the cases, to the second, third and fourth.

- Cases 2 and 4 are explicit solutions (solved analytically). Cases 1 and 3 are implicit solutions (they are simple to solve numerically).
Summary

- We have developed a conceptual framework for logistics cost minimization, where:
  - Decisions are made on an annual basis
    - Shipment size and number of trips are endogenous
  - Not all transport costs are proportional to number of tonnes
    - Unlike conventional inventory theory models, transport costs will affect the choice variables
  - Uncertain demand and uncertain lead time
    - Firms choose a reorder point depending on demand during lead time
  - Solution depends on a set of generic cost parameters
    - Most of these parameters are already available for Norwegian conditions
  - Framework can be used to assess “value of reliability”
    - Which can be problematic in a SP setting
  - Cost minimization can (almost) be solved analytically
    - Short computation time
Summary

According to our judgement, the best way to utilize the commodity flow survey data for discrete choice model predictions

Next steps:

- Collect some missing data regarding parameter values
- Calculate minimized logistics costs for a set of firms chosen from the commodity flow survey
- Estimate a discrete choice model for modal choice
- Embed it in an equilibrium model with network externalities
- Conduct policy simulations

Thank you for your attention!