A MODIFIED PAIRED COMBINATORIAL LOGIT ROUTE CHOICE MODEL WITH PROBIT-BASED EQUIVALENT IMPEDANCE
GUANGZHOU CITY

Guangzhou city

Evening peak of BRT

Guangzhou BRT
1. INTRODUCTION
1. INTRODUCTION

**MNL: Multinomial Logit**

**C-Logit:** C for *Commonality Factor*

**PSL: Path Size Logit**

**GEV: Generalized Extreme Value**

**CNL: or called GNL, Generalized Nested Logit**

**PCL: Paired Combination Logit**
1. INTRODUCTION

PCL (Paired Combination Logit)

- 1: \( L_1 \)
- 2: \( L_2 + L_4 \)
- 3: \( L_3 + L_4 \)

\[
P(L_1')P(1|L_1') = \left[ \exp\left(-\frac{V_1}{1-\sigma_{12}}\right) + \exp\left(-\frac{V_2}{1-\sigma_{12}}\right) \right]^{1-\sigma_{12}}
\]

\[
\sum_{k=1}^{3} \sum_{l=k+1}^{3} \left[ \exp\left(-\frac{V_1}{1-\sigma_{12}}\right) + \exp\left(-\frac{V_2}{1-\sigma_{12}}\right) \right]^{1-\sigma_{12}}
\]

Similarity index

Binary logit

Probability of choosing route 1
1. INTRODUCTION

- How to fix the similarity index?
  - experience

- equivalent impedance

LI Jun and OUYANG Jun. A MODIFIED PAIRED COMBINATORIAL LOGIT ROUTE CHOICE MODEL WITH UNIFIED PARAMETER LPCL (LPCL)

\[ L_1 = 10 \]
\[ L_2 = x \]
\[ L_3 = x \]
\[ L_4 = 10 - x \]
1. INTRODUCTION

The overlapping link IIA of logit

Equivalent impedance

\[
\exp(-\theta T_3)/\sum [\exp(-\theta T_3) + \exp(-\theta T_1) + \exp(-\theta T_2)] = \exp(-\theta T_2)/\sum [\exp(-\theta T_3) + \exp(-\theta T_1) + \exp(-\theta T_2)] + \exp(-\theta T_1)/\sum [\exp(-\theta T_3) + \exp(-\theta T_1) + \exp(-\theta T_2)]
\]
1. INTRODUCTION

\[ \exp(-\theta T \downarrow 3) / \sum \uparrow \left[ \exp(-\theta T \downarrow 3) + \exp(-\theta T \downarrow 1) + \exp(-\theta T \downarrow 2) \right] = \exp(-\theta T \downarrow 2) / \sum \left[ \exp(-\theta T \downarrow 3) + \exp(-\theta T \downarrow 1) + \exp(-\theta T \downarrow 2) \right] \]

\[ T \downarrow 1 = \text{length of L1/length of the shortest route} \]
1. INTRODUCTION

LPCL makes “equivalent impedance + PCL” workable

Fig. probability of choosing route 2

\[ L_1 = 10 \]
\[ L_2 = x \]
\[ L_3 = x \]
\[ L_5 \]
\[ L_4 = 10 - x \]
2. THE PROPOSED MODEL: PPCL
   (PROBIT PCL)
2 THE PROPOSED MODEL: PPCL

Route 1 & 2, 1 & 3 equivalent to $L_1$ and by Probit

$P(L_1)P(1 | L_1)$

The marginal and conditional probability are computed as PCL

Use probit equivalent makes OA to $L_5$
2 THE PROPOSED MODEL: PPCL

- Approximation for 3 options' Probit
  \[ P_3 = \text{Prob}[U_3 > \max(U_1, U_2)] = \Phi \left( \frac{V_3 - E[\max(U_1, U_2)]}{\sqrt{\text{var}(U_3 + E[\max(U_1, U_2)])}} \right) \]
  \[ = \Phi \left( \frac{V_3 - \mu_{12}}{\sqrt{\sigma_3^2 + \sigma_m^2 - 2\rho_3\sigma_3\sigma_m}} \right) \]

- \( P_1 + P_2 + P_3 = 1 \)

- \( P_3 = P_2 + P_1 = \Phi \left( \frac{V_3 - \mu_{12}}{\sqrt{\sigma_3^2 + \sigma_m^2 - 2\rho_3\sigma_3\sigma_m}} \right) = 0.5 \)

- \( \Phi \) is standard normal distribution, \( \phi \) is standard density function of normal distribution
2 THE PROPOSED MODEL: PPCL

\[ V_3 = \mu_{12} = V_1 \Phi \left( \frac{V_1 - V_2}{\sqrt{\sigma_1^2 + \sigma_2^2}} \right) + V_2 \Phi \left( \frac{V_2 - V_1}{\sqrt{\sigma_1^2 + \sigma_2^2}} \right) + \sqrt{\sigma_1^2 + \sigma_2^2} \phi \left( \frac{V_1 - V_2}{\sqrt{\sigma_1^2 + \sigma_2^2}} \right) \]

\[ L_1 = 10 \]

\[ L_2 = x \]

\[ L_3 = x \]

\[ L_4 = 10 - x \]

\[ V_5 = -L_5 = -L_2 \Phi \left( \frac{-L_2 + L_3}{\sqrt{\sigma_2^2 + \sigma_3^2}} \right) - L_3 \Phi \left( \frac{-L_3 + L_2}{\sqrt{\sigma_2^2 + \sigma_3^2}} \right) + \sqrt{\sigma_2^2 + \sigma_3^2} \phi \left( \frac{-L_2 + L_3}{\sqrt{\sigma_2^2 + \sigma_3^2}} \right) \]

\[ L_5 = x \Phi(0) + x \Phi(0) - \sqrt{2} \beta x \phi(0) = x - \frac{\sqrt{\beta x}}{\pi} \]

Compared to LPCL

\[ L_5 = \frac{\theta x}{\log 2 + \theta} \]
2 THE PROPOSED MODEL: PPCL

where

\[
\mu_{12} = E[\max(U_1, U_2)] = V_1 \Phi(\alpha_{12}) + V_2 \Phi(-\alpha_{12}) + \sqrt{\text{var}(U_1 + U_2)} \phi(\alpha_{12})
\]

\[
\alpha_{12} = (V_1 - V_2) / \sqrt{\text{var}(U_1 + U_2)}
\]

\[
\text{var}(U_1 + U_2) = \sigma_1^2 + \sigma_2^2 - 2\rho_{12} \sigma_1 \sigma_2
\]

\[
\rho_3' = \rho[U_3, \max(U_1 + U_2)] = \frac{\sigma_1 \rho_{13} \Phi(\alpha_{12}) + \sigma_2 \rho_{13} \Phi(-\alpha_{12})}{\sqrt{\omega_{12} - \mu_{12}^2}}
\]

\[
\omega_{12} = E[\max(U_1 + U_2)] = (V_1^2 + \sigma_1^2) \Phi(\alpha_{12})
\]

\[
+ (V_2^2 + \sigma_2^2) \Phi(-\alpha_{12}) + (V_1^2 + V_2^2) \sqrt{\text{var}(U_1 + U_2)} \phi(\alpha_{12})
\]

\[
\sigma_m^2 = \text{var}[\max(U_1, U_2)] = \omega_{12} - \mu_{12}^2
\]
3 NUMERICAL EXAMPLES
3 NUMERICAL EXAMPLES

- Probit obeys the normal distribution of $(E, \beta x)$

- Logit obeys the gumbel distribution of $(E, \frac{\pi^2}{6\theta^2})$

- In order to compare the models, assume that the variances of 2 models are equal, so

$$\beta = \frac{\pi^2}{6\theta^2 x}$$
3 NUMERICAL EXAMPLES: NETWORK 1

Figure 2. Choice probabilities of middle route in network 1
### 3 NUMERICAL EXAMPLES: NETWORK 1

<table>
<thead>
<tr>
<th>$x$</th>
<th>LPCL</th>
<th>PPCL</th>
<th>$x$</th>
<th>LPCL</th>
<th>PCL</th>
<th>PPCL</th>
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<td>10</td>
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<td>0.2357</td>
<td>0.2357</td>
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</table>

θ = 1

Equivalent impedance comparison of LPCL and PPCL

Mean: 8.95 | 4.35 | 0.24
3 NUMERICAL EXAMPLES: NETWORK 2

\[ L_1 = x \]
\[ L_2 = 10 - x \]
\[ L_3 = 10 - x \]
\[ L_4 = 10 - 2x \]
\[ L_5 = x \]
\[ L_6 = 10 - x \]
\[ L_7 = 10 - x \]

\[ L_1 = x \]
\[ L_2 = 10 - x \]
\[ L_3 = 10 - x \]
\[ L_4 = 10 - 2x \]
\[ L_5 = x \]
\[ L_6 = 10 - x \]
\[ L_7 = 10 - x \]

\[ \theta = 1 \]
\[ \theta = 1.5 \]
\[ \theta = 2 \]

Figure 4. Choice probabilities of upper route in network 2
### 3 NUMERICAL EXAMPLES: NETWORK 2

**Equivalent impedance**

<table>
<thead>
<tr>
<th></th>
<th>Equivalent impedance</th>
<th>Relative error (compared to Probit, %)</th>
</tr>
</thead>
<tbody>
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<td>5</td>
<td>2.9531</td>
<td>4.2764</td>
</tr>
</tbody>
</table>

**mean**

<table>
<thead>
<tr>
<th></th>
<th>LPCL</th>
<th>PCL</th>
<th>PPCL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 1$</td>
<td>15.66</td>
<td>2.60</td>
<td>1.66</td>
</tr>
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</table>
4. CONCLUSION
PPCL PROS

1. Retains the advantage of the PCL model to resolve the overlapping route problem
2. Better represent the random error term in the route choice decision process
3. Clear theoretical background while original PCL may require different parameter for each OD pair
4. Demonstrate more improvements on the IIA issue of original Logit route choice model
5 RELATIVE IMPEDANCE & DISCUSSION
5.1 RELATIVE IMPEDANCE

$L_1 = 1\text{km}$  
$L_2 = 3\text{km}$  

$P_1 = 82\%$

$L_1 = 1000\text{m}$  
$L_2 = 3000\text{m}$

$P_1 = 100\%$
5.1 RELATIVE IMPEDANCE

- Why?
- Basic assumption of Logit
  variance \( \sigma^2 = \frac{\pi^2}{6\theta^2} \)
- Modification to the variance

Modified variance \( \sigma^2 = \frac{\pi^2}{6\theta^2} V^2 = \frac{\pi^2}{6} \left( \frac{\theta'}{V} \right) \)

\[
P_i = \frac{\exp(-\theta' \frac{V_i}{V_{\min}})}{\sum_j \exp(-\theta' \frac{V_j}{V_{\min}})} = \frac{1}{1 + \sum_{j \neq i} \exp \left[ -\theta' \left( \frac{V_i - V_j}{V_{\min}} \right) \right]}
\]

Perception error regardless of impedance

Min impedance (shortest route) \( V_{\min} \)
5.1 RELATIVE IMPEDANCE

\[ P_i = \frac{\exp(-\theta' \frac{V_i}{V_{\text{min}}})}{\sum_j \exp(-\theta' \frac{V_j}{V_{\text{min}}}) + \sum_{j \neq i} \exp \left[ -\theta' \left( \frac{V_i - V_j}{V_{\text{min}}} \right) \right]} \]

\[ P_1 = 82\% \]

\[ P_1 = 82\% \]
5.2 DISCUSSION

- Relationship between impedance and perception error?
  - Assumption 1: shortest route $x$, other routes are proportionate to $x$
  - $D(cx) = c^2 D(x)$ $\implies \sigma^2 = \frac{\pi^2}{6\theta^2} V^2$
  - Assumption 2: independent perception for each link (reasonnable)
  - $D(a+b) = D(a) + D(b)$ $\implies \sigma^2 = \beta V$

\[
P_i = \frac{1}{1 + \sum_{j \neq i} \exp \left[ -\theta' \left( \frac{V_i - V_j}{V} \right) \right]}
\]

\[
P_i = \frac{1}{1 + \sum_{j \neq i} \exp \left[ -\theta' \left( \frac{V_i - V_j}{\sqrt{V}} \right) \right]}
\]
5.3 FUTURE WORK

- Parameter calibration
- Test the model
  - Source: GPS taxi data of Guangzhou city
MUCH APPRECIATION FOR YOUR PATIENCE