Introduction à l'optimisation et la recherche opérationnelle (2018–2019)

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Optimisation non linéaire (7 décembre 2018)

Question 1:

Considérons la fonction

$$f(x_1, x_2) = x_1^4 - 2x_1^2 + x_2^3 - 3x_2$$

et considérons les points

$$\{ \ (2,2)^T, \ (-1,1)^T, \ (0,-1)^T \ \}$$

a) Vérifier si ces points sont des points stationnaires et, dans l'affirmative, identifier leur nature (maximum local, minimum local, point de selle). Justifier vos réponses.

b) Exécuter deux itérations de la méthode de Newton avec comme point de départ le point $(2,2)^T$.

Question 2:

Considering the following optimization problem :

$$\min f(x_1, x_2) = -x_1^2 + 4x_1x_2^2 - x_2^2.$$

1. Calculate one iteration of the local Newton method using as starting point $x^0 = (-1,0)^T$.
2. Calculate the Cauchy point of f in $x^0 = (-1,0)^T$.

Question 3:

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as

$$f(x_1, x_2) = x_1^2 + 11x_2^2 + x_1x_2,$$

for wich the gradient is

$$\nabla f(x_1, x_2) = \begin{pmatrix} 2x_1 + x_2 \\ 22x_2 + x_1 \end{pmatrix}.$$

We would like to apply the steepest descent method with initial value $x_0 = (4,1)$ after preconditioning the function.

1. Write the change of variables

$$x' = L_k^T x,$$

where L_k^T is such that the Hessian matrix at the step k is $H_k^T = L_k L_k^T$. It corresponds to its cholesky decomposit. This change of variables is used to precondition the function. Then, write the obtained new function.

2. Calculate the direction of the first iteration of the preconditioned steepest descent method.

Question 4:

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as

$$f(x_1, x_2) = x_1^2 + 2x_2^2.$$

1. Apply one iteration of the steepest descent algorithm starting in $x^0 = (9, 1)$. Choose a step α_0 satisfying both Wolfe conditions with $\beta_1 = 1/100$ and $\beta_2 = 0.99$.

2. Apply three iterations of Newton's algorithm starting in $x^0 = (9, 1)$. Choose a step α_0 satisfying both Wolfe conditions with $\beta_1 = 1/100$ and $\beta_2 = 0.99$.