Introduction à l'optimisation et
la recherche opérationnelle (2018–2019)

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Optimisation en nombres entiers (23 novembre 2018)

Question 1:

In order to participate in a cooking competition called *Top Chef*, Benoît must build a team of excellent chefs. He has access to a pool of candidates (see table below). For each of them, he knows the age, the restaurant where they come from, and their cooking specialities (some candidates have only one, some others have two). He also knows the number of points that each of them has been able to collect in previous similar competitions. And he wants to build a team with the maximum number of such points. But several constraints apply :

1. The team must contain at least 3 chefs who are skilled in preparing appetizers.
2. The team must contain at least 4 chefs who are skilled in preparing fish.
3. The team must contain at least 4 chefs who are skilled in preparing meat.
4. The team must contain at least 3 chefs who are skilled in preparing dessert.
5. To promote young people, the team must contain at least 2 chefs who are 20 or less.
6. Yoda and Obi-Wan cannot stand each other, and it is not possible to have both of them in the team.
7. Han and Sheev have so much experience in working together that if one of the two is included in the team, the other must be too.
8. For the sake of fairness, not more than 3 chefs from the same restaurant should be hired in the same team.

Write an integer optimization problem to help Benoît build his team.

Name	Age	Restaurant	Points	First speciality	Second speciality
Anakin	49	Lausanne	124	Fish	Meat
Chewbacca	39	Noirmont	105	Fish	Meat
Dooku	49	Crissier	118	Meat	Dessert
Gial	33	Mézières	134	Dessert	
Han	35	Crissier	160	Appetizer	
Jabba	47	Crissier	184	Fish	
Jubnuk	28	Neuchatel	101	Fish	
Lando	17	Lucens	116	Dessert	
Leia	18	Sierre	187	Appetizer	Fish
Lorth	41	Baulmes	132	Dessert	
Luke	37	Vufflens	181	Appetizer	
Obi-Wan	22	Vufflens	199	Fish	Meat
Padme	38	Cossonay	192	Appetizer	
Qui-Gon	19	Cossonay	136	Fish	
Sebulba	43	Valeyres	123	Meat	
Sheev	21	Crissier	127	Meat	
Teebo	32	Sierre	132	Meat	
Watto	20	Sierre	131	Meat	
Yoda	39	Noirmont	102	Appetizer	Fish
Zuckuss	42	Vufflens	123	Dessert	

Question 2:

The journal *Millenium* needs to schedule the staff for the printing workshop for the five days of the week. During each day, there are eight one-hour time slots. Four employees are available for the tasks. Each employee has reported his or her preference for each time slot and each day on a scale from 0 to 10, where 10 corresponds to the highest preference and 0 corresponds to unavailability (see table below). The following constraints must be verified.

1. Each of the 40 slots must be covered by exactly one employee.
2. An employee cannot be assigned to a time slot if she/he is not available.
3. Every person must take a lunch break either between 12 :00 and 13 :00, or between 13 :00 and 14 :00.
4. Because of the noisy work environment, every person can work only two consecutive time slots. A break of at least one hour must be taken after that.

5. No one can work more than 20 hours per week.

Write an integer optimization problem to help *Millenium* schedule the workshop employees in order to maximize their satisfaction according to the stated preferences, while verifying the constraints.

Name	Slot	Mo	Tu	We	Th	Fr
Mikael	9-10	10	10	10	10	10
Mikael	10-11	9	9	9	9	9
Mikael	11-12	8	8	8	8	8
Mikael	12-13	1	1	1	1	1
Mikael	13-14	1	1	1	1	1
Mikael	14-15	1	1	1	1	1
Mikael	15-16	1	1	1	1	1
Mikael	16-17	1	1	1	1	1
Lisbeth	9-10	10	9	8	7	6
Lisbeth	10-11	10	9	8	7	6
Lisbeth	11-12	10	9	8	7	6
Lisbeth	12-13	10	3	3	3	3
Lisbeth	13-14	1	1	1	1	1
Lisbeth	14-15	1	2	3	4	5
Lisbeth	15-16	1	2	3	4	5
Lisbeth	16-17	1	2	3	4	5
Harriet	9-10	10	10	10	10	10
Harriet	10-11	9	9	9	9	9
Harriet	11-12	8	8	8	8	8
Harriet	12-13	0	0	0	0	0
Harriet	13-14	1	1	1	1	1
Harriet	14-15	1	1	1	1	1
Harriet	15-16	1	1	1	1	1
Harriet	16-17	1	1	1	1	1
Alexander	9-10	10	9	8	7	6
Alexander	10-11	10	9	8	7	6
Alexander	11-12	10	9	8	7	6
Alexander	12-13	10	3	3	3	3
Alexander	13-14	1	1	1	1	1
Alexander	14-15	1	2	3	4	5
Alexander	15-16	1	2	3	4	5
Alexander	16-17	1	2	3	4	5

Question 3:

Considérer le problème suivant :

$$\begin{aligned} & \max x_1 + 2x_2 \\ s.c. & \\ & x_1 + x_2 \leq 8 \\ & -x_1 + x_2 \leq 2 \\ & x_1 - x_2 \leq 4 \\ & x_2 = -4, -2, \text{ ou } 9 \\ & x_1 = 0, 1, 4 \text{ ou } 6 \end{aligned}$$

1. Reformuler le problème comme un problème d'optimisation linéaire binaire équivalent.
2. Comment changerait votre réponse à la question précédente si la fonction objective devenait $\max x_1^2 + 2x_2$?

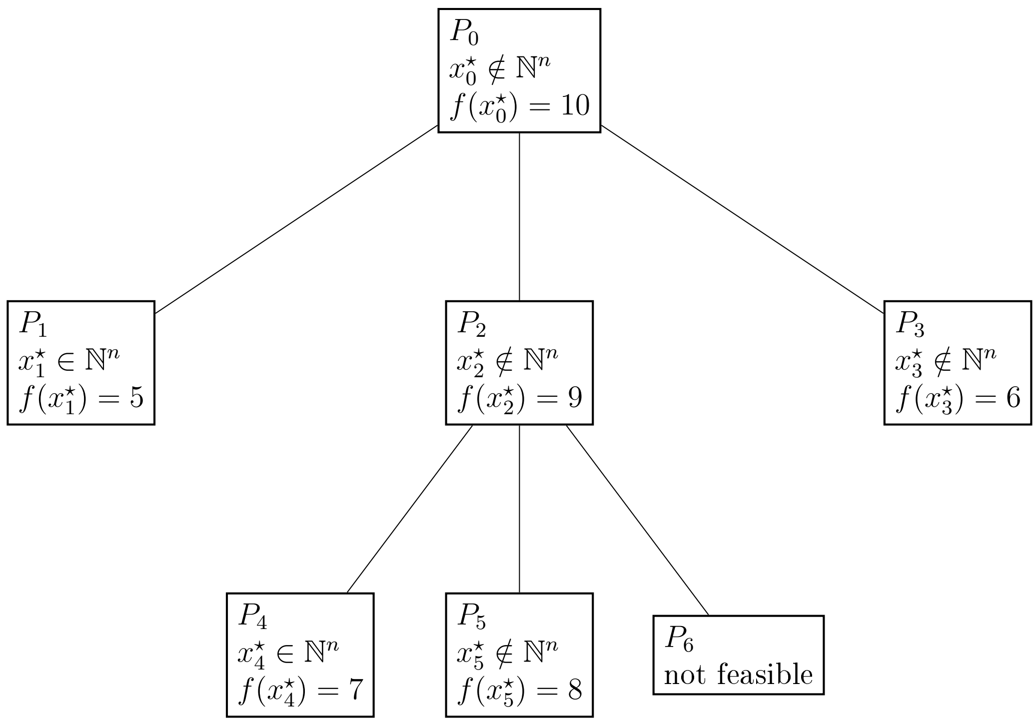
Question 4:

Un ébéniste fabrique des armoires et des tables. Une armoire nécessite $1h$ de travail et $9m^2$ de bois. Une table nécessite $1h$ de travail et $5m^2$ de bois. L'ébéniste dispose de $6h$ de travail et de $45m^2$ de bois. Chaque armoire génère un profit de 8 CHF, et chaque table génère un profit 5 CHF.

1. Formuler le problème P qui maximise le profit de l'ébéniste.
2. Représenter le domaine admissible de ce problème.
3. Donner la solution optimale du problème $R(P)$ qui est la relaxation linéaire du problème P.
4. Résoudre le problème P par la méthode de séparation/évaluation ?

Question 5:

We use the Branch and Bound algorithm to resolve an integer linear optimization problem. We obtain the following Branch and Bound tree for which $x_i^\star$ is the optimal solution for the relaxation problem P_i.



1. Is the corresponding optimization problem a maximization or a minimization problem ?
2. What is the current optimal solution ?
3. Which nodes can we discard ?
4. Is the algorithm terminated ?