Lab VIII: Balancing Bike Sharing Systems

CIVIL-557 Decision Aid Methodologies in Transportation

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1 Problem description

A company has recently started operating a station-based dockless bike sharing system in Lausanne. In the system there are 100 bicycles which can be collected and returned in 11 different locations around the city centre.

Customers can use the bicycles for 24 hours a day. For operational purposes, however, the company divides the 24 hours into 4 time periods: 07:00 - 11:00 (morning peak), 11:00 - 15:00 (offpeak), 15:00 - 19:00 (afternoon peak), 19:00 - 07:00 (night). We assume that the company knows the exact daily demand for each station and time period. We define the imbalance for a given station and a given time period as the difference between the bicycles returned to the station and the bicycles collected from the station (i.e. if the station imbalance is positive, at the end of the period there will be more bicycles than at the beginning; if it is negative, at the end of the period there will be less bicycles than at the beginning).

For the sake of simplicity, we define the following assumptions:

- Bicycles are collected and returned at a constant rate for all stations and time periods.
- Repositioning occurs instantaneously (at 11.00, at 15.00, at 19.00 and at 07.00).
- At the time of the repositioning no bike is being used, i.e. all 100 bikes are parked in one of the 11 stations.

The initial configuration of the system at 07:00 is given. At the end of each period, the company needs to reposition the bicycles in order to be able to satisfy the demand for the following time period. Additionally, after each repositioning, there must be at least 3 bicycles available at all stations. Finally, at the end of the fourth time period (07:00 of the following day) the system must return to the initial configuration.

The goal of the company is to minimize the total daily operational costs of repositioning the bicycles.

2 Network flow formulation

Sets

S	Set of stations in the system.
P	Set of time periods.

Parameters

I_{sp}	Imbalance for station $s \in S$ during period $p \in P$
c_{st}	Cost of transporting a bicycle from station $s \in S$ to station $t \in S$
q_{s0}	Number of bicycles located at station $s \in S$ at the beginning of the day

Variables

$x_{stp} \in Z_{\geq 0}$	Non-negative integer variable indicating the number of bicycles moved from station $s \in S$ to station $t \in S$ at the end of period $p \in P$
$z_{sp} \in Z$	Integer variable indicating how many bicycles are removed from or added to station $s \in S$ at the end of period $p \in P$
$q_{sp} \in Z_{\geq 0}$	Non-negative integer variable indicating how many bicycles are available at station $s \in S$ after repositioning at the end of period $p \in P$

Model

min Define an objective function (1)

s.t. Define constraints (2)

(3)

(4)

(5)

(6)

The objective function (1) aims at minimizing the cost of relocating the bicycles throughout the day. Constraints (2) ensure consistency between the x and the z variables. Constraints (3) define the number of bicycles available at a station at the end of a period, which depends on the number of bicycles available at the end of the previous period, the imbalance during that period and the relocations at the end of that period. Constraints (4) impose that at the end of the fourth period we must return to the initial configuration of the system. Constraints (5-6) make sure that after each relocation at all stations there are enough bicycles to satisfy the demand for the following period, and in no case less than 3 bicycles.

Questions

- Complete the model, implement it OPL and solve it. What is the optimal solution?

- Analyze the information printed thanks to the post-processing script. Represent the flows for one time period on the graph. Do the results make sense?
- How is the decision of relocating bicycles affected by our perfect knowledge of the future demand? Explain through an example.
- Do you think that the current way to define the costs of relocating bicycles is realistic? If not, how would you change it?

3 Routing formulation

Now let us assume that the company owns a single van which is parked at the company's depot and which can carry a maximum of 30 bicycles. At the end of each period, the van performs a route to reposition bicycles. The van starts and ends its routes empty at the depot.

Now the company wants to include in the objective function not only the number of relocated bicycles, but also the cost of the routes driven by the van. The goal remains the minimization of the total daily operational costs of repositioning the bicycles.

For this part, the used notation is as follows:

Sets

N	Set of nodes in the system, which includes the stations and the depot d
S	Set of stations in the system. $S = N \setminus \{d\}$
P	Set of time periods.

Parameters

I_{sp}	Imbalance for station $s \in S$ during period $p \in P$
q_{s0}	Number of bicycles located at station $s \in S$ at the beginning of the day
c_{ij}	Cost of driving the van from node $i \in N$ to node $j \in N$
C	Capacity of the van

Variables

$x_{stp} \in Z_{\geq 0}$	Non-negative integer variable indicating how many bicycles are on the van travelling from $s \in S$ to station $t \in S$ at the end of period $p \in P$
$z_{sp} \in Z$	Integer variable indicating how many bicycles are removed from or added to station $s \in S$ at the end of period $p \in P$
$q_{sp} \in Z_{\geq 0}$	Non-negative integer variable indicating how many bicycles are available at station $s \in S$ after repositioning at the end of period $p \in P$
$y_{ijp} \in \{0, 1\}$	Binary variable indicating whether the van travels on the arc connecting node $i \in N$ to node $j \in N$ in the route driven at the end of period $p \in P$
$f_{ijp} \in Z_{\geq 0}$	Auxiliary flow variables used in the subtour elimination constraints

Model

$$\min \quad \text{Define an objective function} \tag{7}$$

$$s.t. \tag{8}$$

$$\tag{9}$$

$$\tag{10}$$

$$\tag{11}$$

$$\tag{12}$$

$$\sum_{j\in N} y_{jip} = \sum_{j\in N} y_{ijp} \qquad \forall i \in N, \forall p \in P \tag{13}$$

$$\sum_{s\in S} y_{0sp} = 1 \qquad \forall p \in P \tag{14}$$

$$x_{stp} \leq C y_{stp} \qquad \forall s \in S, \forall t \in S, \forall p \in P \tag{15}$$

$$f_{ijp} \leq M y_{ijp} \qquad \forall i \in N, \forall j \in N, \forall p \in P \tag{16}$$

$$f_{ijp} \geq y_{ijp} \qquad \forall i \in N, \forall j \in N, \forall p \in P \tag{17}$$

$$f_{ijp} \leq \sum_{k\in N} \sum_{l\in N} y_{klp} \qquad \forall i \in N, \forall j \in N, \forall p \in P \tag{18}$$

The objective function (7) aims at minimizing the cost of relocating the bicycles throughout the day, taking into account the routing costs and the number of relocated bicycles. Constraints (8-12) are as in (2-6). Constraints (13) are the vehicle flow conservation constraints. Constraints (14) ensure that for each route the vehicle leaves from and returns to the depot. Constraints (15) are the capacity constraints. Constraints (16-18) are the subtour elimination constraints.

Questions

- Complete the model, implement it in OPL and solve it. What is the optimal solution?
- Represent the four solution routes on the graph. Do the results make sense?
- Change the vehicle capacity to $C = 35, 40, 50, 60$. How does the optimal solution change?
- Change the vehicle capacity to $C = 25, 20, 15, 10$. How does the optimal solution change?
- If you were to add more features to the model in order to make it as similar as possible to a real-life problem, how would you change the model? What would you try to add?