CIVIL-557

Decision Aid Methodologies In Transportation

Lab V: Integer programming and Cutting planes

Virginie Lurkin

Transport and Mobility Laboratory (TRANSP-OR)
École Polytechnique Fédérale de Lausanne EPFL

Solution of the previous lab

Assignment #4 – Part A

- Some containers are transferring perishable goods and therefore, need to be in a refrigarated state (file `Containers.txt`, 1 – if a container is having such need). Several blocks in the yard have the refrigarating facility (file `Refrigarators.txt`, 1 – if a block offers such facility).
- To further prevent heavy traffic in the yard area, 2 adjacent blocks cannot exceed 75% of their capacity at the same time.

Assignment #4 – Part A

- Solve the basic model given to you in Lecture 2.
- Solve the model with the refrigaration add-on only.
- Solve the model with the adjacency add-on only.
- Solve the complete model (basic + all add-ons at the same time).
- Report the value of objective function for all the models. Since the LP formulation of the basic model gives fractional solutions, re-run it as an IP (i.e. x variables are integers). Keep the IP settings for all subsequent models. Are the x values changing among the different models?
- What should be the value of big-M for the adjacency constraints?

Assignment #4 – refrigeration constraints

- Parameters:
  - **c_n** – 1 if a container n needs refrigaration, 0 – othersise
  - **r_i** – 1 if a block i offers refrigaration facility, 0 – otherwise

- Decision variables:

y_in – 1 if a container n is stored at block i

Assignment #4 – refrigeration constraints

- Modeling approach:

c_n	r_i	allowed	y_in
0	0	1	free
0	1	1	free
1	0	0	0
1	1	1	free

- Notice that **c_n** ≤ **r_i**
- Notice that when **c_n = 0**, **y_in** is **free** and since 0 times anything is 0, we can have **c_n * y_in ≤ r_i.**
- If **c_n = 1** and **r_i = 0**, **y_in** will be forced to be **0**, which is what we want.
- If **c_n = 1** and **r_i = 1**, **y_in** can be both **0** or **1**.
- **Holds for all i and all n**

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Assignment #4 – refrigeration constraints

- The y_in variable needs to be linked to x_i variable:

$$\sum_{n=1}^{N} y_{in} = x_i, \qquad forall\ i\ in\ I$$

- The optimal solution remains the same as before (1295,88).

Assignment #4 – refrigeration constraints

- Decision variables:

  z_i – 1 if a block i exceeds 75% of capacity, 0 – otherwise

- Mapping decision to its states:

  $x_i + a_i \leq 0.75 * A + M * z_i$, forall i in I

  *M= 0.25*A is big enough

- Adjacency constraints:

  $z_i + z_{(i+1)} \leq 1$, forall i in I: i < |I|

- The optimal solution remains the same as before (1295,88).

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Lab 5

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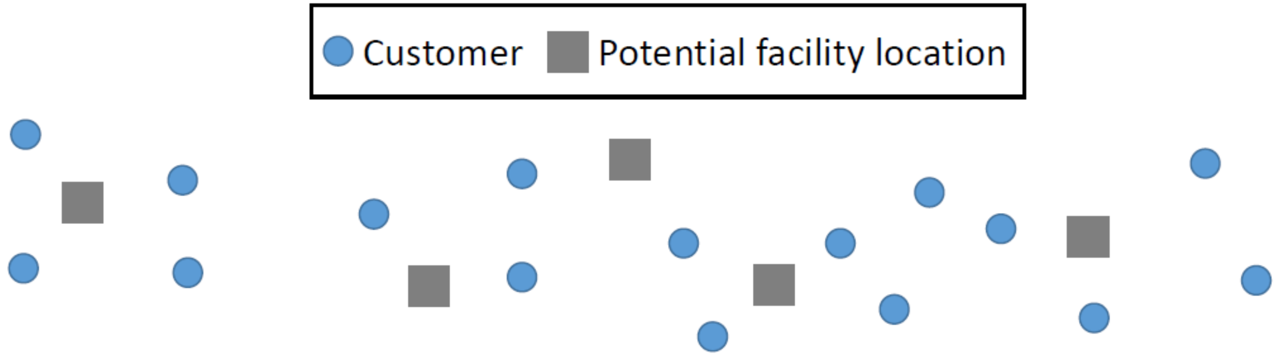
Uncapacitated Facility Location Problem

- f_i - fixed cost of establishing a facility at location i
- c_{ij} - cost of supplying customer j's demand by facility at location i



- y_i - 1 if a facility is established at location i, 0 otherwise,
- x_{ij} - 1 if the demand of customer j is satisfied by the facility i, 0 otherwise.

UFLP – formulation I

- Objective function

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \sum_{i=1}^{m} f_i y_i$$

- Constraints

$$\sum_{i=1}^{m} x_{ij} = 1, \qquad \forall j = 1, \dots, n$$

$$\sum_{j=1}^{n} x_{ij} \leq M y_i, \qquad \forall i = 1, \dots, m$$

$$x_{ij} \in \{0,1\}, \qquad \forall i = 1,2, \dots, m; \forall j = 1,2, \dots, n$$

$$y_i \in \{0,1\}, \qquad \forall i = 1,2, \dots, m$$

UFLP – formulation 2

- Objective function

$$\mathbf{min} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \sum_{i=1}^{m} f_i y_i$$

- Constraints

$$\sum_{i=1}^{m} x_{ij} = 1, \qquad \forall j = 1, \dots, n$$

$$x_{ij} \leq y_i, \qquad \forall i = 1,2, \dots, m; \forall j = 1,2, \dots, n$$

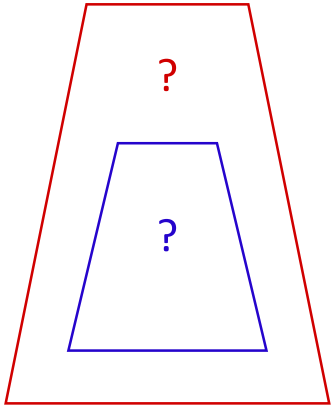
$$x_{ij} \in \{0,1\}, \qquad \forall i = 1,2, \dots, m; \forall j = 1,2, \dots, n$$

$$y_i \in \{0,1\}, \qquad \forall i = 1,2, \dots, m$$

UFLP problem

	Customer (service costs c_{ij})			**Fixed cost (f_i)**
Facility	1	2	3	
1	1000	20	30	200
2	1000	30	40	200
3	1000	160	150	200
4	50	140	120	200

Which formulation is better?



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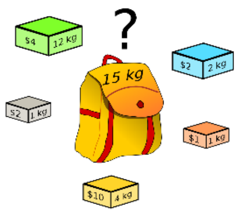
Assignment #5 – Part A

- Solve the LP relaxation corresponding to the UFLP(1)
- Solve the LP relaxation corresponding to the UFLP(2)
- Is the solution integer?
- For each model, report:
    - The value of each variable
    - The value of the objective function,
    - The running time.
- Which formulation is the best?
- Write the dual of the best formulation
- Solve the dual problem and give an interpretation of the solution

Knapsack problem

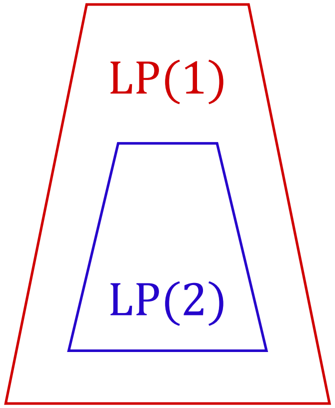


IP(1)

$$
\begin{aligned}
\max \quad & 30x_1 + 17x_2 + 14x_3 + 11x_4 + 9x_5 \\
s.t. \quad & 29x_1 + 20x_2 + 16x_3 + 12x_4 + 10x_5 \leq 37 \\
& x_i \in \{0,1\}, \forall i = 1, \ldots 5.
\end{aligned}
$$

IP(2)

$$
\begin{aligned}
\max \quad & 30x_1 + 17x_2 + 14x_3 + 11x_4 + 9x_5 \\
s.t. \quad & 29x_1 + 20x_2 + 16x_3 + 12x_4 + 10x_5 \leq 37 \\
& x_i \in \{0,1\}, \forall i = 1, \ldots 5. \\
& x_2 + x_3 + x_4 \leq 2 \\
& x_1 + x_2 \leq 1 \\
& x_1 + x_3 \leq 1 \\
& x_1 + x_4 \leq 1 \\
& x_1 + x_5 \leq 1
\end{aligned}
$$



Assignment #5 – Part B

- Solve the IP corresponding to LP(1).
- Solve the LP relaxation associated with LP(1).
- Is the solution integer? If no, deduce some cuts or valid inequalities and create new LPs models.
- Create at least 5 different models, with different cuts.
- Deduce at least one Gomory cut.
- Solve the LP relaxation associated with each model.
- For each model, report:
  - The value of each variable
  - The value of the objective function,
  - The running time.
- What cuts should be added to obtain immediately an integer optimal solution from the LP relaxation?

Assignment #5

- Send a single file containing all the answers including your code (format is up to you) AND mathematical formulations of the different models.
- Group/Individual work
- Send your reports to virginie.lurkin(at)epfl.ch
- Send your reports by 8:00 P.M. next Monday