CIVIL-557

Decision Aid Methodologies In Transportation

Lab III: LP and Duality

Virginie Lurkin

Transport and Mobility Laboratory (TRANSP-OR)
École Polytechnique Fédérale de Lausanne EPFL

Duality

Primal (P)

$$\min_{x \in \mathbb{R}^n} \; c^T x$$

subject to $Ax \geq b$

$x \geq 0,$

where $A \in \mathbb{R}^{m \times n}$, b $\in \mathbb{R}^m$, c $\in \mathbb{R}^n$

Dual (D)

$$\max_{y \in \mathbb{R}^m} \; b^T y$$

subject to $A^T y \leq c$

$y \geq 0,$

where $A \in \mathbb{R}^{m \times n}$, b $\in \mathbb{R}^m$, c $\in \mathbb{R}^n$

Container storage problem

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Exercise

- Solve the dual of the container storage problem for the small instance

Minimize $u_1^+ + u_1^- + u_2^+ + u_2^-$

Subject to

$$
\begin{aligned}
x_1 - u_1^+ + u_1^- &= 4\\
-x_2 + u_2^+ - u_2^- &= 2\\
x_1 + x_2 &= 2\\
x_1, x_2, u_1^+, u_1^-, u_2^+, u_2^- &\geq 0
\end{aligned}
$$

Maximize $4\,y_1 + 2\,y_2 + 2\,y_3$

Subject to

$$
\begin{aligned}
y_1 + y_3 &\leq 0\\
-y_2 + y_3 &\leq 0\\
-y_1 &\leq 1\\
y_1 &\leq 1\\
y_2 &\leq 1\\
-y_2 &\leq 1
\end{aligned}
$$

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Solution

```
dvar float y1;
dvar float y2;
dvar float y3;

maximize 4*y1 + 2*y2 + 2*y3;
subject to {
    y1 + y3 <= 0;
    -y2 + y3 <= 0;
    -y1 <= 1;
    y1 <= 1;
    y2 <= 1;
    -y2 <= 1;
}
```

- Check that you obtain the same solution as for the primal

Exercise

- Update your model and code to be generic (i.e. the model can solve the dual problem for any size)
- Solve the problem for the large instance of Lab2 and check that you obtain the same solution as for the primal

Solution

```
int N=...;
int B=...;
int A=...;
int a[1..100]=...;
range block = 1..B;

float F = (N+sum (i in block) a[i])/(A*B);

dvar float y[block];
dvar float z;

maximize sum (i in block) (A*F - a[i])*y[i] + N*z;

subject to{

    forall (i in block){
        z + y[i] <= 0;
        -y[i] <= 1;
        y[i] <= 1;
    }

}
```

Heuristic

A heuristic technique, often called simply a heuristic, is any approach to problem solving, learning, or discovery that employs a practical method not guaranteed to be optimal or perfect, but sufficient for the immediate goals.

Heuristic

Simple combinatorial scheme:

1. Rearrange blcks so that a_i increases with i

2. Determine x_i in increasing order of i using:

$$x_1 = \min\{N, \max\{0, A \times \mathrm{F} - a_1\}\}$$

$$x_i = \mathrm{m}in\{\max\{0, A \times \mathrm{F} - a_i\}, N - \sum_{r=1}^{i-1} x_r\} \qquad \forall i \geq 2$$

Transportation Problem

Transportation Problem

One of the main products of the P & T COMPANY is canned peas. The peas are prepared at three canneries (near Bellingham, Washington; Eugene, Oregon; and Albert Lea, Minnesota) and then shipped by truck to four distributing warehouses in the western United States (Sacramento, California; Salt Lake City, Utah; Rapid City, South Dakota; and Albuquerque, New Mexico). Because the shipping costs are a major expense, management is initiating a study to reduce them as much as possible. For the upcoming season, an estimate has been made of the output from each cannery, and each warehouse has been allocated a certain amount from the total supply of peas. This information (in units of truckloads), along with the shipping cost per truckload for each cannery-warehouse combination, is given in Table 8.2. Thus, there are a total of 300 truck- loads to be shipped. The problem now is to determine which plan for assigning these shipments to the various cannery-warehouse combinations would *minimize the total shipping cost.*

Transportation Problem – Table 8.2

TABLE 8.2 Shipping data for P & T Co.

		Shipping Cost ($) per Truckload				
		Warehouse				
		1	**2**	**3**	**4**	**Output**
	1	464	513	654	867	75
Cannery	2	352	416	690	791	125
	3	995	682	388	685	100
Allocation		80	65	70	85	

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Transportation Problem – Primal (lab 2)

Minimize $$\sum_i \sum_j cost_{ij}\, x_{ij}$$

Subject to $$\sum_j x_{ij} = output_i, \qquad \forall i$$

$$\sum_i x_{ij} = allocation_j, \qquad \forall j$$

$$x_{ij} \geq 0$$

With x_{ij}, the number of truckloads travelling from cannery i to warehouse j.

Assignment #3 – PART A

- Write the dual LP of the problem
- Implement the problem and report the optimal solution.
- Your model and code have to be generic (i.e. the model can solve the dual of the transportation problem for any size).
- Solve the dual problem for the large instance (file `Large_Instance.txt`).
- What is the economic interpretation of the dual LP?

Assignment #3 – PART B

- Given the following LP

$$
\text{(P)} \qquad \max \quad 2x_1 + x_2
$$
$$
x_1 + 2x_2 \leq 14
$$
$$
2x_1 - x_2 \leq 10
$$
$$
x_1 - x_2 \leq 3
$$
$$
x_1, x_2 \geq 0
$$

- Write its dual
- Check that $x = (\frac{20}{3}, \frac{11}{3})$ is a feasible solution of (P)
- Show that x is also an optimal solution of (P) by applying the complementary slackness conditions
- Derive an optimal solution of the dual

Assignment #3

- Write a short report containing your answers
- Individual work or in group (max 3 students)
- Send your reports to virginie.lurkin(at)epfl.ch
- Send your reports by 8:00PM next Monday

References

- https://www.ibm.com/developerworks/community/blogs/jfp/entry/CPLEX_Is_Free_For_Students?lang=en
- http://www.chkwon.net/julia/book/juliabook-preview.pdf
- http://jump.readthedocs.io/en/latest/refmodel.html
- http://jump.readthedocs.io/en/latest/refvariable.html
- http://jump.readthedocs.io/en/latest/quickstart.html#objective-and-constraints