Aggregation and forecasting

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Aggregation

- So far, prediction of individual behavior
- In practice, not useful
- Need for forecast of aggregate demand:
  - number of trips
  - number of passengers
  - etc.

Aggregation

Linear models

$$t_n = \alpha + \beta p_n$$

where

- t_n: number of trips from zone n
- p_n: population in zone n

- If $\bar{p}$ is the average population
- $\bar{t} = \alpha + \beta\bar{p}$ is the average number of trips

It does not work with choice models, because they are nonlinear

Aggregation

- Discrete "Travel/no travel" model, y_n income

$$
\begin{array}{lrcl}
\text{No travel} & V_1 & = & 0 \\
\text{Travel} & V_2 & = & -3 + 3y_n
\end{array}
$$

	Income	V1	V2	P1	P2
Household 1	1	0	0	50%	50%
Household 2	10	0	27	0%	100%
Avg. income	5.5	0	13.5	0%	100%
Avg. probability H1 & H2				25%	75%

Aggregation

- Choice model

$$P(i|x_n)$$

  where x_n gathers attributes of all alternatives and socio-economic characteristics of n

- If the population is composed of N individuals, the total expected number of individuals choosing i is

$$N(i) = \sum_{n=1}^{N} P(i|x_n)$$

- Hopeless to know x_n for each and every individual
- The sum would involve a lot of terms.
- The distribution of x could be used.

Aggregation

- Assume that the distribution of x is continuous with PDF $p(x)$
- Then the share of the population choosing i is given by

$$\widehat{W}(i) = \int_x P(i|x)p(x)dx$$

- In practice, $p(x)$ is also unknown
- The integral may be cumbersome to compute

Aggregation

- If the population is segmented in S homogeneous segments
- If N_s is the number of individuals in segment s
- Then

$$\widehat{N}(i) = \sum_{s=1}^{S} N_s P(i|x_s)$$

Illustration

The travel model:

- Discrete "Travel/no travel" model, y_n income

$$P(\text{travel}) = \frac{e^{-3+3y_n}}{1 + e^{-3+3y_n}}$$

- Population: $N = 200'000$ persons
- Sample: $S = 500$ persons
- Sampling rate: $S/N = 1/400 = 0.25\%$

Illustration

s	y_s	S_s	N_s	$P(\text{travel})$	PS_s	PN_s
1	0	150	20000	4.7%	7	949
2	0.5	200	30000	18.2%	36	5473
3	1	40	50000	50.0%	20	25000
4	1.5	10	50000	81.8%	8	40879
5	2	50	30000	95.3%	48	28577
6	2.5	50	20000	98.9%	49	19780
		500	200000		169	120657

$$120657 \neq 400 \times 169 = 67542$$

People with low probability of travel are oversampled

Aggregation: sample enumeration

Most practical method: sample enumeration

- Let n be an observation in the sample belonging to segment s
- Let W_s be the weight of segment s, that is

$$W_s = \frac{N_s}{S_s} = \frac{\#\text{ persons in segment } s \text{ in population}}{\#\text{ persons in segment } s \text{ in sample}}$$

- The number of persons choosing alt. i is estimated by

$$\widehat{N}(i) = \sum_{n \in \text{sample}} \sum_s W_s P(i|x_n) I_{ns}$$

where $I_{ns} = 1$ if individual n belongs to segment s, 0 otherwise

Aggregation

We can write

$$\begin{aligned}\widehat{N}(i) &= \sum_{n\in\text{sample}} \sum_s W_s P(i|x_n) I_{ns} \\ &= \sum_{n\in\text{sample}} P(i|x_n) \sum_s W_s I_{ns}\end{aligned}$$

The term $\sum_s W_s I_{ns}$ is the weight of individuals n belonging to segment s. The share of alt. i is estimated by $W(i) =$

$$\frac{1}{N} \sum_{n\in\text{sample}} P(i|x_n) \sum_s W_s I_{ns} = \sum_{n\in\text{sample}} P(i|x_n) \sum_s \frac{N_s}{N} \frac{1}{S_s} I_{ns}$$

Forecasting

Procedure

- Modify x_n in the sample to reflect anticipated modifications
- Apply the sample enumeration again and re-calculate market shares

Example: original

s	y_s	S_s	P(travel)	W_s	Trips
1	0	150	4.74%	133.33	949
2	0.5	200	18.24%	150	5473
3	1	40	50.00%	1250	25000
4	1.5	10	81.76%	5000	40879
5	2	50	95.26%	600	28577
6	2.5	50	98.90%	400	19780
					120657

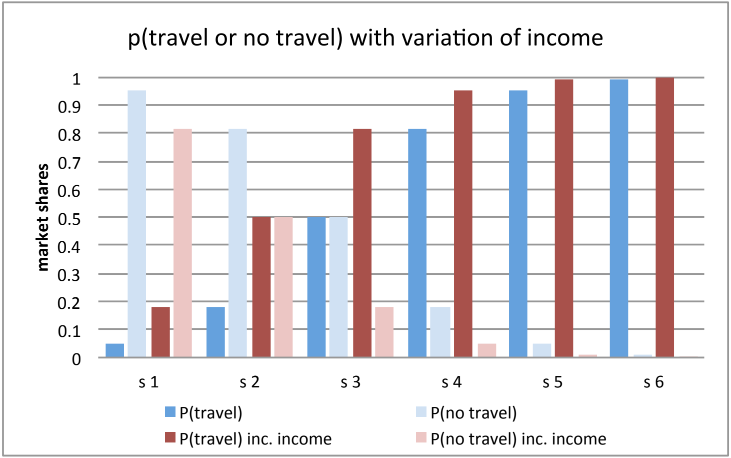
- Increase all salaries by 0.5
- What is the impact on the total number of trips?

Example: modification

s	y_s	S_s	P(travel)	W_s	Trips
1	0.5	150	18.24%	133.33	3649
2	1	200	50.00%	150	15000
3	1.5	40	81.76%	1250	40879
4	2	10	95.26%	5000	47629
5	2.5	50	98.90%	600	29670
6	3	50	99.75%	400	19951
					156777

- Before: 120657
- After: 156777
- Increase: about 30%

Example


p(travel or no travel) with variation of income
market shares
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
s 1
s 2
s 3
s 4
s 5
s 6
P(travel)
P(no travel)
P(travel) inc. income
P(no travel) inc. income

Summary

- Aggregation:
  - Sample enumeration.
  - Correct for sampling errors using weights
  - Enumeration by segments to compare market shares across groups
- Forecasting:
  - Forecast the value of the explanatory variables x.
  - Aggregate.