Choice Theory

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Outline

Choice theory

Choice: outcome of a sequential decision-making process

- Definition of the choice problem: How do I get to EPFL?
- Generation of alternatives: Car as driver, car as passenger, train
- Evaluation of the attributes of the alternatives: Price, time, flexibility, comfort
- Choice: Decision rule
- Implementation: Travel

Building the theory

A choice theory defines

1. Decision maker
2. Alternatives
3. Attributes of alternatives
4. Decision rule

Decision maker

Unit of analysis

- Individual
  - Socio-economic characteristics: age, gender, income, education, etc.
- A group of persons (we ignore internal interactions)
  - Household, firm, government agency
  - Group characteristics
- Notation: n

Alternatives

Choice set

- Mutually exclusive, finite, exhaustive set of alternatives
- Universal choice set ($\mathcal{C}$)
- Individual n: *choice set* ($\mathcal{C}_n$) $\subseteq \mathcal{C}$
- Availability, awareness, feasibility

Example: Choice of transport mode

- $\mathcal{C} = \{car, bus, metro, walk\}$
- ...traveller has no drivers licence, trip is 12km long
- $\mathcal{C}_n = \{bus, metro\}$

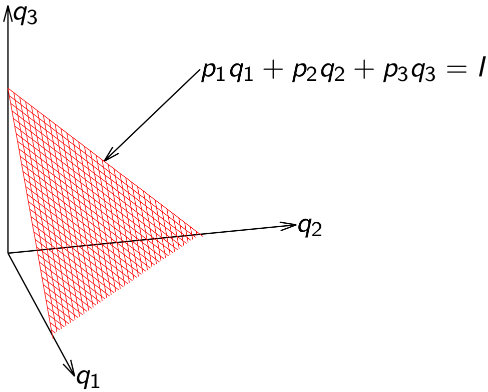
Swait, J. (1984) *Probabilistic Choice Set Formation in Transportation Demand Models* Ph.D. dissertation, Department of Civil Engineering, MIT, Cambridge, Ma.

Continuous choice set

Microeconomic demand analysis

Commodity bundle

- q_1: quantity of milk
- q_2: quantity of bread
- q_3: quantity of butter
- Unit price: p_i
- Budget: I

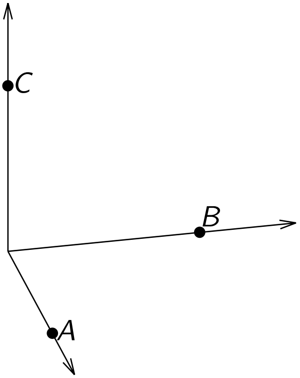


Discrete choice set

Discrete choice analysis

List of alternatives

- Brand A
- Brand B
- Brand C



Alternative attributes

Characterize each alternative i for each individual n

- ➜ cost
- ➜ travel time
- ➜ walking time
- ➜ comfort
- ➜ bus frequency
- ➜ etc.

Nature of the variables

- ✔ Generic or specific
- ✔ Quantitative or qualitative
- ✔ Measured or perceived

Decision rules

Economic man

Grounded in global rationality

- Relevant knowledge of options/environment
- Organized and stable system of preferences
- Evaluates each alternative and assigns precise pay-off (measured through the utility index)
- Selects alternative with highest pay-off

Utility

- Captures attractiveness of alternative
- Allows ranking (ordering) of alternatives
- What decision maker optimizes

A matter of viewpoints

- Individual perspective
  - Individual possesses perfect information and discrimination capacity
- Modeler perspective
  - Modeler does not have full information about choice process
  - Treats the utility as a random variable
  - At the core of the concept of 'random utility'

Consumer theory

Neoclassical consumer theory

- Underlies mathematical analysis of preferences
- Allows us to transform 'attractiveness rankings'...
- into an operational demand functions

Keep in mind

- Utility is a latent concept
- It cannot be directly observed

Figure : Jeremy Bentham

Consumer theory

Continuous choice set

- Consumption bundle

$$Q = \begin{pmatrix} q_1 \\ \vdots \\ q_L \end{pmatrix} ; p = \begin{pmatrix} p_1 \\ \vdots \\ p_L \end{pmatrix}$$

- Budget constraint

$$\sum_{\ell=1}^{L} p_\ell q_\ell \leq I.$$

- No attributes, just quantities

Preferences

Operators $\succ$, $\sim$, and $\succsim$

- $Q_a \succ Q_b$: Q_a is preferred to Q_b,
- $Q_a \sim Q_b$: indifference between Q_a and Q_b,
- $Q_a \succsim Q_b$: Q_a is at least as preferred as Q_b.

To ensure consistent ranking

- Completeness: for all bundles a and b,

$$Q_a \succ Q_b \text{ or } Q_a \prec Q_b \text{ or } Q_a \sim Q_b.$$

- Transitivity: for all bundles a, b and c,

$$\text{if } Q_a \succsim Q_b \text{ and } Q_b \succsim Q_c \text{ then } Q_a \succsim Q_c.$$

- "Continuity": if Q_a is preferred to Q_b and Q_c is arbitrarily "close" to Q_a, then Q_c is preferred to Q_b.

Utility

Utility function

- Parametrized function:

$$\widetilde{U} = \widetilde{U}(q_1, \ldots, q_L; \theta) = \widetilde{U}(Q; \theta)$$

- Consistent with the preference indicator:

$$\widetilde{U}(Q_a; \theta) \geq \widetilde{U}(Q_b; \theta)$$

is equivalent to

$$Q_a \succsim Q_b.$$

- Unique up to an order-preserving transformation

Optimization problem

Optimization

Decision-maker solves the optimization problem

$$\max_{q \in \mathbb{R}^L} U(q_1, \ldots, q_L)$$

subject to the budget (available income) constraint

$$\sum_{i=1}^{L} p_i q_i = I.$$

Demand

Quantity is a function of prices and budget

$$q^* = f(I, p; \theta)$$

Optimization problem

$$\max_{q_1,q_2} U = \beta_0 q_1^{\beta_1} q_2^{\beta_2}$$

subject to

$$p_1 q_1 + p_2 q_2 = I.$$

Lagrangian of the problem:

$$L(q_1, q_2, \lambda) = \beta_0 q_1^{\beta_1} q_2^{\beta_2} - \lambda(p_1 q_1 + p_2 q_2 - I).$$

Necessary optimality condition

$$\nabla L(q_1, q_2, \lambda) = 0$$

where λ is the Lagrange multiplier and β's are the Cobb-Douglas preference parameters

Framework

Optimality conditions

Lagrangian is differentiated to obtain the first order conditions

$$\begin{array}{rcrcl}\partial L/\partial q_1 = \beta_0\beta_1 q_1^{\beta_1-1} q_2^{\beta_2} & - & \lambda p_1 & = & 0 \\ \partial L/\partial q_2 = \beta_0\beta_2 q_1^{\beta_1} q_2^{\beta_2-1} & - & \lambda p_2 & = & 0 \\ \partial L/\partial \lambda = p_1 q_1 + p_2 q_2 & - & I & = & 0\end{array}$$

We have

$$\begin{array}{rcrcl}\beta_0\beta_1 q_1^{\beta_1} q_2^{\beta_2} & - & \lambda p_1 q_1 & = & 0 \\ \beta_0\beta_2 q_1^{\beta_1} q_2^{\beta_2} & - & \lambda p_2 q_2 & = & 0\end{array}$$

Adding the two and using the third optimality condition

$$\lambda I = \beta_0 q_1^{\beta_1} q_2^{\beta_2}(\beta_1 + \beta_2)$$

Framework

Equivalent to

$$\beta_0 q_1^{\beta_1} q_2^{\beta_2} = \frac{\lambda I}{(\beta_1 + \beta_2)}$$

As $\beta_0 \beta_2 q_1^{\beta_1} q_2^{\beta_2} = \lambda p_2 q_2$, we obtain (assuming $\lambda \neq 0$)

$$q_2^* = \frac{I \beta_2}{p_2(\beta_1 + \beta_2)}$$

Similarly, we obtain

$$q_1^* = \frac{I \beta_1}{p_1(\beta_1 + \beta_2)}$$

Demand functions

Product 1

$$q_1^* = \frac{I}{p_1} \frac{\beta_1}{\beta_1 + \beta_2}$$

Product 2

$$q_2^* = \frac{I}{p_2} \frac{\beta_2}{\beta_1 + \beta_2}$$

Comments

- Demand decreases with price
- Demand increases with budget
- Demand independent of β_0, which does not affect the ranking

Marginal rate of substitution

Factoring out λ from first order conditions we get

$$\frac{p_1}{p_2} = \frac{\partial U(q^*)/\partial q_1}{\partial U(q^*)/\partial q_2} = \frac{MU(q_1)}{MU(q_2)}$$

MRS

- Ratio of marginal utilities (right) equals...
- ratio of prices of the 2 goods (left)
- Holds if consumer is making optimal choices

Discrete goods

Discrete choice set

- Calculus cannot be used anymore

$$U = U(q_1, \ldots, q_L)$$

with

$$q_i = \begin{cases} 1 & \text{if product } i \text{ is chosen} \\ 0 & \text{otherwise} \end{cases}$$

and

$$\sum_i q_i = 1.$$

Framework

- Do not work with demand functions anymore
- Work with utility functions
- U is the "global" utility
- Define U_i the utility associated with product i.
- It is a function of the attributes of the product (price, quality, etc.)
- We say that product i is chosen if

$$U_i \geq U_j \quad \forall j.$$

Simple example: mode choice

Attributes

	Attributes	
Alternatives	Travel time (t)	Travel cost (c)
Car (1)	t_1	c_1
Train (2)	t_2	c_2

Utility

$$\widetilde{U} = \widetilde{U}(y_1, y_2),$$

where we impose the restrictions that, for $i = 1, 2$,

$$y_i = \begin{cases} 1 & \text{if travel alternative i is chosen,} \\ 0 & \text{otherwise;} \end{cases}$$

and that only one alternative is chosen: $y_1 + y_2 = 1$.

Simple example: mode choice

Utility functions

$$\begin{aligned} U_1 &= -\beta_t t_1 - \beta_c c_1, \\ U_2 &= -\beta_t t_2 - \beta_c c_2, \end{aligned}$$

where $\beta_t > 0$ and $\beta_c > 0$ are parameters.

Equivalent specification

$$\begin{aligned} U_1 &= -(\beta_t/\beta_c)t_1 - c_1 &= -\beta t_1 - c_1 \\ U_2 &= -(\beta_t/\beta_c)t_2 - c_2 &= -\beta t_2 - c_2 \end{aligned}$$

where $\beta > 0$ is a parameter.

Choice

- Alternative 1 is chosen if $U_1 \geq U_2$.
- Ties are ignored.

Simple example: mode choice

Choice

Alternative 1 is chosen if

$$-\beta t_1 - c_1 \geq -\beta t_2 - c_2$$

or

$$-\beta(t_1 - t_2) \geq c_1 - c_2$$

Alternative 2 is chosen if

$$-\beta t_1 - c_1 \leq -\beta t_2 - c_2$$

or

$$-\beta(t_1 - t_2) \leq c_1 - c_2$$

Dominated alternative

- If $c_2 > c_1$ and $t_2 > t_1$, $U_1 > U_2$ for any $\beta > 0$
- If $c_1 > c_2$ and $t_1 > t_2$, $U_2 > U_1$ for any $\beta > 0$

Simple example: mode choice

Trade-off

- Assume $c_2 > c_1$ and $t_1 > t_2$.
- Is the traveler willing to pay the extra cost $c_2 - c_1$ to save the extra time $t_1 - t_2$?
- Alternative 2 is chosen if

$$-\beta(t_1 - t_2) \leq c_1 - c_2$$

  or

$$\beta \geq \frac{c_2 - c_1}{t_1 - t_2}$$

- β is called the *willingness to pay* or *value of time*

Dominated choice example

Obvious cases:

- $c_1 \geq c_2$ and $t_1 \geq t_2$: 2 dominates 1.
- $c_2 \geq c_1$ and $t_2 \geq t_1$: 1 dominates 2.
- Trade-offs in over quadrants

Illustration

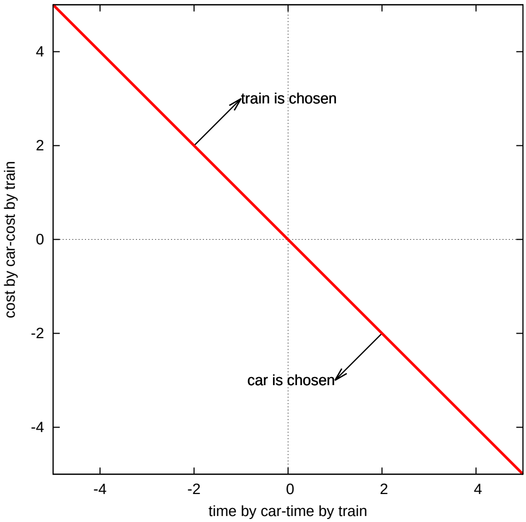
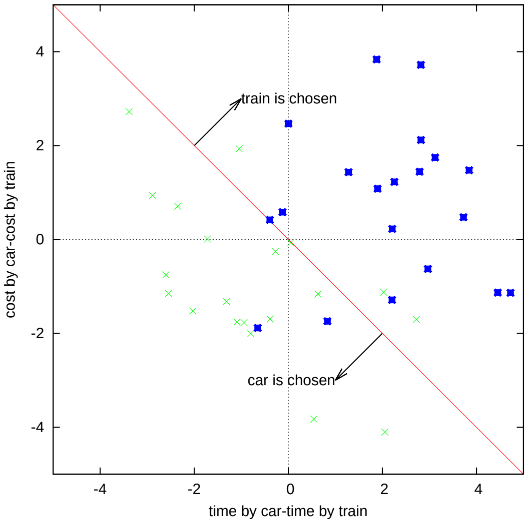


Illustration with real data



Is utility maximization a behaviorally valid assumption?

Assumptions

Decision-makers

- are able to process information
- have perfect discrimination power
- have transitive preferences
- are perfect maximizer
- are always consistent

Relax the assumptions

Use a probabilistic approach: what is the probability that alternative i is chosen?

Introducing probability

Constant utility

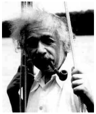
- Human behavior is inherently random
- Utility is deterministic
- Consumer does not maximize utility
- Probability to use inferior alternative is non zero

Random utility

- Decision-maker are rational maximizers
- Analysts have no access to the utility used by the decision-maker
- Utility becomes a random variable

Niels Bohr "Nature is stochastic"

Einstein "God does not throw dice"



Assumptions

Sources of uncertainty

- ☞ Unobserved attributes
- ☞ Unobserved taste variations
- ☞ Measurement errors
- ☞ Instrumental variables

Manski 1973 The structure of Random Utility Models *Theory and Decision* 8:229–254

Random utility model

Probability model

$$P(i|\mathcal{C}_n) = \Pr(U_{in} \geq U_{jn}, \text{ all } j \in \mathcal{C}_n),$$

Random utility

$$U_{in} = V_{in} + \varepsilon_{in}.$$

Random utility model

$$P(i|\mathcal{C}_n) = \Pr(V_{in} + \varepsilon_{in} \geq V_{jn} + \varepsilon_{jn}, \text{ all } j \in \mathcal{C}_n),$$

or

$$P(i|\mathcal{C}_n) = \Pr(\varepsilon_{jn} - \varepsilon_{in} \leq V_{in} - V_{jn}, \text{ all } j \in \mathcal{C}_n).$$

Over to the lab: CM1 112

Further Introduction to Biogeme
Binary Logit Model Estimation
`http://biogeme.epfl.ch/`