TRANSP-OR

EXERCISE SESSION 8

For today's lecture you will have to use material from the textbook[1] that you have not seen in the lectures. Make sure that you have read and understood section 10.5.3 about the incremental logit before doing the exercises.

Exercise 1

1. In a Stated Preferences (SP) mode choice experiment, respondents are exposed to the following three alternatives: (1) biking, (2) walking and (3) taking the metro. You have estimated a model with the following utility functions:

$$U_{1n} = \beta_1 \cdot t_{1n} + \varepsilon_{1n}$$
$$U_{2n} = ASC_2 + \beta_2 \cdot t_{2n} + \varepsilon_{2n}$$
$$U_{3n} = ASC_3 + \beta_3 \cdot t_{3n} + \varepsilon_{3n}$$

where t_{in} is the travel time of respondent n using mode i, $\varepsilon_{1n}, \varepsilon_{2n}, \varepsilon_{3n} \overset{iid}{\sim} EV(0,1)$ and β_i, ASC_i are the parameters that have been estimated, with $i \in \{1, 2, 3\}$. The population of interest consists of exactly *four* individuals: one with travel time t_{11}, two with equal travel times $t_{12} = t_{13}$ and one with travel time t_{14}. Fig. 1 illustrates the probability of choosing alternative 1 (bicycle), as predicted by this model, for each level of travel time.

Compute the fraction of the population choosing alternative 1.

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[1]Ben-Akiva, M., Bierlaire, M., McFadden, D. & Walker, J. Discrete Choice Analysis.

P(bicycle | t_{1n})

disaggregate probability

0.75

0.60

0.25

t_{11} $t_{12}=t_{13}$ t_{14} x
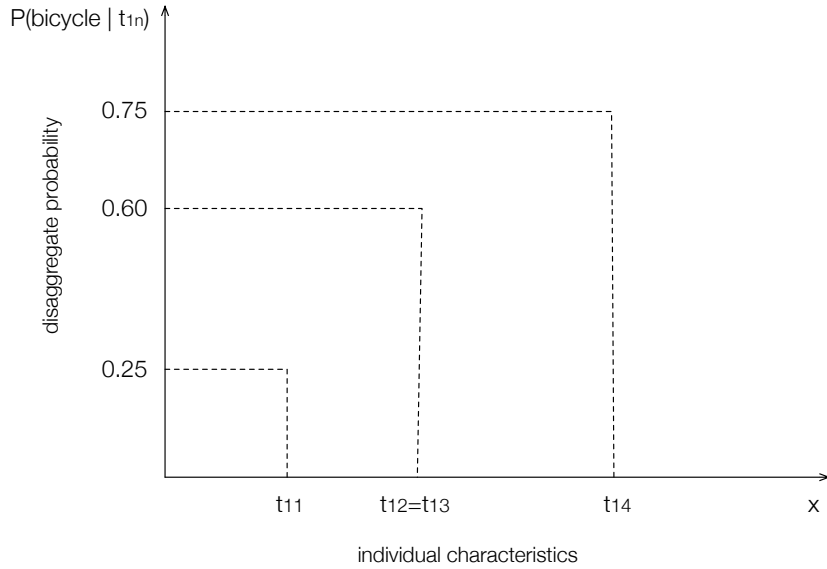
individual characteristics

Figure 1: $P(\text{bicycle}|t_{1n})$

2. Consider now the first individual of the population with travel time by bicycle t_{11}. Your *linear-in-parameters* logit model from the previous sub-question has predicted the probability with which this individual will choose each alternative as shown in column $P(i|t_{11})$ of Table 1. Due to an increase in the number of bike lanes in the city, there is a change in travel time from t_{11} to t'_{11}. The utilities of the three alternatives for this individual change as indicated in column ΔV_{in}.

Mode	$P(i|t_{11})$	ΔV_{in}	$P(i|t'_{11})$
Bicycle	0.25	0.5	
Walk	0.45	0	
Metro	0.30	0	

Table 1: Choice probabilities before and after an increase in number of bike lanes

Complete Table 1 by computing the new choice probabilities $P'(i|t_{11})$ for the three alternatives.

Exercise 2 The data about internal trips for a city of 300'000 individuals is described in Table 2. Three different transportation modes are considered to travel inside the city: car, bus and slow modes (walk and bike). The population is segmented in three different groups, according to their age: young (age $\leq$ 21), adult (21 < age $\leq$ 65) and retired (age > 65).

Mode	Age			
	Young	Adult	Retired	
Car	10'000	120'000	15'000	145'000
Bus	15'000	30'000	25'000	70'000
Slow mode	25'000	50'000	10'000	85'000
	50'000	200'000	50'000	300'000

Table 2: Description of the population of the city

Table 3 contains the data for a sample of 350 individuals from the full population.

Mode	Age			
	Young	Adult	Retired	
Car	20	120	15	155
Bus	30	30	25	85
Slow mode	50	50	10	110
	100	200	50	350

Table 3: Description of the sample

1. Calculate the sampling probabilities. Which sampling strategy has been used to collect the data?

2. Which estimation procedure can be used with this sampling strategy?

3. Express (mathematically) how to estimate the market shares in the population for each alternative using the sample (Table 3) and the choice probabilities $P(i|x_n)$.
   Hint: Consider the weight of each segment W_s ($W_s = \frac{\text{\# persons in segment s in the population}}{\text{\# persons in segment s in the sample}}$) and the indicator I_{ns} (1 if individual n belongs to segment s and 0 otherwise).