Forecasting – 7.3 Indicators

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Willingness to pay and value of time

If the model contains a cost or price variable, it is possible to analyze the trade-off between any variable and money. It reflects the *willingness* of the decision maker *to pay* for a modification of another variable of the model.

A typical example in transportation is the *value of time*, that is the price that a traveler is willing to pay to decrease the travel time.

- Let c_{in} be the cost of alternative i for individual n.
- Let x_{in} be the value of another variable of the model.
- Let $V_{in}(c_{in}, x_{in})$ be the value of the utility function associated by individual n with alternative i.
- Consider a scenario where the variable under interest takes the value $x_{in} + \delta^x_{in}$.
- We denote by δ^c_{in} the additional cost that would achieve the same utility, that is

$$V_{in}(c_{in} + \delta^c_{in}, x_{in} + \delta^x_{in}) = V_{in}(c_{in}, x_{in}). \tag{1}$$

The willingness to pay is the additional cost per unit of x, that is

$$\delta^c_{in}/\delta^x_{in}. \tag{2}$$

Its calculation involves solving equation (1).

If the variable x_{in} is continuous, and if V_{in} is differentiable in x_{in} and c_{in}, we can invoke Taylor's theorem

$$\begin{aligned} V_{in}(c_{in}, x_{in}) &= V_{in}(c_{in} + \delta^c_{in}, x_{in} + \delta^x_{in}) \\ &\approx V_{in}(c_{in}, x_{in}) + \delta^c_{in}\frac{\partial V_{in}}{\partial c_{in}}(c_{in}, x_{in}) + \delta^x_{in}\frac{\partial V_{in}}{\partial x_{in}}(c_{in}, x_{in}) \end{aligned}$$

to obtain

$$\frac{\delta^c_{in}}{\delta^x_{in}} = -\frac{(\partial V_{in}/\partial x_{in})(c_{in}, x_{in})}{(\partial V_{in}/\partial c_{in})(c_{in}, x_{in})}. \tag{3}$$

If x_{in} and c_{in} appear linearly in the utility function, that is if

$$V_{in}(c_{in}, x_{in}) = \beta_c c_{in} + \beta_x x_{in} + \cdots \tag{4}$$

then the willingness to pay involves the ratio of the coefficients:

$$\frac{\delta^c_{in}}{\delta^x_{in}} = -\frac{(\partial V_{in}/\partial x_{in})(c_{in}, x_{in})}{(\partial V_{in}/\partial c_{in})(c_{in}, x_{in})} = -\frac{\beta_x}{\beta_c}. \tag{5}$$

The above equation is the willingness to pay for an *increase* of the value of the variable x_{in}. If this increase improves the utility of the alternative, then $\beta_x > 0$. As $\beta_c < 0$, the willingness to pay is positive. In the context of value of time, we want to calculate the willingness to pay to *decrease* the travel time. Therefore, we have

$$\mathrm{VOT}_{in} = \delta^c_{in}/(-\delta^t_{in}) = \frac{(\partial V_{in}/\partial t_{in})(c_{in}, t_{in})}{(\partial V_{in}/\partial c_{in})(c_{in}, t_{in})}. \tag{6}$$

If V is linear in these variables, we have

$$\mathrm{VOT}_{in} = \delta^c_{in}/(-\delta^t_{in}) = \frac{\beta_t}{\beta_c}. \tag{7}$$

The willingness to pay is negative when a decrease of the cost compensates a modification of another variable that decreases the utility. In this case, it is sometimes called *willingness to accept*.

The above derivation, based on converting trade-offs into monetary units, is the most common one. Similar quantities can be derived by using other variables than cost as the reference.