Testing – 6.4 Likelihood ratio test

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Practice quiz

In a mode choice case study, consider the models with the following utility specifications (where the index n related to the individual has been dropped to simplify the notations):

1. Linear with generic coefficients

$$U_{\text{car}} = ASC_{\text{car}} + \beta_{\text{tt}} \cdot \text{tt}_{\text{car}} + \beta_{\text{tc}} \cdot \text{tc}_{\text{car}} + \varepsilon_{\text{car}},$$
$$U_{\text{pt}} = \beta_{\text{tt}} \cdot \text{tt}_{\text{pt}} + \beta_{\text{tc}} \cdot \text{tc}_{\text{pt}} + \varepsilon_{\text{pt}}.$$

2. Linear with alternative specific coefficients

$$U_{\text{car}} = ASC_{\text{car}} + \beta_{\text{tt,car}} \cdot \text{tt}_{\text{car}} + \beta_{\text{tc,car}} \cdot \text{tc}_{\text{car}} + \varepsilon_{\text{car}},$$
$$U_{\text{pt}} = \beta_{\text{tt,pt}} \cdot \text{tt}_{\text{pt}} + \beta_{\text{tc,pt}} \cdot \text{tc}_{\text{pt}} + \varepsilon_{\text{pt}}.$$

3. Power series

$$U_{\text{car}} = ASC_{\text{car}} + \beta_{\text{tt}} \cdot \text{tt}_{\text{car}} + \beta_{\text{tc}} \cdot \text{tc}_{\text{car}} + \beta_{\text{tc\_squared}} \cdot \text{tc}^2_{\text{car}} + \varepsilon_{\text{car}},$$
$$U_{\text{pt}} = \beta_{\text{tt}} \cdot \text{tt}_{\text{pt}} + \beta_{\text{tc}} \cdot \text{tc}_{\text{pt}} + \beta_{\text{tc\_squared}} \cdot \text{tc}^2_{\text{pt}} + \varepsilon_{\text{pt}}.$$

4. Box-cox

$$U_{\text{car}} = ASC_{\text{car}} + \beta_{\text{tt\_}boxcox} \cdot \frac{(\text{tt}_{\text{car}} - 1)^{\lambda}}{\lambda} + \beta_{\text{tc}} \cdot \text{tc}_{\text{car}} + \varepsilon_{\text{car}},$$
$$U_{\text{pt}} = \beta_{\text{tt\_}boxcox} \cdot \frac{(\text{tt}_{\text{pt}} - 1)^{\lambda}}{\lambda} + \beta_{\text{tc}} \cdot \text{tc}_{\text{pt}} + \varepsilon_{\text{pt}}.$$

5. Logarithm

$$U_{\text{car}} = ASC_{\text{car}} + \beta_{\text{tt}} \cdot \text{tt}_{\text{car}} + \beta_{\text{tc\_log}} \cdot \log(\text{tc}_{\text{car}}) + \varepsilon_{\text{car}},$$
$$U_{\text{pt}} = \beta_{\text{tt}} \cdot \text{tt}_{\text{pt}} + \beta_{\text{tc\_log}} \cdot \log(\text{tc}_{\text{pt}}) + \varepsilon_{\text{pt}}.$$

6. Piecewise linear

$$
\begin{aligned}
U_{\text{car}} =& ASC_{\text{car}} + \beta_{\text{tt},<15} \cdot \text{tt}_{\text{car},<15} + \beta_{\text{tt},[15,60)} \cdot \text{tt}_{\text{car},[15,60)} + \beta_{\text{tt},\geq 60} \cdot \text{tt}_{\text{car},\geq 60} \\
& + \beta_{\text{tc}} \cdot \text{tc}_{\text{car}} + \varepsilon_{\text{car}}, \\
U_{\text{pt}} =& \beta_{\text{tt},<15} \cdot \text{tt}_{\text{pt},<15} + \beta_{\text{tt},[15,60)} \cdot \text{tt}_{\text{pt},[15,60)} + \beta_{\text{tt},\geq 60} \cdot \text{tt}_{\text{pt},\geq 60} + \beta_{\text{tc}} \cdot \text{tc}_{\text{pt}} + \varepsilon_{\text{pt}}.
\end{aligned}
$$

where for $i \in \{\text{car}, \text{pt}\}$

$$
\text{tt}_{i,<15} = \begin{cases} \text{tt}_i, & \text{if} \quad \text{tt}_i < 15 \\ 15, & \text{otherwise,} \end{cases}
$$

$$
\text{tt}_{\text{car},[15,60)} = \begin{cases} 0, & \text{if} \quad \text{tt}_i < 15 \\ \text{tt}_i - 15, & \text{if} \quad \text{tt}_i \in [15,60) \\ 60, & \text{otherwise,} \end{cases}
$$

$$
\text{tt}_{i,\geq 60} = \begin{cases} 0, & \text{if} \quad \text{tt}_i < 60 \\ \text{tt}_i - 60, & \text{otherwise.} \end{cases}
$$

where tt_{car} and tt_{pt} are the travel times in minutes by car and public transportation respectively, tc_{car} and tc_{pt} are the travel costs in CHF of car and public transportation respectively; ASC_{car}, β's and λ are parameters to be estimated; and $\varepsilon_{\text{car}}, \varepsilon_{\text{pt}} \overset{iid}{\sim} EV(0,1)$.

When we want to test two of these models, when can we apply the likelihood ratio test?