Testing

Likelihood ratio test

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Introduction to choice models

ÉCOLE POLYTECHNIQUE
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Applications of the likelihood ratio test

Benchmarking

Unrestricted model

$$V_{in} = \beta_1 x_{ink} + \cdots$$
$$V_{jn} = \beta_2 x_{jnk} + \cdots$$
$$\vdots$$

Restricted model

Equal probability model

$$V_{in} = 0$$
$$V_{jn} = 0$$
$$\vdots$$

Restrictions

$$\beta_k = 0, \ \forall k$$

Benchmarking

Log likelihood of the unrestricted model

$$\mathcal{L}(\widehat{\beta})$$

Log likelihood of the restricted model

$$P_{in} = 1/J_n, \ \forall i \in \mathcal{C}_n, \forall n$$

$$\mathcal{L}(0) = -\sum_{n=1}^{N} \log(J_n)$$

Statistic

$$-2(\mathcal{L}(0) - \mathcal{L}(\widehat{\beta})) \sim \chi^2_K$$

Benchmarking revisited

Unrestricted model

$$
\begin{aligned}
V_{in} &= \beta_1 x_{ink} + \cdots \\
V_{jn} &= \beta_2 x_{jnk} + \cdots \\
&\vdots
\end{aligned}
$$

Restricted model

Only alternative specific constants

$$
\begin{aligned}
V_{in} &= \beta_i \\
V_{jn} &= \beta_j \\
&\vdots
\end{aligned}
$$

Restrictions

All coefficients but the constants are constrained to zero.

Benchmarking revisited

Log likelihood of the unrestricted model

$$\mathcal{L}(\widehat{\beta})$$

Log likelihood of the restricted model

$$P_{in} = N_i/N \; \forall i \in \mathcal{C}, \forall n$$

$$\mathcal{L}(c) = \sum_{i=1}^{J} N_i \log(N_i/N)$$

Statistic

$$-2(\mathcal{L}(c) - \mathcal{L}(\widehat{\beta})) \sim \chi^2_d \text{ with } d = K - J + 1$$

Benchmarking

Classical output of estimation software

Summary statistics

Number of observations = 2544

$$
\begin{aligned}
\mathcal{L}(0) &= -2794.870 \\
\mathcal{L}(c) &= -2203.160 \\
\mathcal{L}(\hat{\beta}) &= -1640.525 \\
-2[\mathcal{L}(0) - \mathcal{L}(\hat{\beta})] &= 2308.689
\end{aligned}
$$

Test of generic attributes

Unrestricted model

Alternative specific

$$\begin{aligned} V_{in} &= \beta_{1i} x_{ink} + \cdots \\ V_{jn} &= \beta_{1j} x_{jnk} + \cdots \\ &\vdots \end{aligned}$$

Restricted model

Generic

$$\begin{aligned} V_{in} &= \beta_{1} x_{ink} + \cdots \\ V_{jn} &= \beta_{1} x_{jnk} + \cdots \\ &\vdots \end{aligned}$$

Restriction

$$\beta_{1i} = \beta_{1j} = \cdots$$

Test of generic attributes

Log likelihood of the unrestricted model

$$\mathcal{L}(\widehat{\beta}_{AS})$$

Log likelihood of the restricted model

$$\mathcal{L}(\widehat{\beta}_{G})$$

Statistic

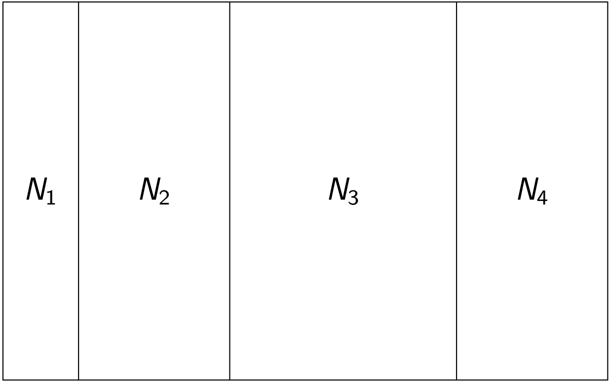
$$-2(\mathcal{L}(\widehat{\beta}_{G}) - \mathcal{L}(\widehat{\beta}_{AS})) \sim \chi^2_d \text{ with } d = K_{AS} - K_{G}$$

Test of taste variations

Segmentation

- Classify the data into G groups. Size of group g: N_g.
- The same specification is considered for each group.
- A different set of parameters is estimated for each group.

Test of taste variations



N

$\mathcal{L}_{N_1}(\widehat{\beta}^1)$ $\mathcal{L}_{N_2}(\widehat{\beta}^2)$ $\mathcal{L}_{N_3}(\widehat{\beta}^3)$ $\mathcal{L}_{N_4}(\widehat{\beta}^4)$ $\sum_{g=1}^{G} \mathcal{L}_{N_g}(\widehat{\beta}^g)$

Test of taste variations

Unrestricted model

Group specific coefficients

$$V_{in} = \sum_{g=1}^{G} (\delta_{ng} \beta_{1g}) x_{ink} + \cdots$$

$$V_{jn} = \sum_{g=1}^{G} (\delta_{ng} \beta_{2g}) x_{jnk} + \cdots$$

$$\vdots$$

Restricted model

Generic coefficients

$$V_{in} = \beta_1 x_{ink} + \cdots$$

$$V_{jn} = \beta_2 x_{jnk} + \cdots$$

$$\vdots$$

Restrictions

$$\beta_{k1} = \beta_{k2} = \cdots = \beta_{kG}, \ \forall k.$$

Test of taste variations

Log likelihood of the unrestricted model

$$\sum_{g=1}^{G} \mathcal{L}_{N_g}(\widehat{\beta}^g)$$

Log likelihood of the restricted model

$$\mathcal{L}_N(\widehat{\beta})$$

Statistic

$$-2\left[\mathcal{L}_N(\widehat{\beta}) - \sum_{g=1}^{G} \mathcal{L}_{N_g}(\widehat{\beta}^g)\right] \sim \chi^2_d \text{ with } d = \sum_{g=1}^{G} K - K = (G-1)K.$$

Tests of nonlinear specifications

Unrestricted model

Power series

$$V_{in} = \sum_{\ell=1}^{L} \beta_{1\ell} \frac{x_{ink}}{x_{\text{ref}}}^{\ell} + \cdots$$
$$V_{jn} = \beta_2 x_{jnk} + \cdots$$
$$\vdots$$

Restricted model

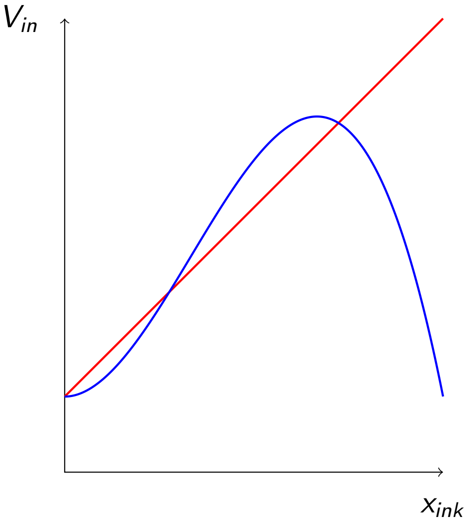
Linear specification

$$V_{in} = \beta_1 x_{ink} + \cdots$$
$$V_{jn} = \beta_2 x_{jnk} + \cdots$$
$$\vdots$$

Restrictions

$$\beta_{12} = \beta_{13} = \cdots = \beta_{1L} = 0$$

Power series



Test of nonlinear specifications

Log likelihood of the unrestricted model

$$\mathcal{L}(\widehat{\beta}_U)$$

Log likelihood of the restricted model

$$\mathcal{L}(\widehat{\beta}_R)$$

Statistic

$$-2\left[\mathcal{L}(\widehat{\beta}_R) - \mathcal{L}(\widehat{\beta}_U)\right] \sim \chi^2_d \text{ with } d = L - 1$$

Notes

- Usually not behaviorally meaningful
- Danger of overfitting
- Polynomials are most of the time inappropriate for extrapolation due to oscillation
- Other nonlinear specifications can be used for testing
  - Piecewise linear
  - Box-Cox