# Testing – 6.4 Likelihood ratio test

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The general motivation of the likelihood ratio test is to investigate parsimonious versions of a given specification, by introducing linear restrictions on the parameters. The null hypothesis of the test is that the parsimonious, or restricted, model is the true model. If it is rejected, the unrestricted model is preferred.

It can be shown (Wilks, 1938) that under the null hypothesis $H_0$, the statistic

$$-2(\mathcal{L}(\hat{\beta}_R) - \mathcal{L}(\hat{\beta}_U)) \sim \chi^2_{(K_U - K_R)}, \tag{1}$$

where

- $\mathcal{L}(\hat{\beta}_R)$ is the log likelihood of the restricted model,
- $\mathcal{L}(\hat{\beta}_U)$ is the log likelihood of the unrestricted model,
- $K_R$ is the number of parameters in the restricted model, and,
- $K_U$ is the number of parameters in the unrestricted model.

The simple hypothesis (that is, the restricted model is correct) and the composite hypothesis (that is, the unrestricted model is correct) are said to be *nested*[1], because the former can be obtained from the latter using linear restrictions. Note that the test can only be applied for nested hypotheses.

The test can be written in terms of likelihood. As

$$\mathcal{L}(\hat{\beta}_R) = \log \mathcal{L}^*(\hat{\beta}_R) \text{ and } \mathcal{L}(\hat{\beta}_U) = \log \mathcal{L}^*(\hat{\beta}_U), \tag{2}$$

we can write (1) as

$$-2 \log \frac{\mathcal{L}^*(\hat{\beta}_R)}{\mathcal{L}^*(\hat{\beta}_U)} \sim \chi^2_{(K_U - K_R)}, \tag{3}$$

that explains the name of the test.

[1] This has nothing to do with the nested logit model.

## References


Wilks, S. S. (1938). The large-sample distribution of the likelihood ratio for testing composite hypotheses, *Ann. Math. Statist.* **9**(1): 60–62.
**URL:** *http://dx.doi.org/10.1214/aoms/1177732360*