Testing

Specification testing

Michel Bierlaire

Introduction to choice models

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Differences from classical hypothesis testing

Classical hypothesis testing: example

Null hypothesis (H_0)

A simple hypothesis contradicting a theoretical assumption.

Lady testing tea

- Theory: a lady is able to tell if the milk has been poured before of after the tea in a cup.
- H_0: the outcome of the taste is purely random.

Specification testing: example

Null hypothesis (H_0)

A simple hypothesis contradicting a theoretical assumption.

Explanatory variable

- Theory: a variable explains the choice behavior.
- H_0: the coefficient of the variable is zero.

Errors in hypothesis testing

Errors in hypothesis testing

Type I error

Errors in hypothesis testing

Type I error

Type II error

Errors in hypothesis testing

Type I error

- H_0 rejected and H_0 true.

Type II error

Errors in hypothesis testing

Type I error

- H_0 rejected and H_0 true.

Type II error

- H_0 accepted and H_0 false.

Errors in hypothesis testing

Type I error

- H_0 rejected and H_0 true.
- Include an irrelevant variable.

Type II error

- H_0 accepted and H_0 false.

Errors in hypothesis testing

Type I error

- H_0 rejected and H_0 true.
- Include an irrelevant variable.

Type II error

- H_0 accepted and H_0 false.
- Omit a relevant variable.

Errors in hypothesis testing

Type I error

- H_0 rejected and H_0 true.
- Include an irrelevant variable.
- Loss of efficiency.

Type II error

- H_0 accepted and H_0 false.
- Omit a relevant variable.

Errors in hypothesis testing

Type I error

- H_0 rejected and H_0 true.
- Include an irrelevant variable.
- Loss of efficiency.

Type II error

- H_0 accepted and H_0 false.
- Omit a relevant variable.
- Specification error.

Errors in hypothesis testing

Type I error

- H_0 rejected and H_0 true.
- Include an irrelevant variable.
- Loss of efficiency.
- Cost: C_I.

Type II error

- H_0 accepted and H_0 false.
- Omit a relevant variable.
- Specification error.

Errors in hypothesis testing

Type I error

- H_0 rejected and H_0 true.
- Include an irrelevant variable.
- Loss of efficiency.
- Cost: C_I.

Type II error

- H_0 accepted and H_0 false.
- Omit a relevant variable.
- Specification error.
- Cost: $C_{II} >> C_I$.

Errors in hypothesis testing

Type I error

- H_0 rejected and H_0 true.
- Include an irrelevant variable.
- Loss of efficiency.
- Cost: C_I.

Type II error

- H_0 accepted and H_0 false.
- Omit a relevant variable.
- Specification error.
- Cost: $C_{II} >> C_I$.

Note

In classical hypothesis testing, $C_I \approx C_{II}$

Impact of an error

Impact of an error

Probability of an error

$$\text{P(Type I)} =$$

Impact of an error

Probability of an error

$$\text{P(Type I)} = \text{P}(H_0 \text{ rejected} | H_0 \text{ true})$$

Impact of an error

Probability of an error

$$\text{P(Type I)} = \quad \text{P}(H_0 \text{ rejected} | H_0 \text{ true}) \quad \text{P}(H_0 \text{ true})$$

Impact of an error

Probability of an error

$$\text{P(Type I)} = \underbrace{\text{P}(H_0 \text{ rejected} | H_0 \text{ true})}_{\alpha} \quad \text{P}(H_0 \text{ true})$$

Impact of an error

Probability of an error

$$\text{P(Type I)} = \quad \text{P}(H_0 \text{ rejected} | H_0 \text{ true}) \quad \text{P}(H_0 \text{ true})$$

$$\alpha \qquad\qquad \lambda$$

Impact of an error

Probability of an error

$$\text{P(Type I)} = \quad \text{P}(H_0 \text{ rejected} | H_0 \text{ true}) \quad \text{P}(H_0 \text{ true})$$

$$\alpha \qquad \lambda$$

$$\text{P(Type II)} =$$

Impact of an error

Probability of an error

$$\mathrm{P}(\text{Type I}) = \quad \mathrm{P}(H_0 \text{ rejected} | H_0 \text{ true}) \quad \mathrm{P}(H_0 \text{ true})$$

$$\alpha \qquad \lambda$$

$$\mathrm{P}(\text{Type II}) = \quad \mathrm{P}(H_0 \text{ accepted} | H_0 \text{ false})$$

Impact of an error

Probability of an error

$$\text{P(Type I)} = \quad \text{P}(H_0 \text{ rejected} | H_0 \text{ true}) \quad \text{P}(H_0 \text{ true})$$

$$\alpha \qquad \lambda$$

$$\text{P(Type II)} = \quad \text{P}(H_0 \text{ accepted} | H_0 \text{ false}) \quad \text{P}(H_0 \text{ false})$$

Impact of an error

Probability of an error

$$P(\text{Type I}) = \underbrace{P(H_0 \text{ rejected} | H_0 \text{ true})}_{\alpha} \quad \underbrace{P(H_0 \text{ true})}_{\lambda}$$

$$P(\text{Type II}) = \underbrace{P(H_0 \text{ accepted} | H_0 \text{ false})}_{\beta} \quad P(H_0 \text{ false})$$

Impact of an error

Probability of an error

$$
\begin{aligned}
\text{P(Type I)} &= \underset{\alpha}{\text{P}(H_0 \text{ rejected} | H_0 \text{ true})} \quad \underset{\lambda}{\text{P}(H_0 \text{ true})} \\
\text{P(Type II)} &= \underset{\beta}{\text{P}(H_0 \text{ accepted} | H_0 \text{ false})} \quad \underset{(1-\lambda)}{\text{P}(H_0 \text{ false})}
\end{aligned}
$$

Impact of an error

Probability of an error

$$\text{P(Type I)} = \underset{\color{red}{\alpha}}{\text{P}(H_0 \text{ rejected} | H_0 \text{ true})} \quad \underset{\color{red}{\lambda}}{\text{P}(H_0 \text{ true})}$$

$$\text{P(Type II)} = \underset{\color{red}{\beta}}{\text{P}(H_0 \text{ accepted} | H_0 \text{ false})} \quad \underset{\color{red}{(1-\lambda)}}{\text{P}(H_0 \text{ false})}$$

Expected cost

$$\text{Expected cost} =$$

Impact of an error

Probability of an error

$$\text{P(Type I)} = \underbrace{\text{P}(H_0 \text{ rejected} | H_0 \text{ true})}_{\alpha} \quad \underbrace{\text{P}(H_0 \text{ true})}_{\lambda}$$

$$\text{P(Type II)} = \underbrace{\text{P}(H_0 \text{ accepted} | H_0 \text{ false})}_{\beta} \quad \underbrace{\text{P}(H_0 \text{ false})}_{(1-\lambda)}$$

Expected cost

$$\text{Expected cost} = \text{P(Type I)} \quad C_I \quad + \quad \text{P(Type II)} \quad C_{II}$$

Impact of an error

Probability of an error

$$
\begin{aligned}
\text{P(Type I)} &= \underset{\color{red}{\alpha}}{\text{P}(H_0 \text{ rejected} \mid H_0 \text{ true})} \quad \underset{\color{red}{\lambda}}{\text{P}(H_0 \text{ true})} \\
\text{P(Type II)} &= \underset{\color{red}{\beta}}{\text{P}(H_0 \text{ accepted} \mid H_0 \text{ false})} \quad \underset{\color{red}{(1-\lambda)}}{\text{P}(H_0 \text{ false})}
\end{aligned}
$$

Expected cost

$$
\begin{aligned}
\text{Expected cost} &= \text{P(Type I)} \quad C_I \quad + \quad \text{P(Type II)} \quad C_{II} \\
&= \alpha\lambda \quad C_I \quad + \quad \beta(1-\lambda) \quad C_{II}
\end{aligned}
$$

Impact of an error

Probability of an error

$$\begin{aligned} \text{P(Type I)} &= \underbrace{\text{P}(H_0 \text{ rejected} | H_0 \text{ true})}_{\alpha} \quad \underbrace{\text{P}(H_0 \text{ true})}_{\lambda} \\ \text{P(Type II)} &= \underbrace{\text{P}(H_0 \text{ accepted} | H_0 \text{ false})}_{\beta} \quad \underbrace{\text{P}(H_0 \text{ false})}_{(1-\lambda)} \end{aligned}$$

Expected cost

$$\begin{aligned} \text{Expected cost} &= \text{P(Type I)} \quad C_I \quad + \quad \text{P(Type II)} \quad C_{II} \\ &= \alpha\lambda \quad C_I \quad + \quad \beta(1-\lambda) \quad C_{II} \end{aligned}$$

Classical hypothesis testing

$\lambda \approx 1,\ C_I \approx C_{II}$

Impact of an error

Probability of an error

$$\begin{aligned}
P(\text{Type I}) &= \underbrace{P(H_0 \text{ rejected} | H_0 \text{ true})}_{\alpha} \quad \underbrace{P(H_0 \text{ true})}_{\lambda} \\
P(\text{Type II}) &= \underbrace{P(H_0 \text{ accepted} | H_0 \text{ false})}_{\beta} \quad \underbrace{P(H_0 \text{ false})}_{(1-\lambda)}
\end{aligned}$$

Expected cost

$$\begin{aligned}
\text{Expected cost} &= P(\text{Type I}) \quad C_I \quad + \quad P(\text{Type II}) \quad C_{II} \\
&= \alpha\lambda \quad C_I \quad + \quad \beta(1-\lambda) \quad C_{II}
\end{aligned}$$

Classical hypothesis testing

$\lambda \approx 1$, $C_I \approx C_{II}$: prefer small α.

Impact of an error

Probability of an error

$$
\begin{aligned}
P(\text{Type I}) &= \underbrace{P(H_0 \text{ rejected} \mid H_0 \text{ true})}_{\alpha} \quad \underbrace{P(H_0 \text{ true})}_{\lambda} \\
P(\text{Type II}) &= \underbrace{P(H_0 \text{ accepted} \mid H_0 \text{ false})}_{\beta} \quad \underbrace{P(H_0 \text{ false})}_{(1-\lambda)}
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$$

Expected cost

$$
\begin{aligned}
\text{Expected cost} &= P(\text{Type I}) \quad C_I \quad + \quad P(\text{Type II}) \quad C_{II} \\
&= \alpha\lambda \quad C_I \quad + \quad \beta(1-\lambda) \quad C_{II}
\end{aligned}
$$

Specification testing

$\lambda \approx 0.5$, $C_{II} >> C_I$: larger α can be used.