Choice with multiple alternatives

Specification of the deterministic part

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Introduction to choice models

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Nonlinear specifications: heteroscedasticity

Heteroscedasticity

Logit is homoscedastic

- ε_{in} i.i.d. across both i and n.
- In particular, they all have the same variance.

Motivation

- People may have different level of knowledge (e.g. taxi drivers)
- Different sources of data

Heteroscedasticity

Data

- G groups in the population.
- Each individual n belongs to exactly one group g.
- Characterized by indicators:

$$\delta_{ng} = \begin{cases} 1 & \text{if } n \text{ belongs to g,} \\ 0 & \text{otherwise} \end{cases}$$

and $\sum_g \delta_{ng} = 1$, for all n.

Heteroscedasticity

Assumption: variance of error terms is different across groups

Consider individual n_1 belonging to group 1, and individual n_2 belonging to group 2.

$$
\begin{aligned}
U_{in_1} &= V_{in_1} + \varepsilon_{in_1} \\
U_{in_2} &= V_{in_2} + \varepsilon_{in_2}
\end{aligned}
$$

and $\mathrm{Var}(\varepsilon_{in_1}) \neq \mathrm{Var}(\varepsilon_{in_2})$

Modeling

Without loss of generality:

$$
\mathrm{Var}(\varepsilon_{in_1}) = \alpha_2^2 \, \mathrm{Var}(\varepsilon_{in_2})
$$

Heteroscedasticity

Modeling: scale parameters

$$\begin{array}{rcrcrcrcl} U_{in_1} & = & V_{in_1} & + & \varepsilon_{in_1} & = & V_{in_1} & + & \varepsilon'_{in_1} \\ \alpha_2 U_{in_2} & = & \alpha_2 V_{in_2} & + & \alpha_2 \varepsilon_{in_2} & = & \alpha_2 V_{in_2} & + & \varepsilon'_{in_2} \end{array}$$

Variance

$$\mathrm{Var}(\varepsilon'_{in_2}) = \mathrm{Var}(\alpha_2 \varepsilon_{in_2})$$

Heterosceadasticity

Modeling: scale parameters

$$\begin{array}{rcrcrcrcl} U_{in_1} & = & V_{in_1} & + & \varepsilon_{in_1} & = & V_{in_1} & + & \varepsilon'_{in_1} \\ \alpha_2 U_{in_2} & = & \alpha_2 V_{in_2} & + & \alpha_2 \varepsilon_{in_2} & = & \alpha_2 V_{in_2} & + & \varepsilon'_{in_2} \end{array}$$

Variance

$$\begin{array}{rcl} \mathrm{Var}(\varepsilon'_{in_2}) & = & \mathrm{Var}(\alpha_2 \varepsilon_{in_2}) \\ & = & \alpha_2^2 \, \mathrm{Var}(\alpha_2 \varepsilon_{in_2}) \end{array}$$

Heteroscedasticity

Modeling: scale parameters

$$
\begin{array}{rcrcrcrcl}
U_{in_1} & = & V_{in_1} & + & \varepsilon_{in_1} & = & V_{in_1} & + & \varepsilon'_{in_1} \\
\alpha_2 U_{in_2} & = & \alpha_2 V_{in_2} & + & \alpha_2 \varepsilon_{in_2} & = & \alpha_2 V_{in_2} & + & \varepsilon'_{in_2}
\end{array}
$$

Variance

$$
\begin{array}{rcl}
\mathrm{Var}(\varepsilon'_{in_2}) & = & \mathrm{Var}(\alpha_2 \varepsilon_{in_2}) \\
 & = & \alpha_2^2 \,\mathrm{Var}(\alpha_2 \varepsilon_{in_2}) \\
 & = & \mathrm{Var}(\varepsilon_{in_1})
\end{array}
$$

Heteroscedasticity

Modeling: scale parameters

$$
\begin{aligned}
U_{in_1} &= V_{in_1} + \varepsilon_{in_1} = V_{in_1} + \varepsilon'_{in_1} \\
\alpha_2 U_{in_2} &= \alpha_2 V_{in_2} + \alpha_2 \varepsilon_{in_2} = \alpha_2 V_{in_2} + \varepsilon'_{in_2}
\end{aligned}
$$

Variance

$$
\begin{aligned}
\mathrm{Var}(\varepsilon'_{in_2}) &= \mathrm{Var}(\alpha_2 \varepsilon_{in_2}) \\
&= \alpha_2^2 \mathrm{Var}(\alpha_2 \varepsilon_{in_2}) \\
&= \mathrm{Var}(\varepsilon_{in_1}) \\
&= \mathrm{Var}(\varepsilon'_{in_1})
\end{aligned}
$$

Heteroscedasticity

Modeling: scale parameters

$$\begin{array}{rcrcrcrcl} U_{in_1} & = & V_{in_1} & + & \varepsilon_{in_1} & = & V_{in_1} & + & \varepsilon'_{in_1} \\ \alpha_2 U_{in_2} & = & \alpha_2 V_{in_2} & + & \alpha_2 \varepsilon_{in_2} & = & \alpha_2 V_{in_2} & + & \varepsilon'_{in_2} \end{array}$$

Variance

$$\begin{array}{rcl} \text{Var}(\varepsilon'_{in_2}) & = & \text{Var}(\alpha_2 \varepsilon_{in_2}) \\ & = & \alpha_2^2 \text{Var}(\alpha_2 \varepsilon_{in_2}) \\ & = & \text{Var}(\varepsilon_{in_1}) \\ & = & \text{Var}(\varepsilon'_{in_1}) \end{array}$$

ε'_{in_1} and ε'_{in_2} can be assumed i.i.d.

Heteroscedasticity

Modeling: utility function

$$\mu_n V_{in} + \varepsilon_{in}$$

where

$$\mu_n = \sum_{g=1}^{G} \delta_{ng} \alpha_g$$

and α_g, $g = 1, \ldots, G$ are unknown parameters to be estimated from data.

Remarks

- Even if $V_{in} = \sum_j \beta_j x_{jin}$ is linear-in-parameters, $\mu_n V_{in} = \sum_j \mu_n \beta_j x_{jin}$ is not.
- Normalization: one α_g must be normalized.