# Binary choice – 3.1 Model specification: the error term

Michel Bierlaire

*Solution of the practice quiz.*

The error terms represent everything that is unknown to the analyst. A possible assumption is that all these elements add up to form the error terms. Then, invoking the central limit theorem, they follow a normal distribution.

Suppose that $\varepsilon_{in}$ and $\varepsilon_{jn}$ are both normal with zero mean, and variance $\sigma_i^2$ and $\sigma_j^2$ respectively. They are possibly correlated with covariance $\sigma_{ij}$. Note that these parameters do not have an index $n$, to reflect the i.i.d. assumption. They are constant across individuals. Under these assumptions the term $\varepsilon_n = \varepsilon_{jn} - \varepsilon_{in}$ is also normally distributed with mean zero and variance $\sigma^2 = \sigma_i^2 + \sigma_j^2 - 2\sigma_{ij}$. We can now solve for the choice probabilities as follows:

$$P_n(i) = \Pr(\varepsilon_{jn} - \varepsilon_{in} \leq V_{in} - V_{jn}) = \Pr(\varepsilon_n \leq V_{in} - V_{jn}), \tag{1}$$

from the random utility model. We now use the fact that $\varepsilon_n$ is normally distributed to obtain

$$P_n(i) = \int_{\varepsilon=-\infty}^{V_{in}-V_{jn}} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\varepsilon}{\sigma}\right)^2\right] \mathrm{d}\varepsilon. \tag{2}$$

By changing the variable $u = \varepsilon/\sigma$ so that $du = d\varepsilon/\sigma$, we obtain a standard normal, and

$$P_n(i) = \frac{1}{\sqrt{2\pi}} \int_{u=-\infty}^{(V_{in}-V_{jn})/\sigma} \exp\left[-\frac{1}{2}u^2\right] \mathrm{d}u, \tag{3}$$

which is

$$P_n(i) = \Phi\left(\frac{V_{in} - V_{jn}}{\sigma}\right), \tag{4}$$

where and $\Phi(\cdot)$ denotes the CDF of a standardized normal distribution.