Theoretical foundations – 2.4 Random utility theory

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Up to now, the theoretical developments have assumed that individuals behave deterministically. Decision-makers are assumed to be all knowing with perfect discriminatory power, able to process information, choose the best choice, and repeat this identical choice under identical circumstances. This is implied by the assumed properties of the preferences, such as completeness, transitivity, and continuity. However, the simple example presented before illustrates that such assumptions may not be fully consistent with real behavior. Actually, there are copious examples both in laboratory experiments and in the field in which it appears that decision-makers do not behave as such. As Tversky (1969) points out, “when faced with repeated choices between x and y, people often choose x in some instances and y in others.” Inspired by the need to explain experimental observations of inconsistent preferences, probabilistic choice theory was developed. In probabilistic choice theory, rather than assuming there is a deterministic process that can be used to establish the choice outcome, it is recognized that the best that can be done is to determine the probability of different choice outcomes given a particular choice situation and decision-maker.

There are several ways of modeling probabilistic choice. In this course, we assume that the source of the stochasticity is due to errors made by the analyst in developing the model. Here the assumption is that while humans *are* deterministic and rational utility maximizers, analysts are unable to understand and model fully all of the relevant factors that affect human behavior. The individual is assumed to be all knowing and rational and select the alternative with the highest utility. However, the utilities are not known to the analyst with certainty and are therefore treated by the analyst as random variables. This is called the *random utility* approach. The value of the random utility approach is that it provides a link with behavioral theory from microeconomics and therefore a link to the concepts and methods that are useful for both developing model specifications and using the models for analysis.

Formally, the utility that individual n associates with alternative i is a random variable denoted U_{in}. The fact that alternative i is chosen is again associated with the fact that U_{in} is the largest utility. The model is now expressed in a probabilistic way, as follows:

$$P(i|\mathcal{C}_n) = \Pr(U_{in} \geq U_{jn}, \forall j \in \mathcal{C}_n). \quad (1)$$

The most common representation for U_{in} is inspired from linear regression. The utility is separated into two additive parts:

$$U_{in} = V_{in} + \varepsilon_{in}, \quad (2)$$

where V_{in} is called the *deterministic* or *systematic* part of the utility, and ε_{in} is the *error term*. Typically, V_{in} involves the explanatory variables, while distributional assumptions are made on the joint distribution of the random vector of error terms $\varepsilon_n = (\varepsilon_{1n}, \ldots, \varepsilon_{J_n n})$

In the rest of the course, we are intuitively deriving concrete models, based on simple assumptions, that are relaxed later on. In the next unit, we provide a general derivation of the model. As it is quite technical, it may be skipped without loss of continuity.

References

Tversky, A. (1969). Intransitivity of preferences, *Psychological Review* **76**(1): 31–48.