Theoretical foundations

Microeconomic consumer theory

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Introduction to choice models

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The case of discrete goods

Microeconomic theory of discrete goods

The consumer

- selects the quantities of continuous goods: $Q = (q_1, \ldots, q_L)$
- chooses an alternative in a discrete choice set $i = 1, \ldots, j, \ldots, J$
- discrete decision vector: $(y_1, \ldots, y_J)$, $y_j \in \{0, 1\}$, $\sum_j y_j = 1$.

Note

- In theory, one alternative of the discrete choice combines all possible choices made by an individual.
- In practice, the choice set will be restricted for tractability

Example

Choices

- House location: discrete choice
- Car type: discrete choice
- Number of kilometers driven per year: continuous choice

Discrete choice set

Each combination of a house location and a car is an alternative

Utility maximization

Utility

$$\widetilde{U}(Q, y, \tilde{z}^T y)$$

- Q: quantities of the continuous good
- y: discrete choice
- $\tilde{z}^T = (\tilde{z}_1, \ldots, \tilde{z}_i, \ldots, \tilde{z}_J) \in \mathbb{R}^{K \times J}$: K attributes of the J alternatives
- $\tilde{z}^T y \in \mathbb{R}^K$: attributes of the chosen alternative
- θ: vector of parameters

Optimization problem

$$\max_{Q,y} \widetilde{U}(Q, y, \tilde{z}^T y)$$

subject to

$$\begin{aligned} & p^T Q + c^T y \leq I \\ & \textstyle\sum_j y_j = 1 \\ & y_j \in \{0, 1\}, \forall j. \end{aligned}$$

where $c^T = (c_1, \ldots, c_i, \ldots, c_J)$ is the cost of each alternative

Solving the problem

- Mixed integer optimization problem
- No optimality condition
- Impossible to derive demand functions directly

Solving the problem

Step 1: condition on the choice of the discrete good

- Fix the discrete good, that is select a feasible y.
- The problem becomes a continuous problem in Q.
- Conditional demand functions can be derived:

$$q_{\ell|y} = \text{demand}(I - c^T y, p, \tilde{z}^T y),$$

  or, equivalently, for each alternative i,

$$q_{\ell|i} = \text{demand}(I - c_i, p, \tilde{z}_i).$$

- $I - c_i$ is the income left for the continuous goods, if alternative i is chosen.
- If $I - c_i < 0$, alternative i is declared unavailable and removed from the choice set.

Solving the problem

Conditional demand functions

$$\text{demand}(I - c_i, p, \tilde{z}_i), \;\; i = 1, \ldots, J$$

Conditional indirect utility functions

Substitute the demand functions into the utility:

$$U_i = \widetilde{U}(\text{demand}(I - c_i, p, \tilde{z}_i), \tilde{z}_i) = U(I - c_i, p, \tilde{z}_i), \;\; i = 1, \ldots, J$$

Solving the problem

Step 2: Choice of the discrete good

$$\max_{y} U(I - c^T y, p, \tilde{z}^T y) \text{ s.t. } \sum_{i=1}^{J} y_i = 1.$$

- Enumerate all alternatives.
- Compute the conditional indirect utility function U_i.
- Select the alternative with the highest U_i.
- Note: no income constraint anymore.

Model for individual n

$$\max_y U(I_n - c_n^T y, p_n, \tilde{z}_n^T y)$$

Simplifications

- S_n: set of characteristics of n, including income I_n.
- Prices of the continuous goods (p_n) are neglected.
- c_{in} is considered as another attribute and merged into $\tilde{z}_n$

$$z_n = \{\tilde{z}_n, c_n\}.$$

$$\max_i U_{in} = U(z_{in}, S_n)$$