EPFL
ENAC TRANSP-OR
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Mathematical Modeling of Behavior
Fall 2017

EXERCISE SESSION 4

Exercise 1 In a case study of transportation mode choice, the parameters of the utility functions have been estimated as follows:

$$\begin{aligned} U_{1n} &= 1 - 0.03 \cdot tt_{1n} - 0.06 \cdot c_{1n} + 0.5 \cdot \text{income}_n + \varepsilon_{1n} \\ U_{2n} &= -0.02 \cdot tt_{2n} - 0.0375 \cdot c_{2n} + 0.5 \cdot \text{university}_n + \varepsilon_{2n} \end{aligned} \tag{1}$$

where tt_{in} is the travel time in minutes and c_{in} is the cost in CHF for respondent n, with $i \in \{\text{car}, \text{train}\}$, $income_n$ takes value 1 if the respondent's monthly income is larger than 6000CHF and 0 otherwise, and $university_n$ takes value 1 if the respondent went to the university and 0 otherwise. $\varepsilon_{1n}, \varepsilon_{2n} \overset{iid}{\sim} \text{EV}(0, 1)$.

1. Compute the probability to choose each mode for the following individuals:

Name	tt_1	tt_2	c_1	c_2	monthly income	university
Eva	22	18	2	2.1	7000	yes
Anna	120	100	10	15	3000	yes
Michel	10	50	3	5	10000	no
Meri	25	9	7	2.1	5000	no

2. What does the alternative specific constant in alternative 1 represent? Interpret one by one all the parameters.

Exercise 2

1. Define the Box-Cox transformation. What modeling assumption are you testing when specifying a Box-Cox transformation of the travel cost in a model of transportation mode choice? Let λ be the parameter of the Box-Cox transformation. What particular cases do you obtain when $\lambda = 1$ or $\lambda = 0$?

2. In a model developed for the transportation mode choice in the Netherlands case study, the deterministic parts of the utilities for the car and rail alternatives are specified as follows:

$$\begin{aligned} V_{Car,n} &= ASC_{CAR} + \beta_{COST} \cdot cost_{car,n} + \beta_{TIME\_CAR} \cdot time_{car,n} \\ V_{Rail,n} &= ASC_{RAIL} + +\beta_{COST} \cdot cost_{rail,n} + \beta_{TIME\_RAIL} \cdot time_{rail,n} + \beta_{FEMALE} \cdot female_n \end{aligned} \tag{2}$$

where $time_{car,n}$ and $time_{rail,n}$ are the travel times for car and rail respectively for individual n, $cost_{car,n}$ and $cost_{rail,n}$ are the travel costs for car and rail for individual n, and $female_n$ takes value 1 if the individual is a female, and 0 if he is a male (note that we can use directly the variable labeled as `gender` from the Netherlands dataset as female is identified with 1 and male with 0). The estimation results for this model are shown in Figure 1.

Formulas

Car utility: *ASC_CAR * one + BETA_COST * car_cost_euro + BETA_TIME_CAR * car_time*
Rail utility: *ASC_RAIL * one + BETA_COST * rail_cost_euro + BETA_TIME_RAIL * rail_time + BETA_FEMALE * gender*

Estimation report

Number of estimated parameters:	5
Sample size:	228
Excluded observations:	1511
Init log likelihood:	-158.038
Final log likelihood:	-115.880
Likelihood ratio test for the init. model:	84.314
Rho-square for the init. model:	0.267
Rho-square-bar for the init. model:	0.235
Akaike Information Criterion:	241.761
Bayesian Information Criterion:	258.908
Final gradient norm:	+1.921e-05
Diagnostic:	CFSQP: Normal termination. Obj: 6.05545e-06 Const: 6.05545e-06
Iterations:	12
Data processing time:	00:00
Run time:	00:00
Nbr of threads:	12

Estimated parameters

Click on the headers of the columns to sort the table [Credits]

Name	Value	Std err	t-test	p-value		Robust Std err	Robust t-test	p-value	
ASC_CAR	2.85	1.09	2.62	0.01		1.02	2.80	0.01	
BETA_COST	-0.130	0.0251	-5.17	0.00		0.0265	-4.89	0.00	
BETA_FEMALE	0.675	0.330	2.05	0.04		0.329	2.05	0.04	
BETA_TIME_CAR	-2.34	0.489	-4.78	0.00		0.495	-4.73	0.00	
BETA_TIME_RAIL	-0.529	0.418	-1.27	0.20	*	0.414	-1.28	0.20	*

Figure 1: Estimation results of the base model

In addition to Model 1 we now estimate a model with a Box-Cox transformation of the cost variables. A snapshot of the estimation results is presented in Figure 2.

Formulas

Car utility: *ASC_CAR * one + (BETA_COST * ((car_cost_euro ^ LAMBDA) - (1))) / LAMBDA + BETA_TIME_CAR * car_time*
Rail utility: *ASC_RAIL * one + BETA_COST * (((rail_cost_euro ^ LAMBDA) - (1)) / LAMBDA) + BETA_TIME_RAIL * rail_time + BETA_FEMALE * gender*

Estimation report

Number of estimated parameters: 6
Sample size: 228
Excluded observations: 1511
Init log likelihood: -158.038
Final log likelihood: -113.265
Likelihood ratio test for the init. model: 89.546
Rho-square for the init. model: 0.283
Rho-square-bar for the init. model: 0.245
Akaike Information Criterion: 238.530
Bayesian Information Criterion: 259.106
Final gradient norm: +5.939e-05
Diagnostic: CFSQP: Normal termination. Obj: 6.05545e-06 Const: 6.05545e-06
Iterations: 24
Data processing time: 00:00
Run time: 00:00
Nbr of threads: 12

Estimated parameters

Click on the headers of the columns to sort the table [Credits]

Name	Value	Std err	t-test	p-value		Robust Std err	Robust t-test	p-value	
ASC_CAR	2.64	1.09	2.41	0.02		1.03	2.56	0.01	
BETA_COST	-0.544	0.266	-2.05	0.04		0.249	-2.19	0.03	
BETA_FEMALE	0.735	0.338	2.18	0.03		0.334	2.20	0.03	
BETA_TIME_CAR	-2.42	0.500	-4.84	0.00		0.509	-4.76	0.00	
BETA_TIME_RAIL	-0.616	0.427	-1.44	0.15	*	0.423	-1.46	0.15	*
LAMBDA	0.400	0.224	1.78	0.07	*	0.211	1.90	0.06	*

Figure 2: Estimation results of model with a Box-Cox transformation

(a) Comment and interpret the values of the estimates of both models using informal tests (i.e., analyze the signs of the coefficients).

(b) Identify what parameters are significantly different from 0 (or 1 in the case of λ) using statistical tests.

(c) Propose two different statistical tests to determine if model 2 provides an improvement compared to model 1.

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