Optimization and Simulation

Laboratory 7

Markov Chain Monte Carlo Methods

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Goals

- Understand and apply:
  - Markov Chain Monte Carlo
  - Metropolis-Hastings algorithm
  - Gibbs sampling

Overview

7.1 Markov Chain Monte Carlo

7.2 Metropolis-Hastings algorithm

7.3 Gibbs sampling

7.1 Markov Chain Monte Carlo

Exercise

Implement a Markov Chains representing the “machine” example introduced in the lecture.

Function to implement (.m)

`MarkovChain`

Test the function (m.)

`MarkovChainTest`

Play with the states, initial state and transition matrix.

Extra question on MSE in `MarkovChainTest`.

7.2 Metropolis-Hastings algorithm

Problem description

Professor M. (former Major M.) sees that the students in his course are extremely emotional.

He records their emotional states, which change every hour. In the total duration of the course, he records the following statistics:

- Sad: 15 hours
- So-so: 22 hours
- Happy: 31 hours

Implement a Metropolis-Hastings algorithm to represent the emotional state of the students.

Exercise

Function to implement (.m)

`MetropolisHastings`

Test the function (m.)

`MetropolisHastingsTest`

7.3 Gibbs sampling

Exercise

Use the Gibbs sampling algorithm to draw from a bivariate normal distribution.

Function to implement (.m)

```
GibbsSamplingBN
```

Test the function (m.)

```
GibbsSamplingBNTest
```

7.3+ Gibbs sampling (extra question)

Gibbs sampling (extra question)

Problem description

Suppose that:

X is a random variable representing the speed of pedestrians

Y is a random variable representing pedestrian sensitivity to congestion

The joint distribution is

$$f_{X,Y}(x, y) = x^2 e^{(-xy^2 - y^2 + 2y - 0.1)}$$

Your task: Implement a method for estimating the expected speed of pedestrians

Gibbs sampling (extra question)

Exercise

Use the Gibbs sampling algorithm to draw from the bivariate normal distribution introduced in the lecture.

Function to implement (.m)

`GibbsSampling`

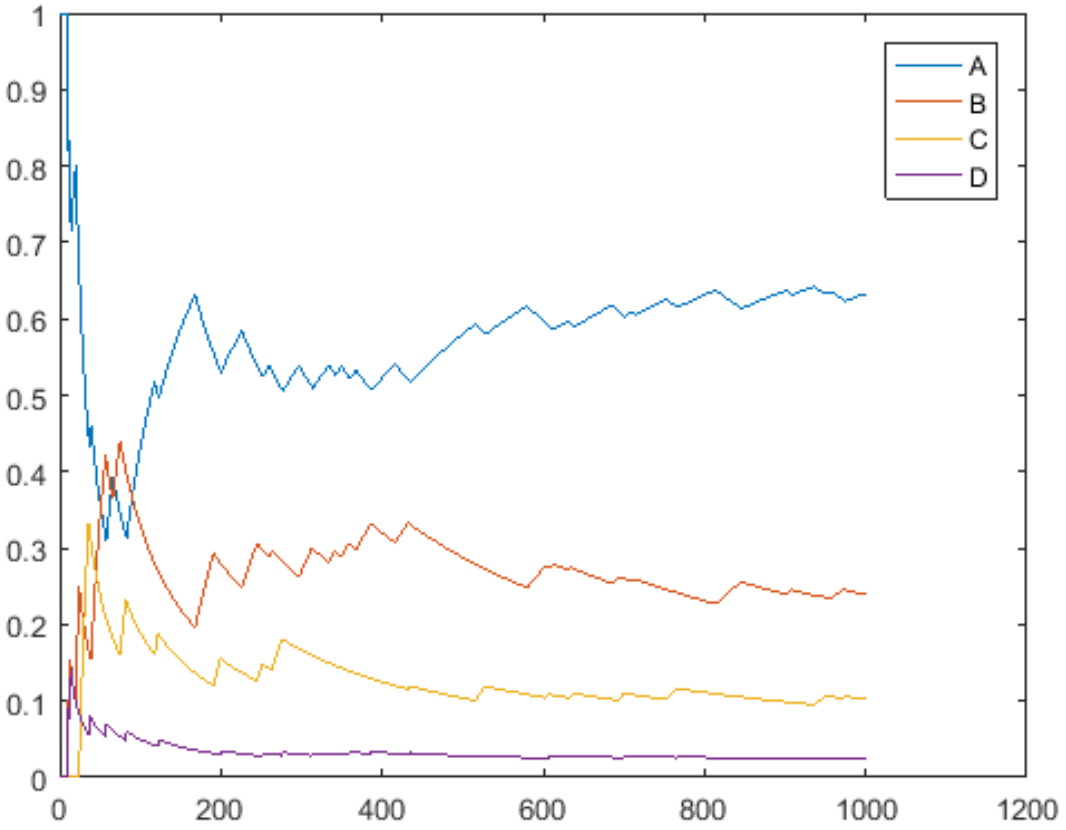
Test the function (m.)

`GibbsSamplingTest`

My results

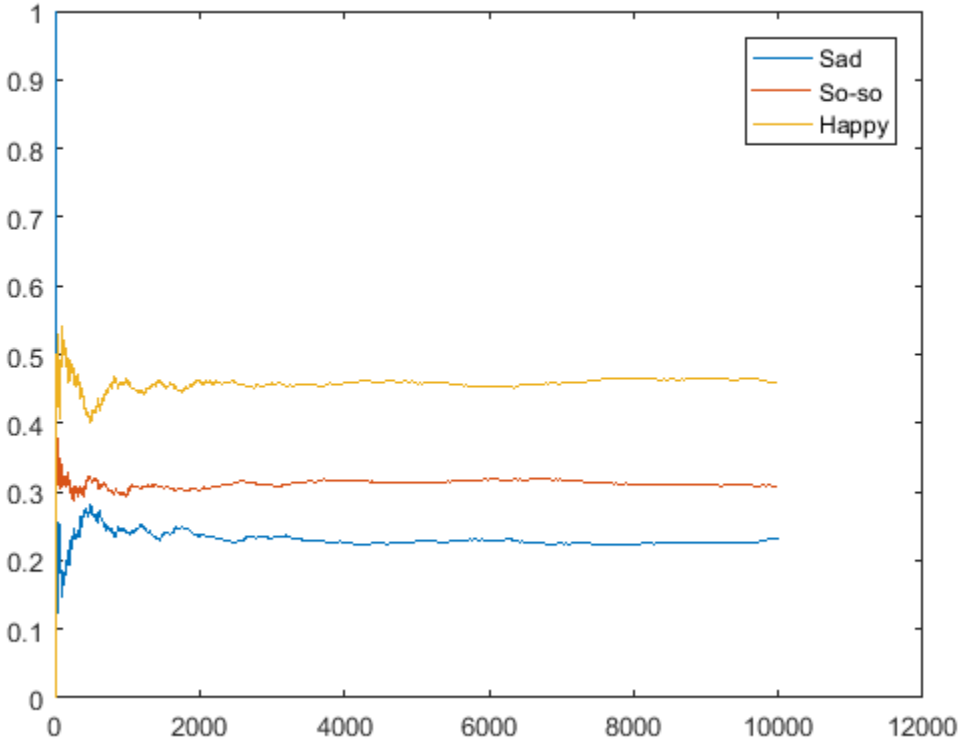
My results

Markov Chain Monte Carlo Methods



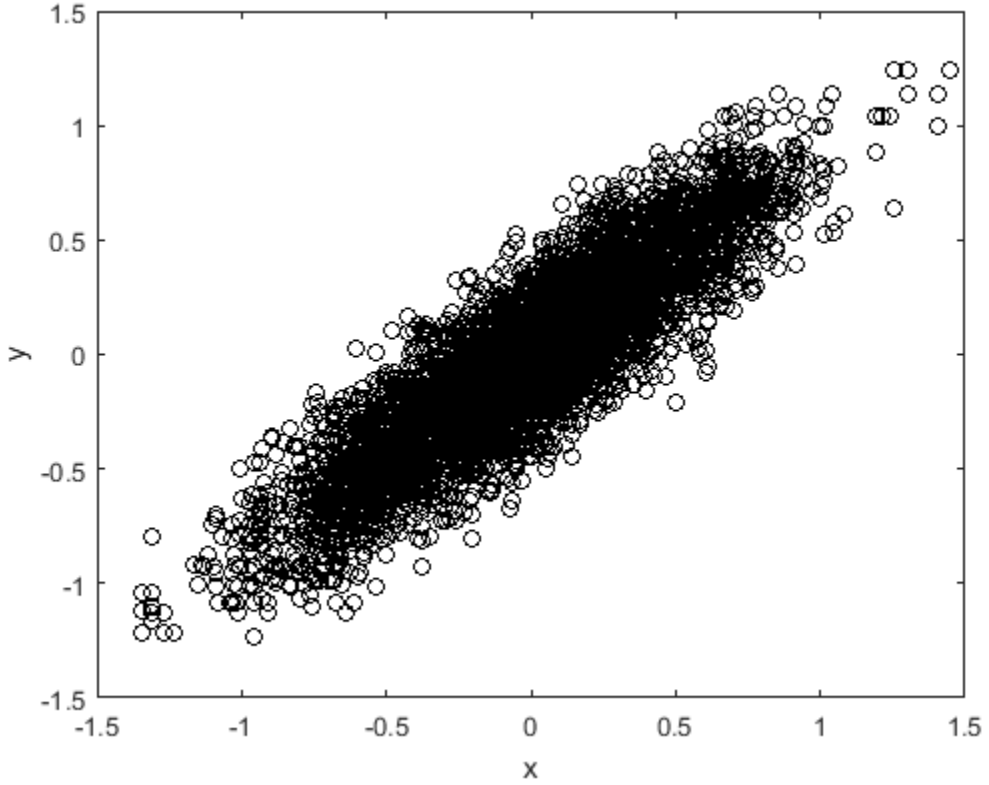
My results

Metropolis-Hastings algorithm



My results

Gibbs sampling



My results

Gibbs sampling (extra question)

The expected speed of pedestrians: 2.03 m/s