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Optimization and simulation
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Project 3: Augmented Lagrangian methods

Objective:
The aim of this project is to implement the augmented lagrangian method for solving different problems and analyze the role of the different parameters on the algorithm's efficiency.
Requirements
The student will implement and apply algorithm 20.1 to the following problems (Hock & Schittkowski, 1981):

$$\begin{aligned} \underset{x\mathbb{R}^2}{\text{minimize}} \quad & ln(1+x_1^2) - x_2 \\ \text{subject to} \quad & (1+x_1^2)^2 + x_2^2 = 4 \\ & -4 \leq x_1 \leq 4 \\ & -4 \leq x_2 \leq 4 \end{aligned}$$

Optimal solution $f(x^*) = -\sqrt{3}$, $x^* = [0, \sqrt{3}]$,$\lambda_1^* = 0.2887$.
Suggested starting point: $x_0 = [2, 2]$.

b)

$$\begin{aligned} \underset{x\mathbb{R}^{10}}{\text{minimize}} \quad & \sum_{i=1}^{10} e^{x_i}(c_i + x_i - ln(\sum_{k=1}^{10} e^{x_k})) \\ & e^{x_1} + 2e^{x_2} + 2e^{x_3} + e^{x_6} + e^{x_{10}} = 2 \\ & e^{x_4} + 2e^{x_5} + e^{x_6} + e^{x_7} = 1 \\ & e^{x_3} + e^{x_7} + e^{x_8} + 2e^{x_9} + e^{x_{10}} = 1 \\ & -100 \leq x_i \leq 100 \quad \forall i = 1, .., 10. \end{aligned}$$

with
$c_1 = -6.089$ $c_2 = -17.164$ $c_3 = -34.054$ $c_4 = -5.914$ $c_5 = -24.721$
$c_6 = -14.986$ $c_7 = -24.1$ $c_8 = -10.708$ $c_9 = -26.662$ $c_{10} = -22.179$
Optimal solution: $f(x^*) = -47.76109026$
$x_1^* = -3.201212$ $x_2^* = -1.912060$ $x_3^* = 0.2444413$ $x_4^* = -6.537489$
$x_5^* = -0.7231524$ $x_6^* = -7.267738$ $x_7^* = -3.596711$ $x_8^* = -4.017769$
$x_9^* = -3.287462$ $x_{10}^* = -2 : 335582$
$\lambda_1^* = 9.78$ $\lambda_2^* = 13$ $\lambda_3^* = 15.2$
Suggested starting point: $x_0 = [-2.3; ...; -2.3]$.

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The student will analyze the impact of the following parameters:

Penalty parameter Initial values $c_0 = 1, 10, 100$ and augmentation factor $\tau = 2, 10, 100$.

Constraints' precision Analyze the impact on the approximation of dual variables with respect to values $\beta = 0.1, 0.5, 0.9$.

We encourage the students to compare their results with the Matlab built-in optimisation algorithms and justify the differences (if there exist any)

Remark Problems' formulation must comply with the required formulation

$$\begin{aligned} \underset{x\mathbb{R}^n}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & h(x) = 0 \end{aligned}$$

Implementation:

- Algorithms 11.5, 20.1.
- Useful Matlab functions for this project: **chol**.