Project 1: Constrained Newton methods

Objective:
The aim of this project is, on the one hand, to implement and analyze the preconditioned projected gradient method and, on the other hand, to implement the Dikin method and to compare it with the simplex method.
Part I - Projected Gradient The student will implement and apply algorithm 18.2 to the following non-linear problem (Jansson & Knüppel, 1992):

$$min \quad x_1^2 - 12x_1 + 11 + 10cos(\pi/2x_1) + 8sin(5\pi x_1) - \frac{exp(-(x_2 - 1/2)^2/2)}{\sqrt{5}}$$

such that

$$-30 \leq x_1 \leq 30$$
$$-10 \leq x_2 \leq 10.$$

The global minimum is $x^* = (5.90133, 0.5)$, $f(x^*) = -43.325862$.

The students will analyze the behaviour of the algorithm for the following families of preconditioners:

1. $H_k = I$ for every k.
2. $H_k = (\bigtriangledown^2 f(x_k) + \tau I)$, where τ is chosen such that H_k is positive-definite (use Cholesky factorization, algorithm 11.4).
3. $H_k = diag(max1, 1/|x_k1|, ..., max1, 1/|x_kn|)$

- We encourage the student to change the value of the step γ (test for example $\gamma = 0.1, 1, 10$) as well as to test different starting points x_0, closer or farther to x^*.
- We encourage the students to compare their results with the Matlab built-in optimisation algorithms and justify the differences (if there exist any)

Part II - Dikin method

The student will implement algorithm 18.3 using several values for β.
For example $\beta = 0.1, 0.5, 0.9, 0.99, 0.999$. The student will compare the performance of Dikin method to the simplex method for different values of n on the following problems (Klee & Minty, 1972):

a)

$$
\begin{aligned}
min - \sum_{i=1}^{n} 2^{n-i} x_i \\
x_1 \leq 5 \\
4x_1 + x_2 \leq 25 \\
8x_1 + 4x_2 + x_3 \leq 125 \\
. \\
. \\
. \\
2^n x_1 + 2^{n-1} x_2 + ... + 4x_{n-1} + x_n \leq 5^n \\
x_1, x_2, ..., x_n \geq 0
\end{aligned}
$$

Use the starting point $(1, 1, ..., 1)^T$. The solution of this problem is $x^* = (0, 0, ..., 0, 5^n)^T$.

b)

$$
\begin{aligned}
min - x_n \\
\epsilon \leq x_1 \leq 1 \\
\epsilon x_{i-1} \leq x_i \leq 1 - \epsilon x_{i-1} \quad \forall i = 2, .., n
\end{aligned}
$$

Use the starting point

$$
\begin{aligned}
x_1 = \frac{1+\epsilon}{2} \\
x_i = \frac{1}{2} \quad \forall i = 2, ..., n.
\end{aligned}
$$

with $0 \leq \epsilon \leq 0.5$ (we suggest $\epsilon = 0.4$).

Implementation

- Algorithm 11.2,11.4,18.2,18.3.
- Usefule Matlab functions for this project: **chol**, **quadprog**.
- You do not need to implement the simplex method; use the Matlab function linprog with option '*LargeScale=off*' and '*Simplex=on*'.