Project assignment
## Groups

<table>
<thead>
<tr>
<th>Group</th>
<th>Members</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Flavio Finger, Jonas Gros, Evanthia Kazagli</td>
</tr>
<tr>
<td>2</td>
<td>Jessie Madrazo, Florian Lucker, Julien Monge</td>
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<tr>
<td>3</td>
<td>Marija Nikolic, Tomas Robenek</td>
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<tr>
<td>4</td>
<td>Paul Anderson, Franz Zeimetz, Alberto Mian</td>
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<tr>
<td>5</td>
<td>Sofia Samoili, Marco Vocialta, Seyedeh Taheri</td>
</tr>
</tbody>
</table>
# Projects

<table>
<thead>
<tr>
<th>Project</th>
<th>Optimization algorithm Matlab coding</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Preconditioned Projected Gradient Method</td>
</tr>
<tr>
<td>2</td>
<td>Interior Point Method</td>
</tr>
<tr>
<td>3</td>
<td>Augmented Lagrangian Method</td>
</tr>
<tr>
<td>4</td>
<td>Local Sequential Quadratic Programming Method</td>
</tr>
<tr>
<td>5</td>
<td>Global Sequential Quadratic Programming Method</td>
</tr>
</tbody>
</table>
Assignment

Project assignment

Group 1  Group 2  Group 3  Group 4  Group 5

Project 1  Project 2  Project 3  Project 4  Project 5

Chen Jiang Hang (Transportation and Mobility Laboratory)  Simulation and Optimization Lab 1  5 / 14
Some tips for Matlab coding
Some tips

- Matrix multiplication: \( y = (AB)x = A(Bx) \), which way is better?
- To solve the linear equation system \( Ax = b \) where \( A \) is square, in Matlab, \( x = A\backslash b \) is more efficient than \( x = \text{inv}(A) \ast b \)
- Matlab discriminates in favor of upper-right triangular matrices for inversion! If your matrix is lower-left triangular, first transpose it, invert the result, and transport back.
Exercises today
Line search

- **Objective:** find a step $\alpha^*$ such that both Wolfe’s conditions are verified.

- **Input:**
  1. Function $f : \mathbb{R}^n \rightarrow \mathbb{R}$, continuously differentiable
  2. Gradient $\nabla f : \mathbb{R}^n \rightarrow \mathbb{R}^n$
  3. Vector $x \in \mathbb{R}^n$
  4. Descent direction $d$ such that $\nabla f(x)^T d < 0$
  5. First approximation of the solution $\alpha_0 > 0$
  6. Parameters $\beta_1$ and $\beta_2$ such that $0 < \beta_1 < \beta_2 < 1$ (e.g., $\beta_1 = 10^{-4}$ and $\beta_2 = 0.99$)
  7. Parameter $\lambda > 1$
Line search

- Initialization: $i = 0, \alpha_l = 0, \alpha_r = +\infty$
- Iterations:
  1. If $\alpha_i$ verify both conditions, then $\alpha^* = \alpha_i$. STOP.
  2. If $\alpha_i$ violates Wolfe 1, then the step is too long and
     \[ \alpha_r = \alpha_i \]
     \[ \alpha_{i+1} = \frac{\alpha_l + \alpha_r}{2} \]
  3. If $\alpha_i$ verifies Wolfe 1 and violate Wolfe 2, then the step is too short and
     \[ \alpha_l = \alpha_i \]
     \[ \alpha_{i+1} = \begin{cases} \frac{\alpha_l + \alpha_r}{2} , & \text{if } \alpha_r < +\infty; \\ \lambda \alpha_i , & \text{otherwise.} \end{cases} \]
  4. $i = i + 1$
Line search

Try your code for the following example:

\[ f(x_1, x_2) = x_1^2 + x_2^2 \]

- Starting point: (1,1)
- \( d = (-0.5, -1) \)
Objective: Modify a matrix in order to make it positive-definite

Input: Symmetric matrix $A \in \mathbb{R}^{n \times n}$

Output: A lower triangular matrix $L$ and $\tau \geq 0$ such that $A + \tau I = LL^T$ is positive-definite
Modified Cholesky Factorization

Frobenius Norm of a matrix:

\[ \| A \|_F = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2} \]

- Initialization: \( k = 0 \); if \( \min_i a_{ii} > 0 \), then \( \tau_k = 0 \), otherwise
  \( \tau_k = \frac{1}{2} \| A \|_F \)

- Iterations:
  1. Compute the Cholesky factorization \( LL^T \) of \( A + \tau_k I \)
  2. If the factorization is successful, STOP.
  3. Else \( \tau_{k+1} = \max\{2\tau_k, \frac{1}{2} \| A \|_F\} \)
  4. \( k = k + 1 \)

Matlab Hint:
- \texttt{norm(A,'fro')}  
- \texttt{chol}
- \texttt{try...catch...end}
Modified Cholesky Factorization

Try your Matlab code on the following matrix:

\[ A = \begin{bmatrix}
6 & 3 & 4 & 8 \\
3 & 6 & 5 & 1 \\
4 & 5 & 10 & 7 \\
8 & 1 & 7 & 5
\end{bmatrix} \]