Decomposition for Network Design

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Outline of lesson 2: Introduction to network design

Network design problems

Multicommodity capacitated network design

Multiperiod capacitated multifacility location

Network design

- Network with multiple commodities
- Each commodity flows between supply and demand points
- Minimization of a “complex” (non-convex) objective function
  - Tradeoff between transportation and investment costs
  - Transportation costs: not necessarily linear, can be piecewise linear
  - Investment costs: “fixed” cost for building, renting, operating “facilities” at nodes or arcs of the network
- Additional constraints: budget, capacity, topology, reliability,...
- Variants:
  - Centralized / Decentralized
  - Static / Dynamic
  - Determinist / Stochastic
  - Strategic / Tactical / Operational

Infrastructure network design: strategic planning

- Planning horizon: years
- Decisions: invest in building roads, warehouses, plants,...
- Typical assumptions:
  - Central control
  - Static network
  - Linear transportation costs
  - Fixed costs for investment decisions
  - Usually no capacities
  - Known demands based on average values
- Robustness is an issue: stochastic demands?

Service network design: tactical planning

- Planning horizon: months
- Decisions: establish or not "services" (vehicles moving between two points) + flows-inventories
- Dynamic network: space-time expansion
  - Node = location-period
  - Transportation arc = (location1-period1, location2-period2) = moving from location1 to location2 in time (period2-period1)
  - Inventory arc = (location-period, location-period+1) = holding inventory at location between two consecutive periods
- Typical assumptions:
  - Central control
  - Linear inventory-transportation costs
  - Fixed costs for service decisions
  - Service capacities
  - Known demands

Adaptive network design: operational planning

- Planning horizon: days
- Decisions: operate or not "facilities" (warehousing or parking space) for fast product delivery + how many vehicles to use on each arc
- Typical assumptions:
  - Central control
  - Dynamic network
  - Piecewise linear transportation costs
  - Fixed costs for facility decisions
  - Facility and vehicle capacities
  - Known demands

Multicommodity capacitated network design

- Directed network $G = (N, A)$, with node set N and arc set A
- Commodity set K: known demand d^k between origin $O(k)$ and destination $D(k)$ for each $k \in K$
- Unit transportation cost c_{ij} on each arc (i, j)
- Capacity u_{ij} on each arc (i, j)
- Cost f_{ij} for each capacity unit installed on arc (i, j)

Problem formulation

$$Z = \min \sum_{(i,j)\in A} \sum_{k\in K} c_{ij} d^k x_{ij}^k + \sum_{(i,j)\in A} f_{ij} y_{ij}$$

$$\sum_{j\in N_i^+} x_{ij}^k - \sum_{j\in N_i^-} x_{ji}^k = \begin{cases} & 1, \quad i = O(k) \\ - & 1, \quad i = D(k) \\ & 0, \quad i \neq O(k), D(k) \end{cases} \qquad i \in N, \; k \in K$$

$$\sum_{k\in K} d^k x_{ij}^k \leq u_{ij} y_{ij} \quad (i,j) \in A$$

$$0 \leq x_{ij}^k \leq 1 \quad (i,j) \in A, \; k \in K$$

$$y_{ij} \; integer \quad (i,j) \in A$$

Extensions

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- Non-bifurcated flows?

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- Multifacility design?

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- Non-bifurcated flows? x_{ij}^k *integer* $\quad (i,j) \in A,\ k \in K$
- Multifacility design? several facilities $t \in T_{ij}$ on each arc, each with capacity u_{ij}^t and cost f_{ij}^t
- Piecewise linear arc flow costs?

Capacitated facility location problem (CFLP)

- K: set of customers
- J: set of locations for potential facilities
- $d_k > 0$: demand of customer k
- $u_j > 0$: capacity at location j
- $f_j \geq 0$: fixed cost for opening facility at location j
- $c_{jk} \geq 0$: unit cost of satisfying the demand of customer k from facility at location j
- **Problem description**: Determine the locations of the facilities to satisfy customers' demands at minimum cost, while respecting the capacity at each facility location

CFLP model

- y_j: 1, if location j is chosen for a facility, 0, otherwise
- x_{jk}: fraction of the demand d_k of customer k satisfied from facility at location j

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$$x_{jk} \leq y_j, \quad j \in J, k \in K$$

$$x_{jk} \in [0, 1], \quad j \in J, k \in K$$

$$y_j \in \{0, 1\}, \quad j \in J$$

Capacitated multifacility location problem (CMFLP)

- K: set of customers
- J: set of locations for potential facilities
- L: set of capacity levels for each facility (including 0)
- $d_k > 0$: demand of customer k
- $u_{jl} > 0$: capacity of level l at location j
- $f_{jl} \geq 0$: fixed cost for opening facility of level l at location j
- $c_{jkl} \geq 0$: unit cost of satisfying the demand of customer k from facility of level l at location j
- **Problem description**: Determine the locations and capacity levels of the facilities to satisfy customers' demands at minimum cost, while respecting the capacity at each facility location (at most one capacity level can be selected at each location)

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$$\sum_{k\in K} d_k x_{jkl} \le u_{jl} y_{jl}, \quad j \in J, l \in L$$

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$$\sum_{k \in K} d_k x_{jkl} \leq u_{jl} y_{jl}, \quad j \in J, l \in L$$

$$x_{jkl} \leq y_{jl}, \quad j \in J, k \in K, l \in L$$

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$$x_{jkl} \leq y_{jl}, \quad j \in J, k \in K, l \in L$$

$$\sum_{l \in L} y_{jl} = 1, \quad j \in J$$

$$x_{jkl} \in [0, 1], \quad j \in J, k \in K, l \in L$$

$$y_{jl} \in \{0, 1\}, \quad j \in J, l \in L$$

Multiperiod capacitated multifacility location problem (MCMFLP)

- K: set of customers
- J: set of locations for potential facilities
- L: set of capacity levels for each facility (including 0)
- $T = \{0, 1, \ldots, |T| + 1\}$: set of time periods
- $d_{kt} > 0$: demand of customer k at period t
- $u_{jl} > 0$: capacity of level l at location j
- $f_{jl'lt} \geq 0$: cost for changing capacity at location j from level l' to l at period t
- $c_{jklt} \geq 0$: unit cost of satisfying the demand of customer k at period t from facility of level l at location j
- **Problem description**: Determine the locations and capacity levels of the facilities to satisfy customers' demands at each time period at minimum cost, while respecting the capacity at each facility location (at most one capacity level can be selected at each location and time period)

MCMFLP model

- $y_{jl'lt}$: 1, if location j is chosen for a facility and changes from capacity level l' to l at period t, 0, otherwise
- x_{jklt}: fraction of the demand d_k of customer k at period t satisfied from facility of level l at location j

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$$\min \sum_{j\in J}\sum_{k\in K}\sum_{l\in L}\sum_{t\in T} d_{kt}c_{jklt}x_{jklt} + \sum_{j\in J}\sum_{l'\in L}\sum_{l\in L}\sum_{t\in T} f_{jl'lt}y_{jl'lt}$$

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$$\sum_{k\in K} d_{kt}x_{jklt} \le u_{jl}\sum_{l'\in L} y_{jl'lt}, \quad j\in J, l\in L, t\in T$$

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$$\sum_{l'\in L} y_{jl'l(t-1)} = \sum_{l^*\in L} y_{jll^*t}, \quad j\in J, l\in L, t\in T\setminus\{0\}$$

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- $y_{jl'lt}$: 1, if location j is chosen for a facility and changes from capacity level l' to l at period t, 0, otherwise
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$$x_{jklt}\in[0,1], \quad j\in J, k\in K, l\in L, t\in T$$
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