A Survey of Different Integer Programming Formulations of the Travelling Salesman Problem

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Summary. Eight distinct (and in some cases little known) formulations of the Travelling Salesman Problem as an Integer Programme are given. Apart from the standard formulation all the formulations are 'compact' in the sense that the number of constraints and variables is a polynomial function of the number of cities in the problem. Comparisons of the formulations are made by projecting out variables in order to produce polytopes in the same space. It is then possible to compare the strengths of the Linear Programming relaxations. These results are illustrated by computational results on a small problem.

Key words: Travelling salesman problem, polytopes

1 Introduction

In this paper we survey eight different formulations of the Asymmetric Travelling Salesman Problem (ATSP) as an Integer Programme (IP). We choose to treat the Asymmetric case as being more general than the Symmetric case. Some of the work has been published elsewhere by other authors. Our purpose is, however, to provide new results as well as present a unifying framework, by projecting all the formulations into the same space.

In Sect. 2 we present the eight formulations classifying them as 'conventional' (**C**), "sequential" (**S**), "flow based" (**F**) and "time staged" (**T**). The reasons for these terms will become apparent. In order to facilitate comparison between the formulations, in some cases we introduce extra variables which equate to expressions within the models. This enables us, in Sect. 3, to compare the Linear Programming (LP) relaxations of all the formulations by projecting out all, but the, common variables. Such comparisons have already been done for some of the formulations by Padberg and Sung (1991), Wong (1980) and Langevin et al. (1990).

Some of the time staged formulations have also been compared by Gouveia and Voss (1995) and discussed by Picard and Queyranne (1978).

The sequential formulation has also been improved by Gouveia and Pires (2001). The extra variables incorporated in this formulation have been used by Sherali and Driscoll (2002) to further tighten the Linear Programming relaxation.

Comparisons have also been made for some formulations of the Symmetric TSP by Carr (1996) and Arthanari and Usha (2000). We unify all these results in the same framework.

In Sect. 4 we present computational results on a small illustrative example in order to verify the results of Sect. 3.

2 Eight Formulations of the ATSP

In all our formulations we will take the set of cities as $N = \{1, 2, ..., n\}$ and define variables

$$\begin{aligned} x_{ij} &= 1 \text{ iff arc } (i,j) \text{ is a link in the tour} \\ &= 0 \text{ otherwise} (i \neq j) \end{aligned}$$

$$c_{ij} \text{ will be taken as the length of arc}(i,j)$$

The objective function will be:

$$\text{Minimise} \sum_{\substack{i,j \\ i \neq j}} c_{ij} x_{ij} \tag{1}$$

2.1 Conventional Formulation (C) (Dantzig, Fulkerson and Johnson (1954))

$$\sum_{\substack{j \\ j \neq i}} x_{ij} = 1 \qquad \forall\, i \in N \tag{2}$$

$$\sum_{\substack{i \\ i \neq j}} x_{ij} = 1 \qquad \forall\, j \in N \tag{3}$$

$$\sum_{\substack{i,j \in M \\ i \neq j}} x_{ij} \leq |M| - 1 \qquad \forall\, M \subset N \textit{ such that} \{1\} \notin M, |M| \geq 2 \tag{4}$$

(*the symbol* '$\subset$' *represents proper inclusion*)

This formulation has $2^{n-1} + n - 1$ constraints and $n(n-1)$ 0–1 variables.

The exponential number of constraints makes it impractical to solve directly. Hence, the usual procedure is to apply the Assignment constraints

(2) and (3) and append only those Subtour Elimination constraints (4) when violated. Alternatively, different relaxations such as the LP relaxation or the Spanning-2 Tree relaxation can be applied and solved iteratively. A reference to these methods is Lawler et al. (1995).

A variant of the above formulation (which we will not classify as a different formulation) is to replace constraints (4) by:

$$\sum_{\substack{i \in M \\ j \in \overline{M}}} x_{ij} \geq 1 \qquad \forall\, M \subset N \text{ where } \{1\} \notin M \textit{ and } \overline{M} = N - M \tag{5}$$

Constraints (5) can be obtained by adding constraints (2) for $i \in M$ and subtracting from (4).

2.2 Sequential Formulation (S) (Miller, Tucker and Zemlin (1960))

Constraints (2) and (3) are retained but we introduce (continuous) variables

$$u_i = \text{sequence in which city } i \text{ is visited } (i \neq 1)$$

and constraints

$$u_i - u_j + n x_{ij} \leq n - 1 \qquad \forall\, i, j \in N - \{1\}, i \neq j \tag{6}$$

This formulation has $n^2 - n + 2$ constraints, $n(n-1)$ 0–1 variables and $(n-1)$ continuous variables.

2.3 Flow Based Formulations

Single Commodity Flow (**F1**) (Gavish and Graves (1978))
Constraints (2) and (3) are retained but we also introduce (continuous) variables:

$$y_{ij} = \text{'Flow' in an arc } (i,j) i \neq j$$

and constraints:

$$y_{ij} \leq (n-1) x_{ij} \qquad \forall\, i, j \in N, i \neq j \tag{7}$$

$$\sum_{\substack{j \\ j \neq 1}} y_{1j} = n - 1 \tag{8}$$

$$\sum_{\substack{i \\ i \neq j}} y_{ij} - \sum_{\substack{k \\ i \neq k}} y_{jk} = 1 \qquad \forall\, j \in N - \{1\} \tag{9}$$

Constraints (8) and (9) restrict $n-1$ units of a single commodity to flow into city 1 and 1 unit to flow out of each of the other cities. Flow can only take place in an arc if it exists by virtue of constraints (7).

It is possible to improve this formulation (**F1'**) by tightening constraints (7) for $i \neq 1$ to:

$$y_{ij} \leq (n-2) x_{ij} \qquad \forall\, i, j \in N - \{1\}, i \neq j \tag{10}$$

This relies on the observation that at most $n-2$ units can flow along any arc not out of city 1. We are not aware of any other authors having recognised this improvement.

This formulation has $n(n+2)$ constraints, $n(n-1)$ 0–1 variables and $n(n-1)$ continuous variables.

Two Commodity Flow (**F2**) (Finke, Claus and Gunn (1983))
Constraints (2) and (3) are retained but we also introduce (continuous) variables:

$$y_{ij} = \text{'Flow' of commodity 1 in arc } (i,j) i \neq j$$

$$z_{ij} = \text{'Flow' of commodity 2 in arc } (i,j) i \neq j$$

and constraints:

$$\sum_{\substack{j \\ j \neq 1}} (y_{1j} - y_{j1}) = n - 1 \tag{11}$$

$$\sum_{j} (y_{ij} - y_{ji}) = 1 \qquad \forall\, i \in N - \{1\}, i \neq j \tag{12}$$

$$\sum_{\substack{j \\ j \neq 1}} (z_{1j} - z_{j1}) = -(n - 1) \tag{13}$$

$$\sum_{j} (z_{ij} - z_{ji}) = -1 \qquad \forall\, i \in N - \{1\}, i \neq j \tag{14}$$

$$\sum_{j} (y_{ij} + z_{ij}) = n - 1 \qquad \forall\, i \in N \tag{15}$$

$$y_{ij} + z_{ij} = (n-1)x_{ij} \qquad \forall\, i, j \in N \tag{16}$$

Constraints (11) and (12) force $(n-1)$ units of commodity 1 to flow in at city 1 and 1 unit to flow out at every other city. Constraints (13) and (14) force $(n-1)$ units of commodity 2 to flow out at city 1 and 1 unit to flow in at every other city. Constraints (15) force exactly $(n-1)$ units of combined commodity in each arc. Constraints (16) only allow flow in an arc if present.

This formulation has $n(n+4)$ constraints, $n(n-1)$ 0–1 variables and $2n(n-1)$ continuous variables.

Multi-Commodity Flow (**F3**) (Wong (1980) and Claus (1984))
Constraints (2) and (3) are retained but we also introduce (continuous) variables:

$$y^k_{ij} = \text{'Flow' of commodity } k \text{ in arc } (i,j)\ \kappa \in N - \{1\}$$

and constraints:

$$y_{ij}^k \le x_{ij} \qquad \forall\ i,j,k \in N, k \neq 1 \tag{17}$$

$$\sum_i y_{1i}^k = 1 \qquad \forall\ k \in N - \{1\} \tag{18}$$

$$\sum_i y_{i1}^k = 0 \qquad \forall\ k \in N - \{1\} \tag{19}$$

$$\sum_i y_{ik}^k = 1 \qquad \forall\ k \in N - \{1\} \tag{20}$$

$$\sum_j y_{kj}^k = 0 \qquad \forall\ k \in N - \{1\} \tag{21}$$

$$\sum_i y_{ij}^k - \sum_i y_{ji}^k = 0 \qquad \forall\ j,k \in N - \{1\}, j \neq k \tag{22}$$

Constraints (17) only allow flow in an arc which is present. Constraints (18) force exactly one unit of each commodity to flow in at city 1 and constraints (19) prevent any commodity out at city 1. Constraints (20) force exactly one unit of commodity k to flow out at city k and constraints (21) prevent any of commodity k flowing in at city k. Constraints (22) force 'material' balance for all commodities at each city, apart from city 1 and for commodity k at city k.

This formulation has $n^3 + n^2 + 6n - 3$ constraints, $n(n-1)$ $0-1$ variables and $n(n-1)^2$ continuous variables.

2.4 Timed Staged Formulations

1st Stage Dependent **T1** (Fox, Gavish and Graves (1980))
In order to facilitate comparisons with the other formulations it is convenient, but not necessary, to retain the variables x_{ij} (linked to the other variables by constraints (25)) and constraints (2) and (3). We introduce 0–1 integer variables:

$$\begin{aligned} y_{ij}^t &= 1 \qquad \text{if arc } (i,j) \text{ is traversed at stage } t \\ &= 0 \qquad \text{otherwise} \end{aligned}$$

and constraints:

$$\sum_{i,j,t} y_{ij}^t = n \tag{23}$$

$$\sum_{\substack{j,t \\ t \ge 2}} t y_{ij}^t - \sum_{k,t} t y_{ki}^t = 1 \qquad \forall\ i \in N - \{1\} \tag{24}$$

$$x_{ij} - \sum_t y_{ij}^t = 0 \qquad \forall\ i,j \in N, i \neq j \tag{25}$$

In addition we impose the conditions:

$$y^t_{il} = 0 \forall\ t \neq n, \quad y^t_{ij} = 0 \forall\ t \neq 1, \quad y^l_{ij} = 0 \forall\ i \neq 1, \quad i \neq j \tag{26}$$

Constraints (24) guarantee that if a city is entered at stage t it is left at stage $t + 1$. Removing certain variables by conditions (26) forces city 1 to be left only at stage 1 and entered only at stage n.

It is not necessary to place upper bounds of 1 on the variables x_{ij}, and this condition may be violated in the LP relaxation.

This model has $n(n + 2)$ constraints and $n(n - 1)(n + 1)$ 0–1 variables. Clearly, but for constraints (25) and variables x_{ij} this model would be even more compact having only n constraints and $n(n - 1)$ variables. This is a remarkable formulation for this reason although, as will be shown in the next section it is also remarkably bad in terms of the strength of its Linear Programming relaxation and therefore the slowness of its overall running time.

2nd Stage Dependent **T2** (Fox, Gavish and Graves (1980))
We use the same variables as in **T1** and constraints (2), (3) and (25) together with:

$$\sum_{\substack{i,t \\ i \neq j}} y^t_{ij} = 1 \qquad \forall\ j \in N \tag{27}$$

$$\sum_{\substack{j,t \\ j \neq i}} y^t_{ij} = 1 \qquad \forall\ i \in N \tag{28}$$

$$\sum_{i,j \neq i} y^t_{ij} = 1 \qquad \forall\ t \in N \tag{29}$$

$$\sum_{\substack{j,t \\ t \geq 2}} t y^t_{ij} - \sum_{k,t} t y^t_{ki} = 1 \qquad \forall\ i \in N - \{1\} \tag{30}$$

Clearly this is a disaggregated form of **T1.**

This model has $4n - 1$ constraints and $n(n-1)(n+1)$ 0–1 variables. Again but for the constraints (25) and variables x_{ij} this would be smaller. In fact the y^t_{ij} variables can, in this formulation, be regarded as continuous.

3rd Stage Dependent **T3** (Vajda (1961))
We use the same variables as in **T1** and **T2** and constraints (2), (3) and (25) together with:

$$\sum_j y^1_{1j} = 1 \tag{31}$$

$$\sum_i y^n_{i1} = 1 \tag{32}$$

$$\sum_j y_{ij}^t - \sum_k y_{ki}^{t-1} = 0 \qquad \forall\ i, t \in N - \{1\} \tag{33}$$

Constraint (31) forces city 1 to be left at stage 1 and constraint (32) forces it to be entered at stage n. Constraints (33) have the same effect as (24).

This model has $2n^2 - n + 3$ constraints and $n(n-1)(n+1)$ 0–1 variables which again could be reduced by leaving out constraints (25) and variables x_{ij} . Again the y_{ij}^t variables can be regarded as continuous.

All the formulations, apart from **C**, have a polynomial (in n) number of constraints. This makes them superficially more attractive than **C**. However, the number of constraints may still be large, for practically sized n, and the LP relaxations weaker. These considerations are discussed in the next section.

3 Comparison of LP Formulations

All formulations presented in Sect. 2 can be expressed in the form:

$$\begin{aligned} &\text{Minimise} \quad \underline{c}.\underline{x} \\ &\text{subject to } \mathrm{A}\underline{x} + \mathrm{B}\underline{y} \sim b \quad \text{where '}\sim\text{' represents '<=' and '=' relations.} \\ &\qquad\qquad \underline{x}, \underline{y} \geq 0 \end{aligned} \tag{34}$$

$\underline{x}$ is the vector of variables $x_{i,j}$ and $\underline{y}$ the different vectors used in the formulations **S**, **F** and **T**. In the case of **S** and **F** $\underline{y}$ represents continuous variables but in the case of **T** integer variables.

In order to facilitate comparisons between the formulation **S**, **F** with **C** we can project out the continuous variables $\underline{y}$ to create a model involving only $\underline{x}$. The size of the polytopes of the associated LP relaxations can then be compared. We will denote the polytope of the resultant LP relaxation of a (projected) model **M** as **P**(**M**). In the case of formulation **T1** the variables $\underline{y}$ must be integer. The projection out of such variables is more complex and may not even result in an IP (see Kirby and Williams (1997)). However, we can still project out the variables $\underline{y}$ from the LP relaxation and return an IP. The LP relaxation of this IP will be weaker than that resulting from the true projection. It will still, however, be a valid comparator of computational difficulty when LP based IP methods are used. Therefore we will continue to use the notation **P**(**M**) for the resulting polytope when projecting out the LP relaxations of the variables $\underline{y}$ in **T1.**

In order to project out the variables $\underline{y}$ in all the formulations we can use Fourier-Motzkin elimination (see Williams (1986)) or equivalently full Benders Decomposition (1962). Martin (1999) gives a full general description of the methods of projection. We do not reproduce the derivation of the methods here but simply restate them. The projection out of the variables $\underline{y}$ is effected by finding all real vectors $\underline{w}$, of appropriate dimension, such that,

$$\underline{w}'B \geq \underline{0} \tag{35}$$

Where $\underline{w}$ has non-negative entries corresponding to rows of (34) with '$\leq$' constraints and unconstrained entries in rows with '=' constraints. The set of $\underline{w}$ satisfying (35) form a convex polyhedral cone and can be characterised by its extreme rays. It is therefore sufficient to seek the finite set of $\underline{w}$ representing extreme rays, which are what would be obtained by (restricted) Fourier-Motzkin elimination. We denote these as rows of the matrix Q. Applying Q to (34) gives:

$$QA\underline{x} \leq Q\underline{b} \tag{36}$$

as an alternative formulation to **C**. Of course, as would be expected, (36) will have an exponential number of constraints, unlike (34), but is in the same space as **C**.

We present the effect of the matrix Q for each of the formulations **S**, **F** and **T** of Sect. 2.

Formulation **S**

The effect of Q is to eliminate $u_2, u_3, ..., u_n$ from all the inequalities in (6). This is done by adding those inequalities around each directed cycle $M \subset N$, where $1 \notin M$. This results in inequalities (for each subset $M \subset N$ by virtue of (2) and (3))

$$x_{i1i2} + x_{i2i3} + \cdots + x_{i|M|i1} \leq |M| - \frac{|M|}{n} \tag{37}$$

(together with (2), (3) and non-negativity).

$$Clearly\, |M| - 1 < |M| - \frac{|M|}{n} \text{ since } M \subset N$$

Since cycles are subsets of their associated sets this demonstrates that

$$\mathbf{P(S)} \supset \mathbf{P(C)} \tag{38}$$

(strict inclusion can be proved by numerical examples).

Therefore the LP relaxation associated with **S** will be weaker than that associated with **C**. This result has already been obtained by Wong (1980) and Padberg and Sung (1991).

Formulation **F1**

The effect of Q is to, for each (subset) $M \subset N$, where $1 \notin M$, create

$$\sum_{i,j \in M} x_{ij} \leq |M| - \frac{|M|}{n-1} \tag{39}$$

$$\text{Clearly}\, |M| - 1 < |M| - \frac{|M|}{(n-1)} < |M| - \frac{|M|}{n}$$

demonstrating that

$$\mathbf{P}(\mathbf{S}) \supset \mathbf{P}(\mathbf{F1}) \supset \mathbf{P}(\mathbf{C}) \tag{40}$$

(Strict inclusion can again be proved by numerical examples).

This result is also obtained by Wong (1980).

Applying the same elimination procedure to the modified formulation (**F1'**) we obtain

$$\frac{1}{n-1} \sum_{\substack{i \in \overline{M}-\{1\} \\ j \in \overline{M}}} x_{ij} + \sum_{i,j \in \overline{M}} x_{ij} \leq |M| - \frac{|M|}{n-1} \tag{41}$$

Clearly, by virtue of (2) and (3), $\frac{1}{n-1} \sum_{\substack{i \in \overline{M}-\{1\} \\ j \in M}} x_{ij} \leq 1 - \frac{|M|}{n-1}$

$$\text{Hence } \mathbf{P}(\mathbf{F1}) \supset \mathbf{P}(\mathbf{F1'}) \tag{42}$$

(Strict inclusion can again be proved by numerical examples).

FORMULATION **F2**

If z_{ij} are interpreted as the 'slack' variables in (16) we can use (16) to substitute them out reducing this formulation to **F1**. This demonstrates that

$$\mathbf{P}(\mathbf{F2}) = \mathbf{P}(\mathbf{F1}) \tag{43}$$

This result is also given by Langevin et al. (1990).

FORMULATION **F3**

The effect of Q is to, for each $M \subset N$,where $1 \notin M$, create

$$\sum_{i,j \in M} x_{ij} \leq |M| - 1 \tag{44}$$

i.e. constraints (4) of formulation **C**.

$$\text{Hence } \mathbf{P}(\mathbf{F3}) = \mathbf{P}(\mathbf{C}) \tag{45}$$

This remarkable result is also obtained by Wong (1980) and Padberg and Sung (1991)

FORMULATION **T1**

The effect of Q is to, for each $M \subset N$, where $1 \notin M$, create

$$\sum_{\substack{i \in M \\ j \in \overline{M}}} x_{ij} \geq \frac{|M|}{n} \tag{46}$$

and

$$\sum_{i,j \in N} x_{ij} = n \tag{47}$$

In the absence of assignment constraints, in this formulation, it is not possible to convert (46) to a form similar to (4). We therefore express it in a form similar to (5). Representing constraints (37) in a similar form to (5) demonstrates that

$$\mathbf{P}(\mathbf{S}) \subset \mathbf{P}(\mathbf{T1}) \tag{48}$$

FORMULATION **T2**
The effect of Q is to, for each subset M of $N - \{1\}$, create

$$\frac{1}{n-1} \sum_{\substack{i \in M \\ j \in \overline{M}-\{1\}}} x_{ij} + \frac{1}{n-1} \sum_{\substack{i \in \overline{M}-\{1\} \\ j \in M}} x_{ij} + \sum_{i,j \in M} x_{ij} \leq |M| - \frac{|M|}{n-1} \tag{49}$$

However, other constraints are also created which, to date, it has not been possible to obtain through the combinatorial explosion resulting from projection. Padberg and Sung give constraints equivalent to (49) as the projection of **T1.** This is clearly wrong.

$$\text{Hence } \mathbf{P}(\mathbf{T2}) \subset \mathbf{P}(\mathbf{F1'}) \tag{50}$$

Again strict inclusion can be proved by numerical examples.

FORMULATION **T3**
We have again not been able to discover the full effect of Q. However, one of the effects of the projection is to produce constraints (49) but there are others

$$\text{Hence } \mathbf{P}(\mathbf{C}) \subset \mathbf{P}(\mathbf{T3}) \subset \mathbf{P}(\mathbf{T2}) \tag{51}$$

Numerical examples demonstrate that the inclusion is strict.

4 Computational Results

In order to demonstrate the comparative sizes of different formulations and the relative strengths of their LP relaxations we give results below for a 10 city TSP.

These results were obtained using the NEWMAGIC modelling language and EMSOL optimiser.

Model	Size	LP Obj.	Iterations	Time (secs)	IP Obj.	Nodes	Time (secs)
C	502*90						
Conventional		766	37	1	766	0	1
	Ass.	804	40	1	804	0	1
	relaxation	835	43	1	835	0	1
	+ subtours (5)	878	48	1	881	9	1
	+ subtours (3)						
	+ subtours (2)						
S	92*99	773.6	77	3	881	665	16
Sequential							
F1	120*180	794.22	148	1	881	449	13
1 Commodity							
F1'	120*180	794.89	142	1	881	369	11
Modified							
F2	140*270	794.22	229	2	881	373	12
2 Commodity							
F3	857*900	878	1024	7	881	9	13
Multi Commodity							
T1	10*990	364.5	25	1	No solution after 12 hours		
1st Stage Dependent							
T2	120*990	799.46	246	18	881	2011	451
2nd Stage Dependent							
T3	193*990	804.5	307	5	881	145	27
3rd Stage Dependent							

5 Concluding Remarks

Eight formulations of the ATSP as an IP have been compared. Unlike other published work in this area the authors provide a unifying framework, in the form of projection, to conduct the comparison. Verification of the results are obtained through a numerical example.

The authors are now investigating, in the first instance, strategies for the manual introduction of the sub-tour elimination constraints with a view to developing a fully automated procedure. This work is being done using the NEWMAGIC modelling language.

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