Decomposition for network design: Exercises 1 — Bernard Gendron

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1 (50 points)

Consider the capacitated facility location problem with single-assignment (CFLPS): given a set of customers K and a set of facility locations J, each customer k having a demand $d_k > 0$ and each facility location j having a capacity $u_j > 0$, we want to satisfy the demand of each customer *from a single facility location*, while respecting the capacity of each facility location, in such a way that we minimize the total costs comprised of:

- $f_j \geq 0$, fixed cost of using facility location $j \in J$;
- $c_{jk} \geq 0$, cost of assigning facility location $j \in J$ to customer $k \in K$ to satisfy its demand.

1. Consider the following binary variables: $y_j = 1$, if a facility is used at location $j \in J$, 0, otherwise; $x_{jk} = 1$, if the demand of customer $k \in K$ is satisfied by a facility at location $j \in J$, 0, otherwise. By using only these variables, formulate this problem as an integer program.
2. Now, assume the demand of each customer can be satisfied by facilities at multiple locations. How would this assumption modify your model?
3. Assume that you know the subset of facility locations that are used in an optimal solution. Formulate the subproblem that would determine the facility location from which each customer's demand is satisfied, given that subset of facility locations. What is the structure of this subproblem?
4. If the demand of each customer can be satisfied by facilities at multiple locations, how would the subproblem be modified? What is the structure of the subproblem then?

2 (50 points)

Consider the uncapacitated network design problem defined over a directed network $G = (N, A)$ with a set of commodities K such that each commodity k has a demand $d^k > 0$ between one origin $O(k)$ and one destination $D(k)$. The problem is to satisfy the demands at minimum cost, given a fixed cost $f_{ij} \geq 0$ for using arc $(i, j) \in A$ and a cost $c_{ij} \geq 0$ per unit of flow on arc $(i, j) \in A$.

1. Consider the following variables: $y_{ij} = 1$, if arc (i, j) is used to carry some flow, 0, otherwise (design variables); x^k_{ij} is the fraction of the demand d^k that flows on arc (i, j). By using only these variables, formulate this problem as a mixed-integer program.
2. If we add the constraint that the demand for each commodity must be satisfied by a single path, would that change your model? Is your model still valid in this case?

3. Now, assume that you know $\mathcal{P}^k$, the set of *all circuit-free paths* connecting $O(k)$ to $D(k)$ for each commodity $k \in K$. Consider the following variables: v_p^k is the fraction of the demand d^k that flows on path $p \in \mathcal{P}^k$. By using only these variables, as well as the design variables, formulate the problem as a mixed-integer program.

4. Assume that $c_{ij} = 0$ for each $(i, j) \in A$. Using this hypothesis, reformulate the problem as an integer program that uses only the design variables.