Tests

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Introduction

- Impossible to determine the most appropriate model specification
- A good fit does not mean a good model
- Formal testing is necessary, but not sufficient
- No clear-cut rules can be given
- Subjective judgments of the analyst
- Good modeling = good judgment + good analysis

Introduction

- Informal tests
- Asymptotic t-test, Confidence interval
- Likelihood ratio tests
  - Test of generic attributes
  - Test of taste variations
  - Test of heteroscedasticity
- Goodness-of-fit measures
- Non nested hypotheses, Nonlinear specifications
- Prediction tests
  - Outlier analysis
  - Market segmentation tests

Informal tests

Sign of the coefficient

Example: Netherland Mode Choice Case

Name	Value	Std err	t-test	Robust Std err	Robust t-test
ASC_CAR	-0.80	0.27	-2.95	0.28	-2.90
BETA_COST	-0.05	0.01	-4.85	0.01	-4.67
BETA_TIME	-1.33	0.34	-3.86	0.35	-3.75

Informal tests

Value of trade-offs

- How much are we ready to pay for an improvement of the level-of-service?
- Example: reduction of travel time
- The increase in cost must be exactly compensated by the reduction of travel time

$$\beta_C(C+\Delta C)+\beta_T(T-\Delta T)+\ldots=\beta_C C+\beta_T T+\ldots$$

Therefore,

$$\frac{\Delta C}{\Delta T}=\frac{\beta_T}{\beta_C}$$

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Informal tests

Value of trade-offs

In general:

- Trade-off: $\frac{\partial V / \partial x}{\partial V / \partial x_C}$
- Units: $\frac{1/\text{Hour}}{1/\text{Guilder}} = \frac{\text{Guilder}}{\text{Hour}}$

Name	Value	Guilders	Euros	CHF	
ASC_CAR	-0.80	15.97	7.25	11.21	
BETA_COST	-0.05				
BETA_TIME	-1.33	26.55	12.05	18.64	(/Hour)

t-test

Is the estimated parameter $\hat{\theta}$ significantly different from a given value θ^*?

- $H_0 : \hat{\theta} = \theta^*$
- $H_1 : \hat{\theta} \neq \theta^*$

Under H_0, if $\hat{\theta}$ is normally distributed with known variance σ^2

$$\frac{\hat{\theta} - \theta^*}{\sigma} \sim N(0, 1).$$

Therefore

$$P(-1.96 \leq \frac{\hat{\theta} - \theta^*}{\sigma} \leq 1.96) = 0.95 = 1 - 0.05$$

t-test

$$P(-1.96 \leq \frac{\hat{\theta} - \theta^*}{\sigma} \leq 1.96) = 0.95 = 1 - 0.05$$

H_0 can be rejected at the 5% level if

$$\left| \frac{\hat{\theta} - \theta^*}{\sigma} \right| \geq 1.96.$$

- If $\hat{\theta}$ asymptotically normal
- If variance unknown
- A t test should be used with n degrees of freedom.
- When $n \geq 30$, the Student t distribution is well approximated by a $N(0, 1)$

Estimator of the asymptotic variance for ML

- Cramer-Rao Bound with the estimated parameters

$$\hat{V}_{CR} = -\nabla^2 \ln L(\hat{\theta})^{-1}$$

- Berndt, Hall, Hall & Haussman (BHHH) estimator

$$\hat{V}_{BHHH} = \left(\sum_{i=1}^{n} \hat{g}_i \hat{g}_i^T \right)^{-1}$$

where

$$\hat{g}_i = \frac{\partial \ln f_X(x_i;\theta)}{\partial \theta}$$

Estimator of the asymptotic variance for ML

Robust estimator:

$$\hat{V}_{CR}\hat{V}_{BHHH}^{-1}\hat{V}_{CR}$$

- The three are asymptotically equivalent
- This one is more robust when the model is misspecified
- Biogeme uses Cramer-Rao and the robust estimators

t-test

Example: Netherland Mode Choice

Name	Value	Std err	t-test	Robust Std err	Robust t-test
ASC_CAR	-0.80	0.27	-2.95	0.28	-2.90
BETA_COST	-0.05	0.01	-4.85	0.01	-4.67
BETA_TIME	-1.33	0.34	-3.86	0.35	-3.75

t-test

Warning with the ASCs (ex: residential telephone)

Name	Value	Robust t-test	Value	Robust t-test
ASC_1			-1.22	-1.52
ASC_2	0.75	4.82	-0.48	-0.58
ASC_3	0.90	1.33	-0.32	-1.48
ASC_4	0.66	0.66	-0.57	-0.81
ASC_5	1.23	1.52		
B1_FCOST	-1.71	-6.25	-1.71	-6.25
B2_MCOST	-2.17	-8.90	-2.17	-8.90

t-test

Comparing two coefficients:
$H_0 : \beta_1 = \beta_2$. The t statistic is given by

$$\frac{\beta_1 - \beta_2}{\sqrt{\operatorname{var}(\beta_1 - \beta_2)}}$$

$$\operatorname{var}(\beta_1 - \beta_2) = \operatorname{var}(\beta_1) + \operatorname{var}(\beta_2) - 2\operatorname{cov}(\beta_1, \beta_2)$$

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t-test

Coefficient1	Coefficient2	Rob. cov.	Rob. corr.	Rob. t-test
ASC_2	ASC_4	0.08	0.14	0.09
ASC_2	ASC_3	0.12	0.66	-0.22
ASC_3	ASC_4	0.03	0.21	0.36
ASC_1	ASC_4	0.08	0.14	-0.66
ASC_1	ASC_3	0.12	0.68	-1.33
B1_FCOST	B2_MCOST	0.02	0.36	1.56
ASC_1	ASC_2	0.65	0.98	-4.82

Confidence intervals

$$\Pr\left(-t_{\alpha/2} \leq \frac{\hat{\beta}_k - \beta_k}{\sqrt{\mathrm{var}(\hat{\beta}_k)}} \leq t_{\alpha/2}\right) = 1 - \alpha$$

or, equivalently,

$$\Pr\left(\hat{\beta}_k - t_{\alpha/2}\sqrt{\mathrm{var}(\hat{\beta}_k)} \leq \beta_k \leq \hat{\beta}_k + t_{\alpha/2}\sqrt{\mathrm{var}(\hat{\beta}_k)}\right) = 1 - \alpha$$

for 95%, $\alpha = 0.05$, and $t_{0.025} = 1.96$.

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Confidence intervals

When more than one parameter is considered, the quadratic form

$$(\hat{\beta} - \beta)^T \Sigma^{-1} (\hat{\beta} - \beta) \sim \chi^2_K$$

where

- $\beta \in \mathbb{R}^K$ is the vector of true parameters,
- $\hat{\beta} \in \mathbb{R}^K$ is the vector of estimates, and
- $\Sigma \in \mathbb{R}^{K \times K}$ is the covariance matrix.

$$\Pr\left((\hat{\beta} - \beta)^T \Sigma^{-1} (\hat{\beta} - \beta) \leq \chi^2_{K,\alpha} \right) = 1 - \alpha.$$

In two dimensions, the “confidence interval” is an ellipse.

Likelihood ratio test

- Used for “nested” hypotheses
- One model is a special case of the other
- H_0: the two models are equivalent

$$-2(\mathcal{L}(\hat{\beta}_R) - \mathcal{L}(\hat{\beta}_U)) \sim \chi^2_{(K_U - K_R)}$$

- $\mathcal{L}(\hat{\beta}_R)$ is the loglikelihood of the restricted model
- $\mathcal{L}(\hat{\beta}_U)$ is the loglikelihood of the unrestricted model

Likelihood ratio test

Example: Netherland Mode Choice Case. 3 models:

- Null model (equal probability): $K = 0$, $\mathcal{L} = -158.04$
- Constants only (reproduces the sample shares): $K = J - 1 = 1$, $\mathcal{L} = -148.35$
- Model with cost and time: $K = 3$, $\mathcal{L} = -123.13$

Likelihood ratio test

$-2(\mathcal{L}(\beta_R) - \mathcal{L}(\beta_U))$			Unrestricted model	
			1	3
			-148.35	-123.13
Restricted	0	-158.04	19.38	69.81
model	1	-148.35		50.43

χ^2	1	3
0	3.84	7.81
1		5.99

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Likelihood ratio test

Test of generic attributes (ex: residential telephone)

	ASC_BM	ASC_SM	ASC_LF	ASC_EF	BETA_C
BM	1	0	0	0	$\ln$(cost(BM))
SM	0	1	0	0	$\ln$(cost(SM))
LF	0	0	1	0	$\ln$(cost(LF))
EF	0	0	0	1	$\ln$(cost(EF))
MF	0	0	0	0	$\ln$(cost(MF))

	ASC_BM	ASC_SM	ASC_LF	ASC_EF	BETA_CM	BETA_CF
BM	1	0	0	0	$\ln$(cost(BM))	0
SM	0	1	0	0	$\ln$(cost(SM))	0
LF	0	0	1	0	0	$\ln$(cost(LF))
EF	0	0	0	1	0	$\ln$(cost(EF))
MF	0	0	0	0	0	$\ln$(cost(MF))

Likelihood ratio test

- Loglikelihood of the restricted model: -477.557
- Loglikelihood of the unrestricted model: -476.608
- Test: 1.898
- Threshold 95% χ^2_1: 3.841
- Cannot reject that the two models are equivalent
- The simplest model is preferred

Note about the t-test: If we test BETA_CM=BETA_CF, we obtain 1.56, which is below the 1.96 threshold

Likelihood ratio test

Test of taste variations (ex: residential telephone)

- Estimate a different model for each of the 5 income groups
- Pool the results together. $K = 6 \times 5 = 30$.
- Estimate a model for the whole sample. $K = 6$
- The test is performed with 24 degrees of freedom

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Likelihood ratio test

		data	loglike
Income group	1	115	-124.67
Income group	2	117	-120.86
Income group	3	104	-114.98
Income group	4	54	-59.23
Income group	5	44	-47.80
Pooled model		434	-467.55
Original model		434	-476.61
Test			18.11
Threshold	χ^2_{24}		36.42

Likelihood ratio test

- We cannot reject the hypothesis that the two models are equivalent
- There is no sign of segmentation per income
- The simplest model is preferred

Likelihood ratio test

Test of heteroscedasticity (ex: residential telephone)
Model 1:

$$
\begin{array}{rcccl}
V_{\text{BM}} & = & \beta_1 & + & \beta_5 \ln(\text{cost}_{\text{BM}}) \\
V_{\text{SM}} & = & \beta_2 & + & \beta_5 \ln(\text{cost}_{\text{SM}}) \\
V_{\text{LF}} & = & \beta_3 & + & \beta_6 \ln(\text{cost}_{\text{LF}}) \\
V_{\text{EF}} & = & \beta_4 & + & \beta_6 \ln(\text{cost}_{\text{EF}}) \\
V_{\text{MF}} & = & & & \beta_6 \ln(\text{cost}_{\text{MF}})
\end{array}
$$

Model 2: scale for perimeter area and non-metropolitan area

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Likelihood ratio test

$$
\begin{aligned}
\mathcal{L}(\text{model1}) &= \text{-476.608} \quad K = 6 \\
\mathcal{L}(\text{model2}) &= \text{-464.068} \quad K = 8 \\
\text{Test} &= 25.08 \\
\text{Threshold 95\%} &= 5.99
\end{aligned}
$$

- We reject the hypothesis that the models are equivalent
- Homoscedasticity across individuals is rejected

Non-nested hypotheses

- Need to compare two different models
- If none of the models is a restricted version of the other, we talk about non-nested models
- The likelihood ratio test cannot be used
- Two possible tests:
  - Composite model
  - Davidson-MacKinnon J-test

Composite model

- We want to test model 1 against model 2
- We generate a composite model C such that both models 1 and 2 are restricted cases of model C.
- We test 1 against C using the likelihood ratio test
- We test 2 against C using the likelihood ratio test
- Possible outcomes:
  - Only one of the two models is rejected. Keep the other.
  - Both models are rejected. Better models should be developed.
  - Both models are accepted. Use $\bar{\rho}^2$ to choose.

Goodness-of-fit

$$\rho^2 = 1 - \frac{\mathcal{L}(\hat{\beta})}{\mathcal{L}(0)}$$

- $\rho^2 = 0$: trivial model, equal probabilities
- $\rho^2 = 1$: perfect fit.

Warning: $\mathcal{L}(\hat{\beta})$ is a biased estimator of the expectation over all samples. Use $\mathcal{L}(\hat{\beta}) - K$ instead.

$$\bar{\rho}^2 = 1 - \frac{\mathcal{L}(\hat{\beta}) - K}{\mathcal{L}(0)}$$

Composite model

Example: residential telephone

	ASC_BM	ASC_SM	ASC_LF	ASC_EF	BETA_C
BM	1	0	0	0	$\ln$(cost(BM))
SM	0	1	0	0	$\ln$(cost(SM))
LF	0	0	1	0	$\ln$(cost(LF))
EF	0	0	0	1	$\ln$(cost(EF))
MF	0	0	0	0	$\ln$(cost(MF))

	ASC_BM	ASC_SM	ASC_LF	ASC_EF	BETA_C
BM	1	0	0	0	cost(BM)
SM	0	1	0	0	cost(SM)
LF	0	0	1	0	cost(LF)
EF	0	0	0	1	cost(EF)
MF	0	0	0	0	cost(MF)

Composite model

Composite model

	ASC_BM	ASC_SM	ASC_LF	ASC_EF	BETA_CL	BETA_C
BM	1	0	0	0	$\ln$(cost(BM))	cost(BM)
SM	0	1	0	0	$\ln$(cost(SM))	cost(SM)
LF	0	0	1	0	$\ln$(cost(LF))	cost(LF)
EF	0	0	0	1	$\ln$(cost(EF))	cost(EF)
MF	0	0	0	0	$\ln$(cost(MF))	cost(MF)

Model	$\mathcal{L}$	K	test	conclusion
Composite	-476.80	6		
log	-477.56	5	1.51	No reject
linear	-482.72	5	11.84	Reject

Model with log is preferred

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Davidson-MacKinnon J-test

$$
\begin{aligned}
M_0: \quad U &= f(X,\beta) + \varepsilon_0 \\
M_1: \quad U &= g(Z,\gamma) + \varepsilon_1
\end{aligned}
$$

- Estimate M_1 to obtain $\hat{\gamma}$
- Consider the model obtained by convex combination

$$
U = (1-\alpha)f(X,\beta) + \alpha g(Z,\hat{\gamma}) + \varepsilon_0
$$

- Note that α and β are estimated, not γ
- If M_0 is true, the true value of α is zero
- Perform a t-test to test α against 0.

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Davidson-MacKinnon J-test

Example: residential telephone

- M_0 model with log(cost)
- M_1 model with cost

Estimate M_1

Name	Value	Robust Std err	Robust t-test
ASC_1	-0.53	0.15	-3.61
ASC_3	0.89	0.15	5.87
ASC_4	0.76	0.71	1.07
ASC_5	1.83	0.39	4.67
B1_COST	-0.15	0.02	-6.28

Davidson-MacKinnon J-test

```
[Expressions]
ASCLIN1 = -5.2704884e-01
ASCLIN3 = +8.9308708e-01
ASCLIN4 = +7.5874800e-01
ASCLIN5 = +1.8310079e+00
BETALIN = -1.4908464e-01
UTILLIN1 = ASCLIN1 + BETALIN * cost1
UTILLIN2 =           BETALIN * cost2
UTILLIN3 = ASCLIN3 + BETALIN * cost3
UTILLIN4 = ASCLIN4 + BETALIN * cost4
UTILLIN5 = ASCLIN5 + BETALIN * cost5
[Utilities]
1       BM      avail1  ALPHA * UTILLIN1
2       SM      avail2  ALPHA * UTILLIN2
3       LF      avail3  ALPHA * UTILLIN3
4       EF      avail4  ALPHA * UTILLIN4
5       MF      avail5  ALPHA * UTILLIN5
```

Davidson-MacKinnon J-test

```
[GeneralizedUtilities]
1 (1 - ALPHA ) * ( ASC_1 + B1_COST * logcost1 )
2 (1 - ALPHA ) * ( ASC_2 + B1_COST * logcost2 )
3 (1 - ALPHA ) * ( ASC_3 + B1_COST * logcost3 )
4 (1 - ALPHA ) * ( ASC_4 + B1_COST * logcost4 )
5 (1 - ALPHA ) * ( ASC_5 + B1_COST * logcost5 )
```

Name	Value	Robust Std err	Robust t-test
ALPHA	0.23	0.21	1.10
ASC_1	-0.72	0.19	-3.70
ASC_3	1.22	0.22	5.67
ASC_4	1.05	0.93	1.12
ASC_5	1.77	0.38	4.68
B1_COST	-2.07	0.31	-6.73

Davidson-MacKinnon J-test

Conclusion:

- Cannot reject the hypothesis that $ALPHA = 0$.
- Cannot reject the hypothesis that the log specification is correct

Davidson-MacKinnon J-test

- M_0 model with cost
- M_1 model with log(cost)

Estimate M_1

Name	Value	Robust Std err	Robust t-test
ASC_1	-0.72	0.15	-4.76
ASC_3	1.20	0.16	7.56
ASC_4	1.00	0.70	1.42
ASC_5	1.74	0.27	6.51
B1_COST	-2.03	0.21	-9.55

Davidson-MacKinnon J-test

```
[Expressions]
ASCLOG1 = -7.2124491e-01
ASCLOG3 = +1.2012643e+00
ASCLOG4 = +9.9917468e-01
ASCLOG5 = +1.7364214e+00
COSTLOG = -2.0261980e+00
UTILLOG1 = ASCLOG1 + COSTLOG * logcost1
UTILLOG2 =           COSTLOG * logcost2
UTILLOG3 = ASCLOG3 + COSTLOG * logcost3
UTILLOG4 = ASCLOG4 + COSTLOG * logcost4
UTILLOG5 = ASCLOG5 + COSTLOG * logcost5
[Utilities]
1       BM      avail1  ALPHA * UTILLOG1
2       SM      avail2  ALPHA * UTILLOG2
3       LF      avail3  ALPHA * UTILLOG3
4       EF      avail4  ALPHA * UTILLOG4
5       MF      avail5  ALPHA * UTILLOG5
```

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Davidson-MacKinnon J-test

```
[GeneralizedUtilities]
1 (1 - ALPHA ) * ( ASC_1 + B1_COST * cost1 )
2 (1 - ALPHA ) * ( ASC_2 + B1_COST * cost2 )
3 (1 - ALPHA ) * ( ASC_3 + B1_COST * cost3 )
4 (1 - ALPHA ) * ( ASC_4 + B1_COST * cost4 )
5 (1 - ALPHA ) * ( ASC_5 + B1_COST * cost5 )
```

Name	Value	Robust Std err	Robust t-test
ALPHA	0.79	0.21	3.70
ASC_1	-0.51	0.69	-0.73
ASC_3	0.95	0.69	1.38
ASC_4	0.91	3.37	0.27
ASC_5	1.96	1.44	1.36
B1_COST	-0.16	0.09	-1.88

Davidson-MacKinnon J-test

Conclusions:

- Reject the hypothesis that ALPHA=0
- Reject the hypothesis that the linear specification is correct

Non linear specification

Three approaches

- Piecewise linear specifications
- Power series expansion
- Box-Cox transforms

Piecewise linear specification

- A coefficient may have different values
- For example

$$V_i = \beta_{T1} x_{T1} + \beta_{T2} x_{T2} + \beta_{T3} x_{T3} + \beta_{T4} x_{T4} + \ldots$$

where

$$x_{T1} = \begin{cases} t & \text{if } t < 90 \\ 90 & \text{otherwise} \end{cases} \qquad x_{T2} = \begin{cases} 0 & \text{if } t < 90 \\ t - 90 & \text{if } 90 \leq t < 180 \\ 90 & \text{otherwise} \end{cases}$$

$$x_{T3} = \begin{cases} 0 & \text{if } t < 180 \\ t - 180 & \text{if } 180 \leq t < 270 \\ 90 & \text{otherwise} \end{cases} \qquad x_{T4} = \begin{cases} 0 & \text{if } t < 270 \\ t - 270 & \text{otherwise} \end{cases}$$

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Piecewise linear specification

Note: coding in Biogeme

$$x_{Ti} = \begin{cases} 0 & \text{if } t < a \\ t-a & \text{if } a \leq t < a+b \\ b & \text{otherwise} \end{cases} \qquad x_{Ti} = \max(0, \min(t-a, b))$$

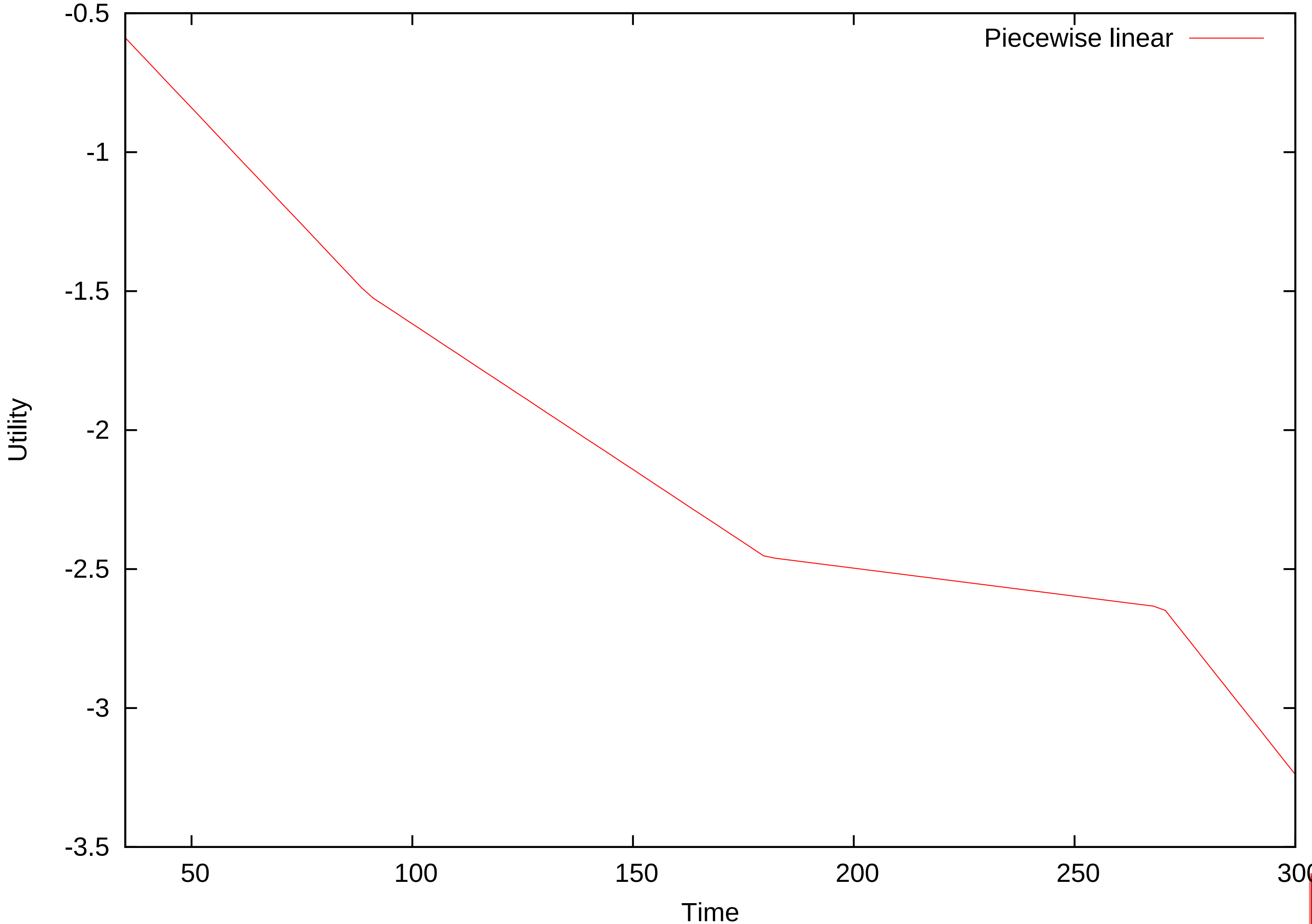
$$\begin{aligned} x_{T1} &= \min(t, 90) \\ x_{T2} &= \max(0, \min(t-90, 90)) \\ x_{T3} &= \max(0, \min(t-180, 90)) \\ x_{T4} &= \max(0, t-270) \end{aligned}$$

Piecewise linear specification

Examples:

t	TT1	TT2	TT3	TT4
40	40	0	0	0
100	90	10	0	0
200	90	90	20	0
300	90	90	90	30

Piecewise linear specification



Piecewise linear specification

- Perform a likelihood ratio test
- Example: Swissmetro
- Linear model: $\mathcal{L} =$-5031.87 ($K = 12$)
- Piecewise linear model: $\mathcal{L} =$-5025 ($K = 15$)
- Test = -2(-5031.87 + 5025) = 13.74
- Threshold 95% χ^2_3= 7.81
- Reject the linear model

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Power series

$$V_i = \beta_1 T + \beta_2 T^2 + \beta_3 T^3 + \ldots$$

- In practice, these terms can be very correlated
- Example: Swissmetro with 2 terms
  - Linear model: $\mathcal{L} =$-5031.87 ($K = 12$)
  - Power series model: $\mathcal{L} =$ -5031.36 ($K = 13$)
  - Test = -2(-5031.87 +5031.36) = 1.02
  - Threshold 95% χ^2_1= 3.84
  - Cannot reject the linear model

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Power series

- Example: Swissmetro with 3 terms
  - Linear model: $\mathcal{L} =$-5031.87 ($K = 12$)
  - Power series model: $\mathcal{L} =$ -5023.79 ($K = 14$)
  - Test = -2(-5031.87 +5023.79) = 16.16
  - Threshold 95% χ^2_2= 5.99
  - Reject the linear model

Box-Cox transforms

- Box-Cox transforms

$$\beta\frac{x^\lambda - 1}{\lambda}, \; x > 0$$

- Box-Tukey transforms

$$\beta\frac{(x+\alpha)^\lambda - 1}{\lambda}, \; x+\alpha > 0$$

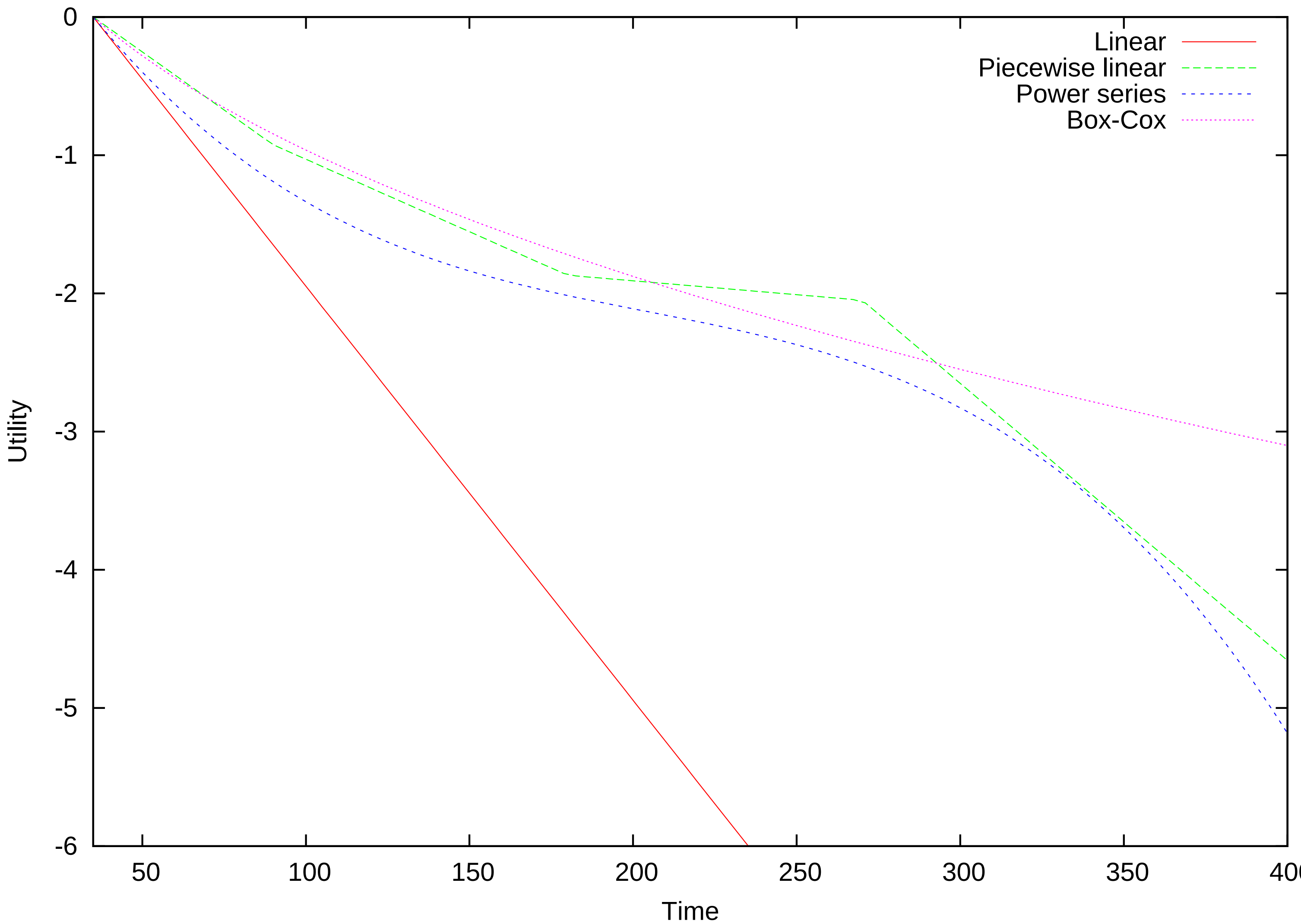
where β, α and λ must be estimated

Box-Cox transforms

Example: Swissmetro

- Linear model: $\mathcal{L}=$-5031.87 ($K=12$)
- Box-Cox model: $\mathcal{L}=$ -5029.83 ($K=13$)
- Test = -2(-5031.87+5029.83) = 4.08
- Threshold 95% χ^2_3= 3.84
- Reject the linear model

Comparison



Linear
Piecewise linear
Power series
Box-Cox
Utility
Time
0
-1
-2
-3
-4
-5
-6
50
100
150
200
250
300
350
400

Outlier analysis

- Apply the model on the sample
- Examine observations where the predicted probability is the smallest for the observed choice
- Test model sensitivity to outliers, as a small probability has a significant impact on the loglikelihood
- Potential causes of low probability:
  - Coding or measurement error in the data
  - Model misspecification
  - Unexplainable variation in choice behavior

Outlier analysis

- Coding or measurement error in the data
  - Look for signs of data errors
  - Correct or remove the observation
- Model misspecification
  - Seek clues of missing variables from the observation
  - Keep the observation and improve the model
- Unexplainable variation in choice behavior
  - Keep the observation
  - Avoid overfitting of the model to the data

Outlier analysis

Example: residential telephone

	ASC_BM	ASC_SM	ASC_LF	ASC_EF	BETA_C
BM	1	0	0	0	$\ln$(cost(BM))
SM	0	1	0	0	$\ln$(cost(SM))
LF	0	0	1	0	$\ln$(cost(LF))
EF	0	0	0	1	$\ln$(cost(EF))
MF	0	0	0	0	$\ln$(cost(MF))

Outlier analysis

- Observation with lowest probability of choice = 3.83%
- Choice: Metro Area Flat
- Costs: BM (5.39), SM (5.78), LF (8.48), EF (n.a.), MF (38.28)
- Area of residence: perimeter (without extended)
- Number of users in the household: 2 (20-29 years)
- Income: 30K–40K
- Conclusion: the model can be improved

Market segments

- Compared predicted vs. observed shares per segment
- Let N_j be the set of samples individuals in segment j
- Observed share for alt. i and segment j

$$S(i,j) = \sum_{n \in N_j} y_{in}/N$$

- Predicted share for alt. i and segment j

$$\hat{S}(i,j) = \sum_{n \in N_j} P_n(i)/N$$

Market segments

Example: residential telephone

	ASC_BM	ASC_SM	ASC_LF	ASC_EF	BETA_C
BM	1	0	0	0	$\ln$(cost(BM))
SM	0	1	0	0	$\ln$(cost(SM))
LF	0	0	1	0	$\ln$(cost(LF))
EF	0	0	0	1	$\ln$(cost(EF))
MF	0	0	0	0	$\ln$(cost(MF))

- Two segments: up to 2 users, more than 2 users

Market segments

	Predicted				Observed		
	<=2	> 2	Total		<=2	> 2	Total
1	57	16	73	1	61	12	73
2	92	31	123	2	102	21	123
3	120	58	178	3	108	70	178
4	2	1	3	4	3	0	3
5	33	24	57	5	29	28	57
	303	131	434		303	131	434

Market segments

Error	$<=2$	>2
1	-7.0%	35.8%
2	-10.2%	49.5%
3	11.2%	-17.3%
4	-37.6%	∞
5	12.9%	-13.4%

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Market segments

Note:

- With a full set of constants: $\sum_{n \in N_j} y_{in} = \sum_{n \in N_j} P_n(i)$
- Do not saturate the model with constants