A short introduction to discrete choice models

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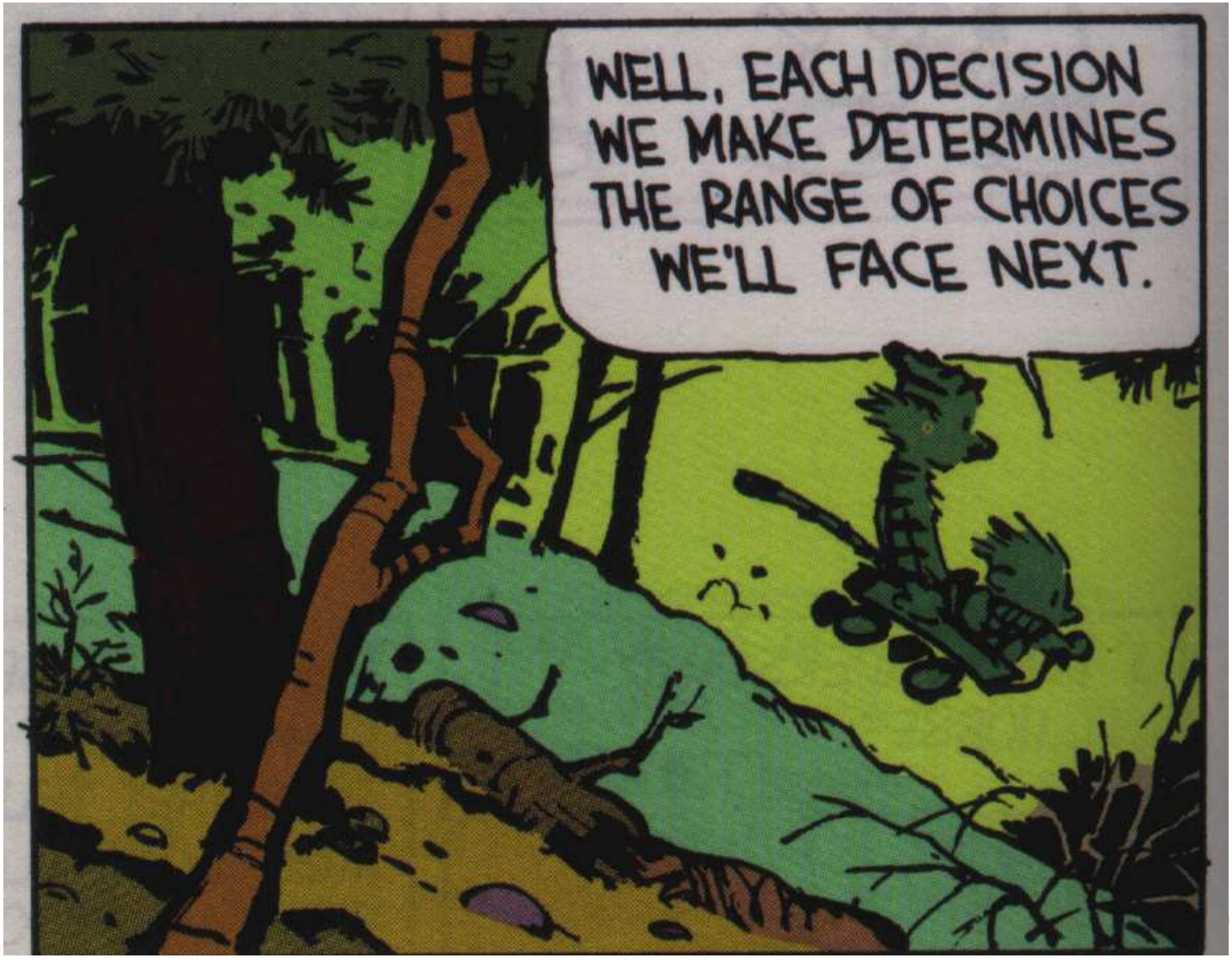
Objectives

- Mathematical modeling of behavior
- Individual behavior (vs. aggregate behavior)
- Theory of behavior which is
  - descriptive: how people behave and not how they should
  - abstract: not too specific
  - operational: can be used in practice for forecasting
- Type of behavior: choice

Motivations



Motivations

"It is our choices that show what we truly are, far more than our abilities" Albus Dumbledore
"Liberty, taking the word in its concrete sense, consists in the ability to choose." Simone Weil

Field :

- Marketing
- Transportation
- Politics
- Management
- New technologies

Type of behavior:

- Choice of a brand
- Choice of a transportation mode
- Choice of a president
- Choice of a management policy
- Choice of investments

Importance

Daniel L. McFadden

1937–

- UC Berkeley 1963, MIT 1977, UC Berkeley 1991
- Laureate of *The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel 2000*
- Owns a farm and vineyard in Napa Valley
- “Farm work clears the mind, and the vineyard is a great place to prove theorems”

Example

Voice over internet protocol (VoIP)

- What is the market penetration?
- How will the penetration change in the future?
- Assumption: level of education is an important explanatory factor

Data collection

- sample of 600 persons, randomly selected
- Two questions:
  1. Do you subscribe to voice over IP? (yes/no)
  2. How many years of education have you had? (low/medium/high)

Example

- Contingency table

	Education			
VoIP	Low	Medium	High	
Yes	10	100	120	230
No	140	200	30	370
	150	300	150	600

- Penetration in the sample: $230/600 = 38.3\%$
- Forecasting: need for a model

Example: a model

- Dependent variable:

$$y = \begin{cases} 1 & \text{if subscriber} \\ 2 & \text{if not subscriber} \end{cases}$$

Discrete dependent variable

- Independent or explanatory variable

$$x = \begin{cases} 1 & \text{if level of education is low} \\ 2 & \text{if level of education is medium} \\ 3 & \text{if level of education is high} \end{cases}$$

Example: a model

- Market penetration in the sample: $\hat{p}(y = 1)$
- Market penetration in the population: $p(y = 1)$ estimated by $\hat{p}(y = 1)$
- Joint probabilities: $\hat{p}(y = 1, x = 2) = 100/600 = 0.1667$
- Marginal probabilities:$\hat{p}(y = 1) = \sum_{k=1}^{3} \hat{p}(1, k) = 10/600 + 100/600 + 120/600 = 0.383$
- Conditional probabilities: $\hat{p}(y = 1|x = 2)$

$$\begin{aligned} \hat{p}(y = 1, x = 2) &= \hat{p}(y = 1|x = 2)\hat{p}(x = 2) \\ \hat{p}(y = 1|x = 2) &= \hat{p}(y = 1, x = 2)/\hat{p}(x = 2) \\ &= 0.1667/0.5 = 0.333 \end{aligned}$$

Example: a model

Similarly, we obtain

$$\begin{aligned}\hat{p}(y=1|x=1) &= 0.067\\ \hat{p}(y=1|x=2) &= 0.333\\ \hat{p}(y=1|x=3) &= 0.8\end{aligned}$$

We obtain a causal relationship.

- Behavioral model: $\hat{p}(y=i|x=j)$
- Forecasting assumption: stable over time

Example: forecasting

- Model:

$$\begin{aligned} p(y=1|x=1) &= \pi_1 &= 0.067 \\ p(y=1|x=2) &= \pi_2 &= 0.333 \\ p(y=1|x=3) &= \pi_3 &= 0.8 \end{aligned}$$

  where π_1, π_2, π_3 are estimated parameters

- Assumption: future level of education: 10%-60%-30%

$$\begin{aligned} p(y=1) &= \sum_{i=1}^{3} p(y=1|x=i)p(x=i) \\ &= 0.1\pi_1 + 0.6\pi_2 + 0.3\pi_3 \\ &= 44.67\% \end{aligned}$$

Example: forecasting

- If the level of education increases
- from 25%-50%-25% to 10%-60%-30%
- Market penetration of VoIP will increase
- from 38.33 % to 44.67%

In summary

- $p(x = j)$ can be easily obtained and forecasted
- $p(y = i|x)$ is the behavioral model to be developed

Outline

- Introduction
- Review of relevant concepts in probability and statistics
- Choice theory
- Binary choice
- Multinomial choice
- Tests
- Data
- Nested Logit model

Bibliography

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